Complex networks

Random (Erdős-Rényi) networks, degree distribution, clustering, Watts-Strogratz network

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Graph measaures: Clustering coefficient

- **Clustering coefficient:** fraction of triangles realized out all possible ones at node i

$$C_i = \frac{2n_{\Delta,i}}{k_i(k_i - 1)},$$

where $n_{\Delta,i}$ is the number of triangles at node i.

- Average clustering coefficient:

$$\langle C \rangle = \frac{1}{N} \sum_{i=1}^{N} C_i$$

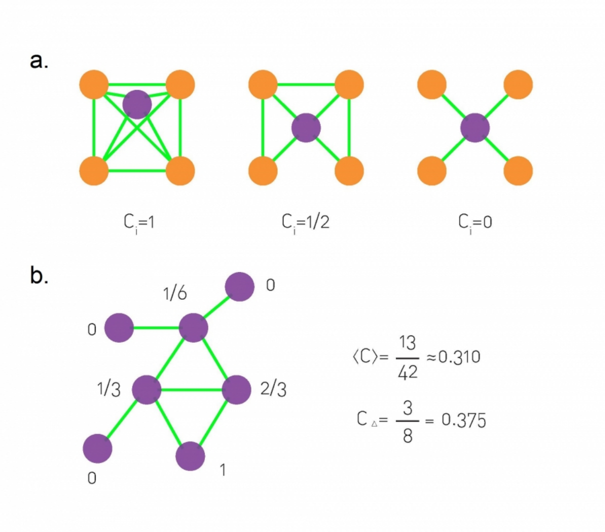
- Global clustering coefficient: fraction of triangles realized out all possible ones.

$$C = \frac{|\{(i,j,k) \text{ circle}, \ i \neq j \neq k\}|}{|\{(i,j,k) \text{ path}, \ i \neq j \neq k\}|}$$

$$C = \frac{3 \times \text{Numberoftriangles}}{\text{Numberofconnectedtriples}}$$

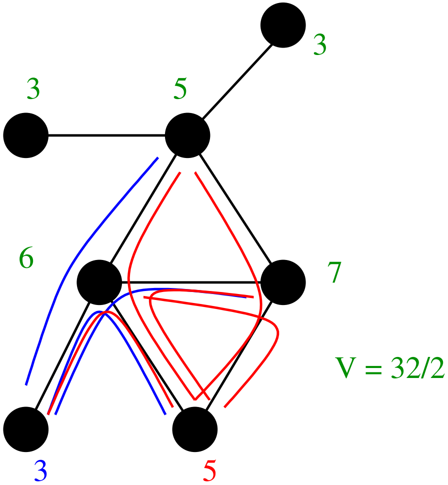
Graph measaures: Clustering coefficient

- Example from Barabasi's http://networksciencebook.com/



The latter is $C_\Delta = 6/16$

Graph measaures: Clustering coefficient



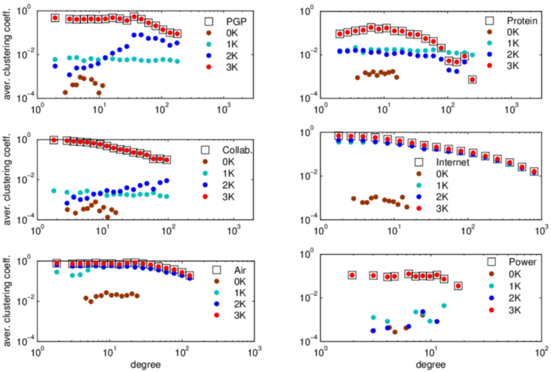
The latter is $C_\Delta = 6/16$

Graph measures: Conditional probability

- **Conditional probability:** $P(x|c)$ normalized distribution of x for cases when condition c holds.
- Example: Clustering coefficient of nodes of degree k:

$$\langle C_k \rangle = \frac{1}{n_k} \sum_{i|k_i=k} C_i(k) = \sum_C CP(C|k)$$

https://arxiv.org/pdf/0908.1143



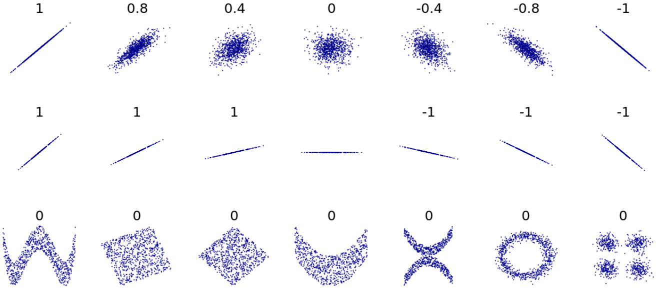
Graph measaures: Pearson correlation coefficient

- **Pearson correlation coefficient:**

$$r_{X,Y} = \frac{E[(X - \mu_X)(Y - \mu_Y)]}{\sigma_X \sigma_Y}$$

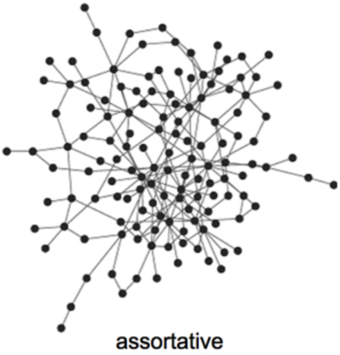
μ_Z mean of Z

σ_Z standard deviation of Z



Graph measures: Assortativity

- **Assortativity:** $\langle k_{nn}(k) \rangle$ average degree of the neighbors of nodes with degree k
- $\langle k_{nn}(k) \rangle$ increasing $\rightarrow$ assortative mixing
- $\langle k_{nn}(k) \rangle$ decreasing $\rightarrow$ disassortative mixing
- **Assortativity coefficient:** Pearson correlation coefficient of degree between pairs of linked nodes, with $r > 0$ for assortative and $r < 0$ for disassotative mixing.



