Statistical Physics

1. Fundamentals of quantum statistical physics
   - Canonical ensembles: partition function, canonical distribution, density operator and free energy.
   - Density matrices and density operators. Expectation values, pure and mixed states.
   - Neumann equation and the equilibrium density operator. Discussion of independent spins in an external field.
   - Neumann entropy, mutual entropy, entanglement. Maximum entropy principle.
2. Quantum gases and fluids
   - Ideal Fermi and Bose systems: energy, specific heat of various Bose and Fermi systems, Bose-Einstein condensation.
   - Superfluidity: experiments, two fluid model, excitations, superfluid wave function, vortices.
3. Phase transitions
   - Classification of phase transitions, order parameter.
   - Mean field theory, Ising model / Landau theories.
   - Characterisation of second-order phase transitions, critical exponents. Spatial inhomogeneities, correlation length, structure of critical correlations.
   - Scaling, generalised homogeneous functions, scaling laws, hyperscaling, universality. Self-similarity, Kadanoff block transformation.
4. Linear response theory
   - Static linear response of classical systems and correlations. Elastic scattering experiments and the structure factor.
   - Time dependent response function, dissipative and elastic response.
   - Kubo formula, dissipation, causality, Kramers-Kronig relation.
5. Fluctuations and noise
   - Time dependent equilibrium correlation functions, classical and quantum noise, Wiener-Khinchin theorem.
   - Fluctuation-dissipation theorem and its classical limit.
   - Onsager's regression hypothesis, classical time dependent response and noise. Johnson-Nyquist noise of electric circuits.
   - Time reversal invariance and Onsager's relations.
6. Stochastic processes
   - Brownian motion, diffusion equation and coefficient. Langevin equation, fluctuating force, white noise.
   - Second fluctuation-dissipation theorem. Einstein relation between mobility and diffusion coefficient.
   - Markov processes, Chapman-Kolmogorov equation, master equation, Fokker-Planck equation. Diffusion and drift.
7. Direction of macroscopic processes
   - Direction of processes: ergodicity, time and ensemble averages. Paradoxes of irreversibility and their resolution.
   - Equilibrium distributions as stable limiting distributions in ergodic and microscopically reversible Markov dynamics.
   - Boltzmann's H theorem, relation to entropy, entropy generation. Maximum entropy principle.
8. Simulation methods
   - Monte Carlo method. Generation of random numbers, importance sampling. Metropolis algorithm. Boundary conditions, ensembles, averages, characteristic time scales.
   - Critical dynamics. Finite size scaling, critical slowing down. Speed-up techniques.
   - Molecular dynamics. Interactions, solution methods. Event directed MD, instabilities.
9. Elective topics: can be chosen from
   a) Complex networks
      - Generative network models, simple graph measures on them: Erdős-Rényi, Watts-Strogratz, configuration model, preferential attachment, clustering coefficient, mean path length, assortativity. Percolation on Erdös-Rényi graphs.
      - Diffusion, spreading and cascades on networks.

- Temporal graphs, motifs, burstiness, spreading on temporal graphs.
- Centrality and node similarity measures, sampling of networks.
- Stochastic block model, inference, modularity community structure.
- Mesoscopic measures, hierarchy, core-periphery.

b) Scaling and Critical Phenomena

- Self-similarity, Kadanoff block transformation. Fundamentals of renormalisation group transformations, universality. Ginzburg-criterion.
- An example of real space renormalisation group, Wilson's RG (relevant and irrelevant operators), and critical exponents, critical correlations.
- Finite size scaling, universal cross-overs.
- Quantum criticality, quantum-classical mapping.
- Computing the critical exponents: □-expansion OR high temperature expansion.
- Kosterlitz-Thouless phase transitions.

c) Random matrices

- Dyson ensemble, level spacing statistics, pair correlation function.
- Thermodynamical model of levels, level transition dynamics.
- Universality. Classically integrable/chaotic systems at the quantum level. Decoherence.
- Quasi-one-dimensional mesoscopic systems. Universal conductance fluctuations. Electrons in quantum dots.
- Chiral and hybrid (metal-superconductor) systems.

Recommended courses

BMETE15AF29 Statistical Physics 1
BMETE15MF44 Statistical Physics 2
BMETE15MF45 Computer Simulations in Physics

For the elective topics:

a) BMETE15MF38 Complex Networks
b) BMETE15MF48 Phase Transitions and Criticality
c) BMETE15MF10 Random Matrix Theory with Physical Applications

Recommended Literature

Basic statistical physics, in English:

- David Wu and David Chandler: Introduction to Modern Statistical Mechanics (Oxford University Press, 1988)
- L.E. Reichl: A Modern Course in Statistical Physics (Wiley-VCH Verlag, 2009)
- M. Toda et al.: Statistical Physics I-II (Springer Series in Solid-State Sciences, 1992)
- K. Huang: Introduction to Statistical Physics (Taylor and Francis)

Simulation methods:

- K. Binder and D.Heerman: Monte Carlo simulation in Statistical Physics (good intro)
- J. Kertész and I. Kondor (eds.): Advances is computer simulation (Springer Lecture Notes) (a good intro by W. Kraut and some good chapters on advances)
- D. Rapaport: The art of molecular dynamics simulation (Cambridge UP) (a number of case studies)

In Hungarian:

- R. Kubo: Statisztikus mechanika (Tankönyvkiadó)
- Kertész János, Zaránd Gergely, Deák András: Statisztikus Fizika jegyzet
- Török János, Kertész János: Statisztikus fizika 1 (Kivonat a Török János, Orosz László, Kertész János: Elméleti Fizika 2 jegyzetből)

A jegyzetek elérhetők a BME TTK Tankönyvtárban (http://tankonyvtar.ttk.bme.hu).

For the elective topics:

a) John Cardy: Scaling and Renormalization in Statistical Physics (Cambridge University Press, 1997).
N. Goldenfeld: Lectures on phase transitions and the renormalization group (Addison-Wesley, 1992).
b) A.L. Barabási and M. Pósfai: Network Science (Cambridge University Press, 2016).
c) Madan Lal Mehta: Random Matrices (Academic Press, 2004).