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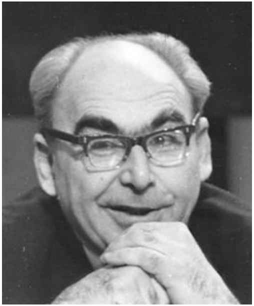
Network	n	r
Physics coauthorship (a)	52 909	0.363
Biology coauthorship (a)	1 520 251	0.127
Mathematics coauthorship (b)	253 339	0.120
Film actor collaborations (c)	449 913	0.208
Company directors (d)	7 673	0.276
Internet (e)	10 697	−0.189
World-Wide Web (f)	269 504	−0.065
Protein interactions (g)	2 115	−0.156
Neural network (h)	307	−0.163
Marine food web (i)	134	−0.247
Freshwater food web (j)	92	−0.276
Random graph (u)		0
Callaway *et al.* (v)		$\delta/(1+2\delta)$
Barabási and Albert (w)		0

Erdős-Rényi model

- Creating networks from random model
- Most basic construction: Take N nodes and L links and place the links randomly
- This is the **Erdős-Rényi model** of random networks (1960)

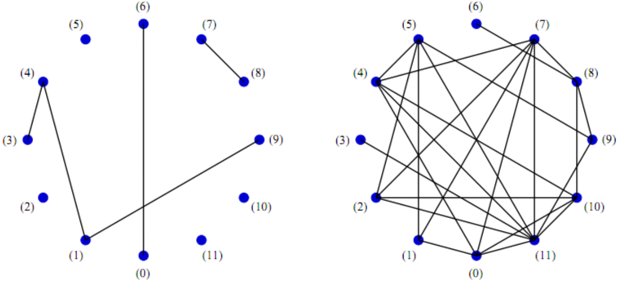
Erdős-Rényi model, versions

- N nodes and L links placed randomly: (N, L)
- N nodes and links with probability p: (N, p)
  - Number of links in a complete graph:

$$L_c = \binom{N}{2} = \frac{N(N-1)}{2}$$

  - Relation between p and L:

$$p = \frac{2L}{N(N-1)}$$

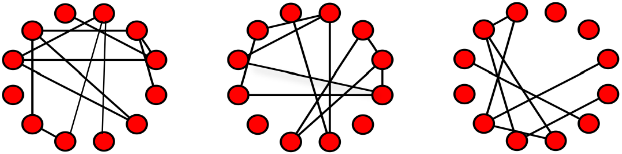


 $N = 12$ $p = 0.3788$ $p = 0.758$ http://tetelwiki.mafihe.hu/

Erdős-Rényi: equivalence of different versions

- Equivalence of the two definition is only in ensemble average
- $N = 12$, $p = 1/6$,

$$\langle L \rangle = \frac{12 \cdot 11}{2 \cdot 6} = 11$$



$L = 12$ $L = 11$ $L = 8$

Erdős-Rényi: equivalence of different versions

- How precisely can we get networks with L links?
- Number of different ways L links can be placed ($L_c = N(N-1)/2$ is number of links in the complete graph):

$$P(L, L_c) = \binom{L_c}{L}$$

- The probability of finding a graph with exactly L links using probability p:

$$P(N, p; L) = \binom{L_c}{L} p^L (1-p)^{L_c - L}$$

- Binomial distribution

Erdős-Rényi: equivalence of different versions

- The probability of finding a graph with exactly L links using probability p:

$$P(N, p; L) = \binom{L_c}{L} p^L (1-p)^{L_c - L}$$

- Average number of links in a graph of (N, p)

$$\langle L \rangle = \sum_{L=\not{0}1}^{L_c} L \binom{L_c}{L} p^L (1-p)^{L_c - L} = \sum_{L=1}^{L_c} L \frac{L_c!}{L!(L_c - L)!} p^L (1-p)^{L_c - L} =$$

$$= pL_c \sum_{L=1}^{L_c} \frac{(L_c - 1)!}{(L-1)!(L_c - L)!} p^{L-1} (1-p)^{L_c - L} = pL_c \cdot 1$$

Erdős-Rényi: equivalence of different versions

- Average number of links in a graph of (N, p): $\langle L \rangle = pL_c$
- Similar derivation (in the second round two terms: $L = (L-1) + 1$):

$$\langle L^2 \rangle = p^2 L_c^2 + p(1-p)L_c$$

- Variance:

$$\sigma^2 = \langle L^2 \rangle - \langle L \rangle^2 = L_c p(1-p)$$

- Relative variance:

$$\frac{\sigma}{\langle L \rangle} = \sqrt{\frac{L_c p(1-p)}{p^2 L_c^2}} = \sqrt{\frac{(1-p)}{pL_c}}$$

- Very sharp for large graphs

$$\frac{\sigma}{\langle L \rangle} = \sqrt{\frac{(1-p)}{pL_c}} \simeq \sqrt{\frac{2(1-p)}{p}\frac{1}{N}}$$

Erdős-Rényi: degree distribution

- Probability of a node to have k links:

$$P(k) = \binom{N-1}{k} p^k (1-p)^{N-1-k}$$

- Average degree:

$$\langle k \rangle = p(N-1)$$

- Variance of node degree

$$\sigma_k^2 = p(1-p)(N-1) = \langle k \rangle (1-p)$$

- Relative variance

$$\frac{\sigma_k}{\langle k \rangle} = \sqrt{\frac{(1-p)}{p(N-1)}} = \mathcal{O}(N^{-1/2})$$

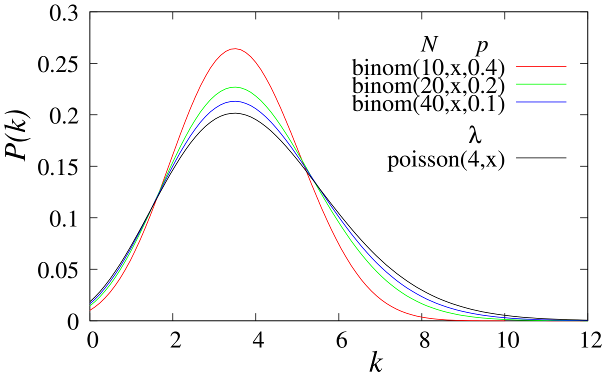
- Narrow distribution

Erdős-Rényi: degree distribution

- Poisson limit theorem, $\lambda \equiv pN$

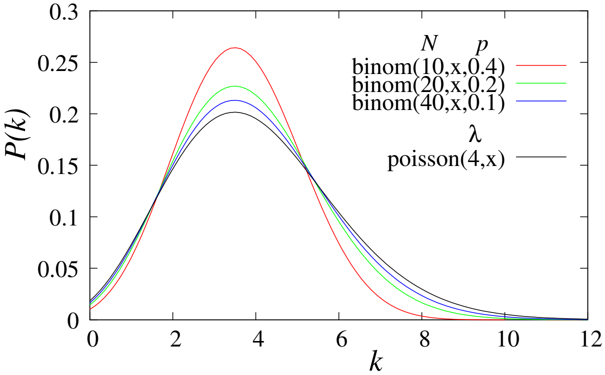
$$\lim_{N\to\infty} \binom{N}{k} p^k (1-p)^{N-k} = e^{-\lambda}\frac{\lambda^k}{k!}$$

- Poisson distribution: mean: λ, variance: λ



Erdős-Rényi: degree distribution

- Poisson distribution: mean: $\lambda \simeq \langle k \rangle$, variance: $\lambda \simeq \langle k \rangle$
- In real life average degree seems to be the quantity that characterizes best the network, so one can define an Erdős-Rényi as $(N, \langle k \rangle)$.



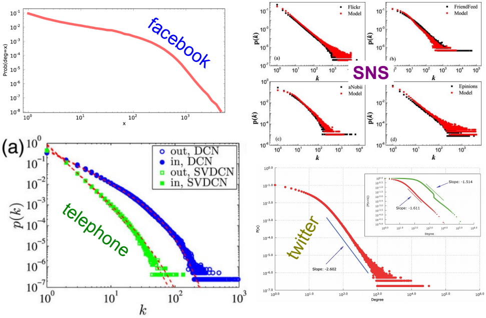
Erdős-Rényi: degree distribution

- Degree distribution of the large Erdős-Rényi graphs is narrow:

$$\frac{\sigma_k}{\langle k \rangle} = \mathcal{O}(N^{-1/2})$$

- It attracts the main criticism points
- BUT
  - Are networks really scale free for which the data looks so?

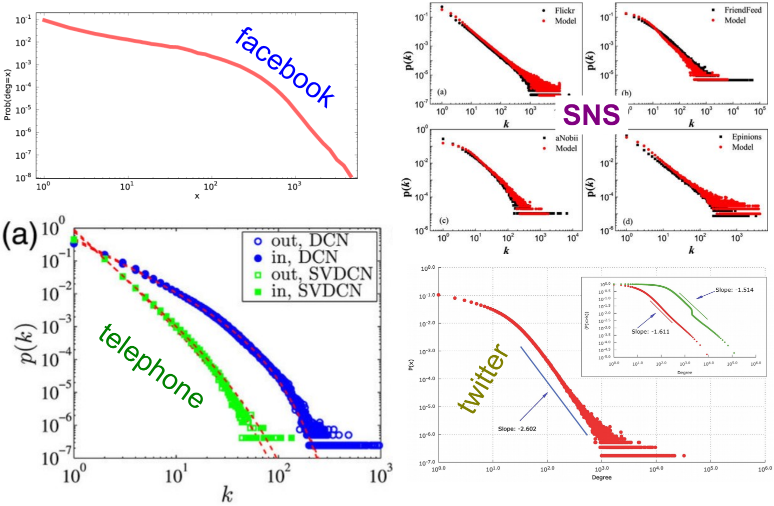
Degree distribution



Erdős-Rényi: degree distribution

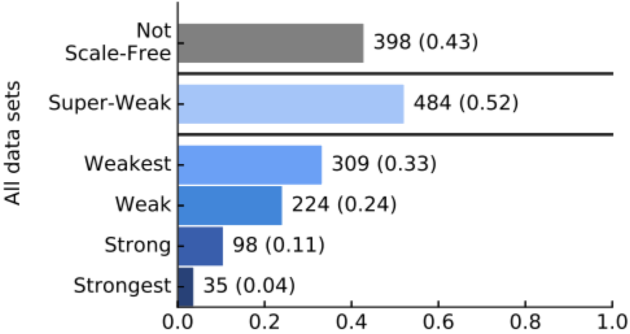
- Empirical social network site degree distributions

Degree distribution



Erdős-Rényi: degree distribution

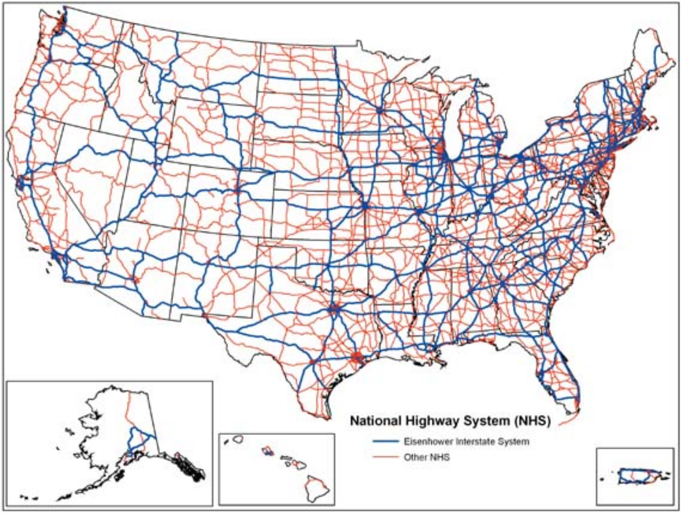
- Test of scale freeness on ~ 1000 empirical networks



Broido and Clause (2018)

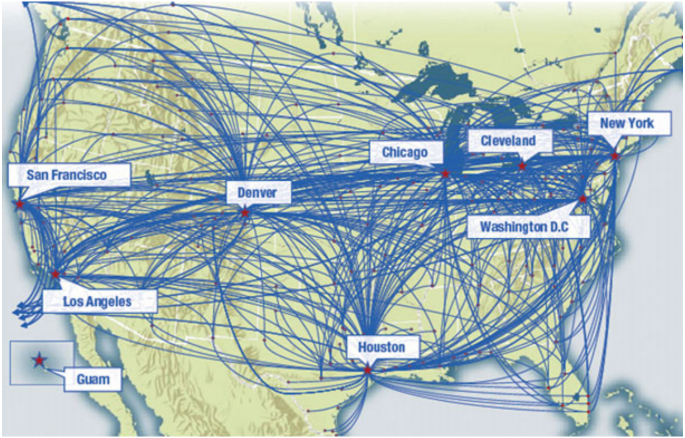
Erdős-Rényi: degree distribution

- Some networks have indeed narrow degree distribution



Erdős-Rényi: degree distribution

- Some networks do not have narrow degree distribution



Erdős-Rényi: the null model

- Some other measures can be of importance
- Erdős-Rényi graphs are truly random and uncorrelated
- Other quantities should show these baseline values
- This makes ER graphs a good basic null model candidate

Erdős-Rényi: clustering coefficient

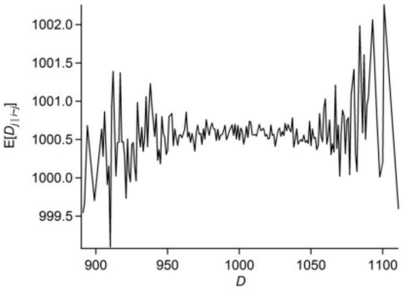
- Clustering coefficient:
- Let us consider two links of a node. The probability that it is a triangle is proportional to the probability that the missing link exists
- Thus

$$C = p = \frac{\langle k \rangle}{N-1}$$

- In large ER graphs the clustering coefficient is almost zero
- There are hardly any triangles in the ER graphs

Erdős-Rényi: assortativity

- The Erdős-Rényi graphs should be non-assortative:
- Reasoning: The link between nodes are established in an independent way without any correlation so the actual node with degree k randomly samples the graph, thus the average degree of the friends is also k
- Funnily the probability to be connected to a node with degree k is not proportional to k.



Noldus,Mieghem 2015

Erdős-Rényi: Percolation

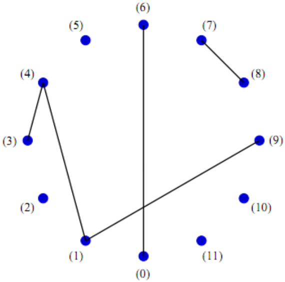
- **Connected components**: There is a path between any two node of a connected component.
- **Percolation**: The network percolates if

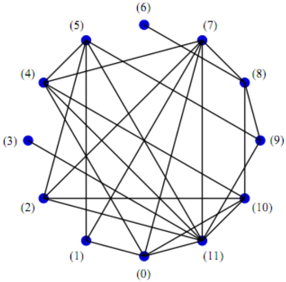
$$\lim_{N\to\infty} |S_\infty|/N = \lim_{N\to\infty} P_\infty > 0,$$

  where S_∞ is the largest connected component and $|S_\infty|$ is its size, and P_∞ is the probability of node belonging to the largest connected component.
- Which means that macroscopic fraction of the nodes belongs to the largest connected components.
- Importance: functioning system cannot fall into (infinitely) many pieces

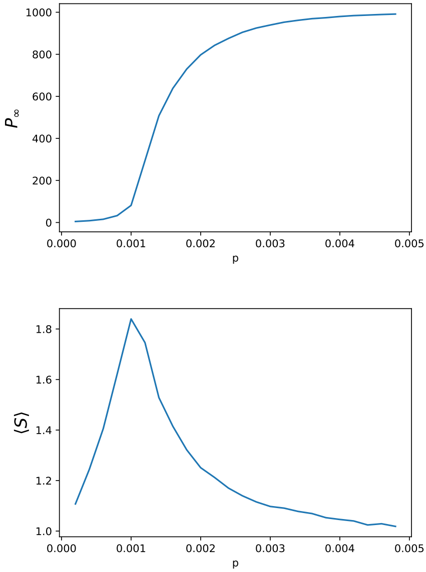
Erdős-Rényi: Percolation

- Percolation transition: Analogous to thermodynamic phase transition.
- Can be continuous (e.g. ER) or discontinuous (e.g. interconnected networks)
- Susceptibility: diverges at the transition
- Susceptibility: average cluster size without the giant component





Erdős-Rényi: Percolation



$N = 1000$

Erdős-Rényi: Percolation threshold

- Where is the percolation threshold?
- u probability, that a node does not belong to the giant component
- A node does not belong to the giant component if all its links are either nonexistent $(1-p)$ or connect to a node not belonging to the giant component (pu):

$$u = (1-p+pu)^{N-1}$$
$$\log u = (N-1)\log\left(1 - \frac{\langle k\rangle}{N-1}(1-u)\right)$$
$$\log u \simeq -\langle k\rangle(1-u)$$

- Probability for a node to be in the giant component:

$$P_\infty = 1 - u = 1 - \exp(-\langle k\rangle P_\infty)$$

Erdős-Rényi: Percolation threshold

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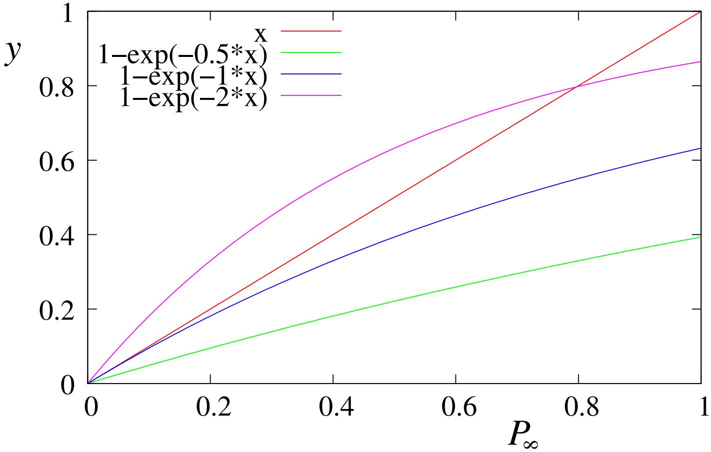
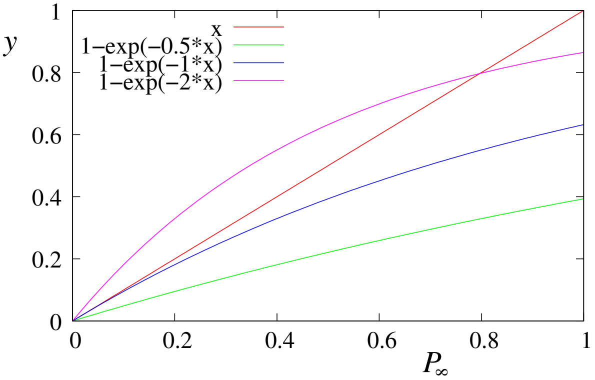


Erdős-Rényi: Percolation threshold

- Where is the percolation threshold?

$$P_\infty = 1 - u = 1 - \exp(-\langle k \rangle P_\infty)$$

- Trivial solution at $P_\infty = 0$
- Non-trivial solution if $\langle k \rangle > 1$



Erdős-Rényi: Percolation threshold

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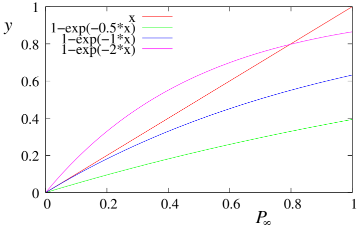
$$P_\infty = 1 - u = 1 - \exp(-\langle k \rangle P_\infty)$$

- Trivial solution at $P_\infty = 0$
- Non-trivial solution if $\langle k \rangle > 1$

$$\frac{d}{dP_\infty}[1 - \exp(-\langle k \rangle P_\infty)] = 1$$
$$\langle k \rangle \exp(-\langle k \rangle P_\infty) = 1$$

- At the transition $P_\infty = 0$, so $\langle k \rangle_c = 1$



Erdős-Rényi: Percolation exponents

- Control parameter $\tilde{k} = \langle k \rangle - \langle k \rangle_c = \langle k \rangle - 1 \ll 1$
(We are close to the transition)
- Order parameter

$$P_\infty = 1 - \exp(-\langle k \rangle P_\infty)$$

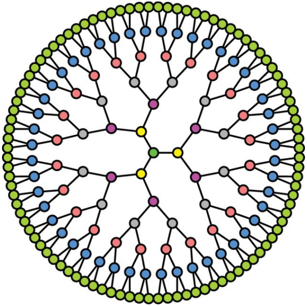
- We assume that $\exp(P_\infty)$ is small, so up to second order

$$P_\infty = 1 - \left[1 - \langle k \rangle P_\infty + \frac{1}{2} \langle k \rangle^2 P_\infty^2 \right]$$
$$1 - \langle k \rangle = \tilde{k} = \frac{1}{2} P_\infty$$

- This gives $\beta = 1$

Erdős-Rényi: Pathlengths

- ER graphs have few circles
- They can make paths shorter
- Consider the worst case: no circles
- The graph is a tree with average degree $\langle k \rangle$
- Note that k has a narrow distribution for large N
- Assume a perfect tree, with degree exactly $\langle k \rangle$
- This is called a Cayley tree (Bethe lattice)

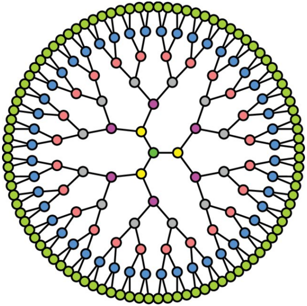


Cayley tree

- Degree $z = \langle k \rangle$, generation n
- Number of nodes:

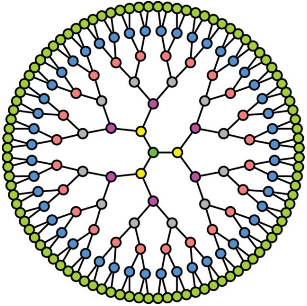
$$N = \frac{z(z-1)^n - 1}{z-2}$$

- Percolation threshold $z = 2$ (Note that there are more nodes in the last layer with degree 1 then the rest)



Cayley tree

- Average path length $\langle s \rangle \simeq 2n$
- $n \sim \log N$
- The length of the average path increases logarithmically: Small world



Erdős-Rényi: Summary

- Ensemble of random graphs
- No correlations
- Sharp degree distribution (Poisson)
- Small clustering coefficient
- Non-assortative
- Percolation threshold at $\langle k \rangle = 1$
- Small world

Small World

- Karinthy: *A fascinating game grew out of this discussion. One of us suggested performing the following experiment to prove that the population of the Earth is closer together now than they have ever been before. We should select any person from the 1.5 billion inhabitants of the Earth – anyone, anywhere at all. He bet us that, using no more than five individuals, one of whom is a personal acquaintance, he could contact the selected individual using nothing except the network of personal acquaintances.*
- Six degrees of separation

Small World

- Stanley Milgram experiment:
  - Letters addressed to a Boston broker
  - People in the Midwest were selected randomly and the packet was sent to them with instructions
  - Letters mast have had to pass on to someone with whom the recipient was on a first-name basis

Small World

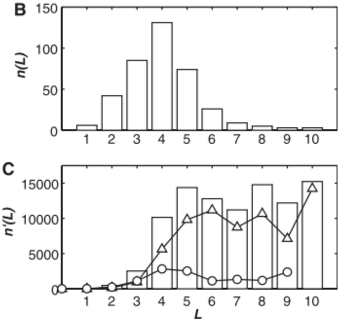
- Stanley Milgram experiment:
  - Letters addressed to a Boston broker
  - People in the Midwest were selected randomly and the packet was sent to them with instructions
  - Letters mast have had to pass on to someone with whom the recipient was on a first-name basis
  - 64 of 296 arrived at the destination
  - Average number of hops was between 5.5 and 6

Small World

- Second experiment
- 24 out of 160 arrived
- Letters reached fast the location
- Then circled around
- Picture below is wrong

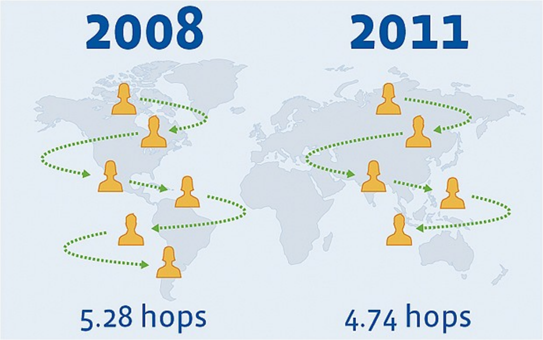
Small World

- Modern version: Duncan J. Watts
- Send emails, same rules
- $\sim$ 60000 emails were sent
- 37% arrived
- The average number of hops was 4.01
- They corrected for incomplete chains and found number of hops between 5 and 7
- Correction was needed due to bad representation of long chains



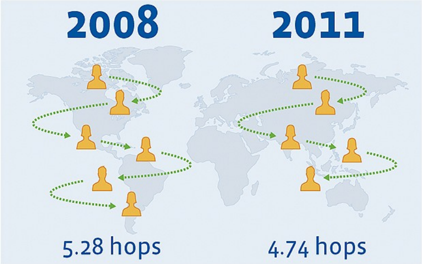
Small World

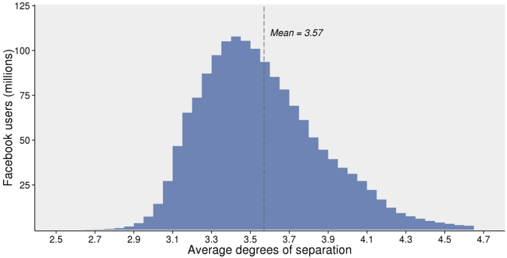
- What about facebook?
- It can be easily measured (by facbook only argh...)



Small World

- What about facebook?

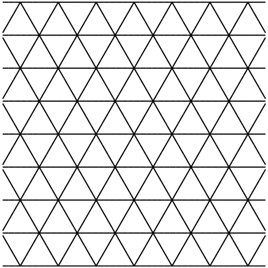
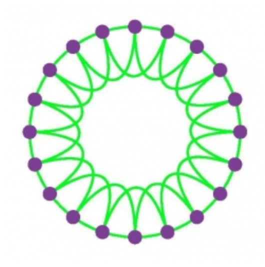
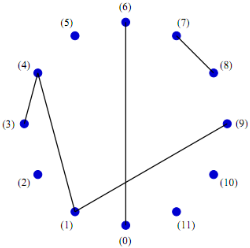


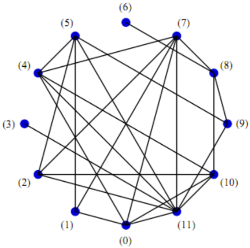


2016

Small World and clustering

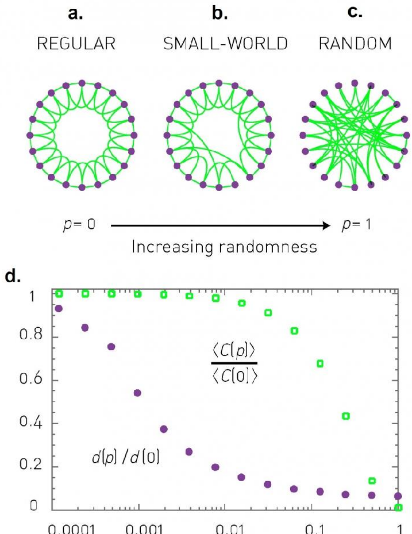
- Erdős-Rényi networks are small words with low clustering
- Triangle lattices are large words with high clustering



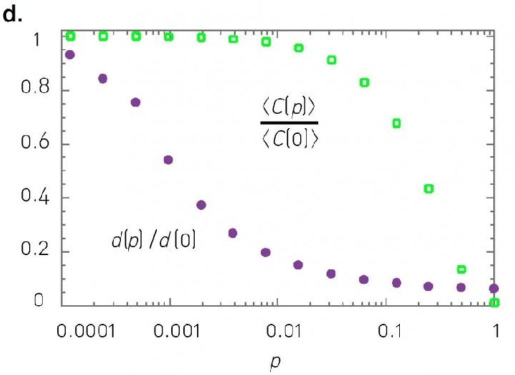
Watts-Strogatz model

- Take a lattice with high clustering
- Introduce shortcuts (rewire)
- Parameter p fraction of rewired links



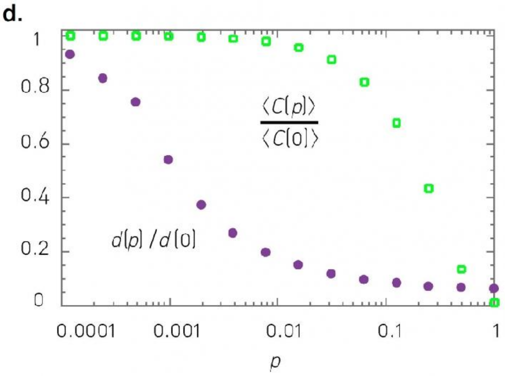
Watts-Strogatz model

- Take a lattice with high clustering
- Introduce shortcuts (rewire)
- Parameter p fraction of rewired links
- Can be high clustering and small world



Watts-Strogatz model

- Degree distribution: shifted Poisson
- This is the major criticism towards the model
- On the other hand tunable randomness.



Scale-free function

- What does it mean?
- Must not have scale included
- Problem: most mathematical functions require dimensionless arguments, e.g. $\exp(x/x_0)$, $\log(x/x_0)$, $\sin(x/x0)$
- Single exception: power law x^α
- Mathematically: scale invariance

$$f(\alpha x) = \alpha^k f(x)$$

- Solution:

$$f(x) = Ax^k$$

Scale-freeness

$$P(x) \sim x^{-\gamma}$$

- What does it mean?
- Normalization? Must have minimum, or maximum value depending on γ (or both!)
- Very uneven distribution: High probability of small value, but very large values are also possible
- Few very rich and a lot of poor
- Origin? Bible: Matt. 25:29, *For whoever has will be given more, and they will have an abundance. Whoever does not have, even what they have will be taken from them.*

Power law distribution

$$P(x) = Cx^{-\gamma}$$

- Two cutoffs: $x \in [a, b]$, C is set to

$$\int_a^b P(x)dx = 1$$

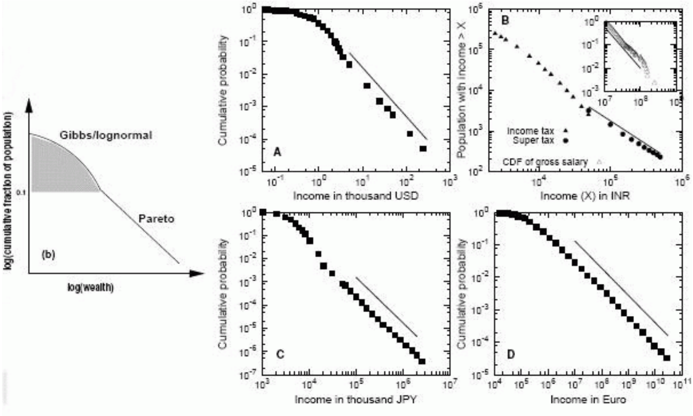
- Cumulative distribution:

$$P(x' > x) = \int_x^b P(x')dx = \frac{C}{\gamma - 1} x^{-(\gamma-1)}$$

- The cumulative distribution decays with a smaller $\gamma - 1$ exponent

Scale-freeness

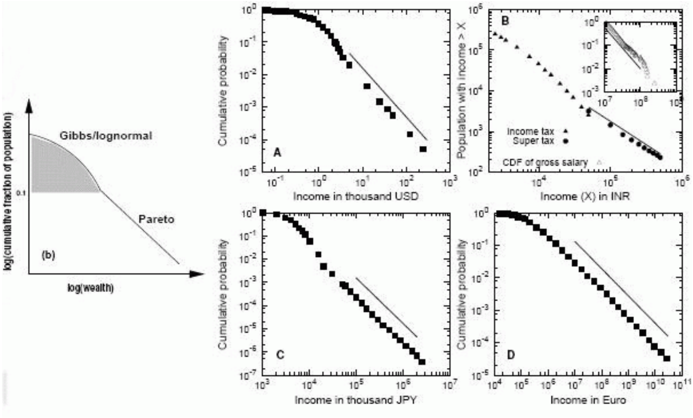
- Economic inequality, Pareto (1890) distribution $P(x) \sim x^{-\alpha}$, $a \simeq 2.5$



Zyga Phys.org 2007

Scale-freeness

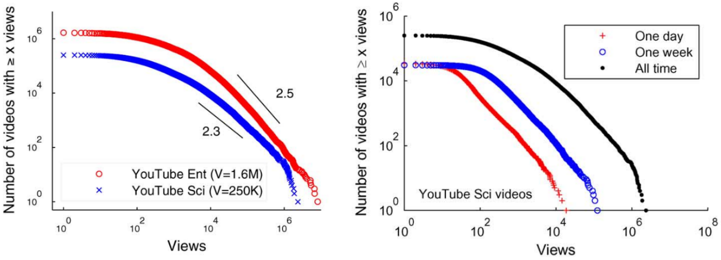
- Pareto principle: 20-80 rule:
- 80% of wealth is in the hands of 20% of the population
- 80 % of land is owned by 20% of people
- 80% of the sales is due to 20% clients



Zyga Phys.org 2007

Scale-freeness

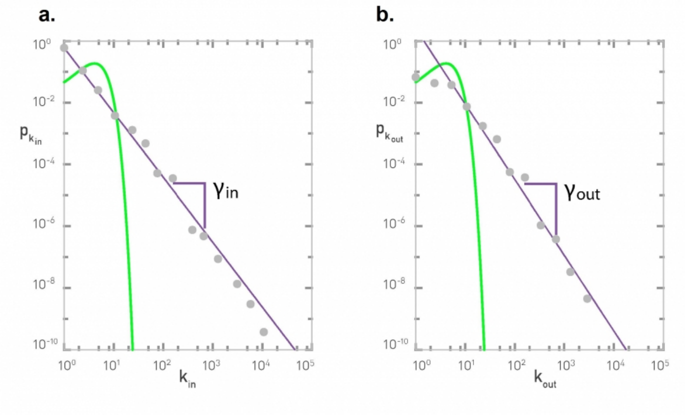
- Views of youtube videos



Cha et al. 2009

Scale-freeness

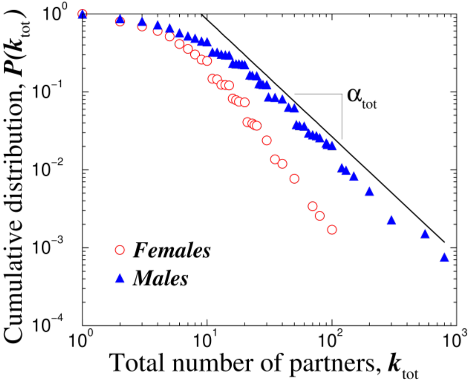
- WWW page popularity
- Exponents are $\gamma_{\text{in}} \simeq 2.1$ $\gamma_{\text{out}} \simeq 2.45$



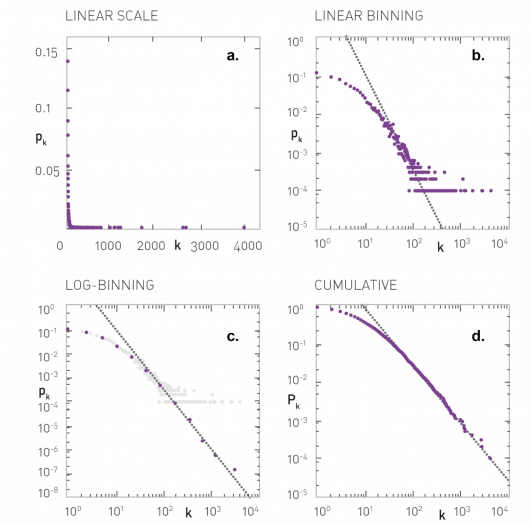
networksciencebook.com

Scale-freeness

- Number of sexual partners in Sweden



Power law: plotting



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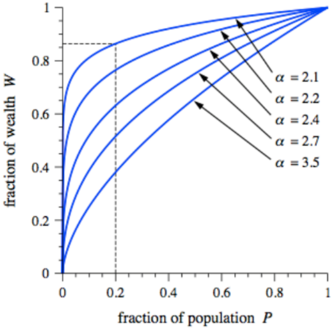
Pareto principle

- Cumulative distribution is:

$$P_>(x) = \int_x^\infty P(x')dx' = \left(\frac{x}{x_{min}}\right)^{-\gamma+1}$$

- For $\gamma > 2$ the fraction of wealth larger than x is

$$W(x) = \frac{\int_x^\infty x'P(x')dx'}{\int_{x_min}^\infty x'P(x')dx'} = \left(\frac{x}{x_{min}}\right)^{-\gamma+1} = P_>(x)^{\frac{\gamma-2}{\gamma-1}}$$



Zipf plots

- George K. Zipf linguist
- Ordered the words according to their occurrence frequency (1935)
- Plotted the frequency against the rank
- Zipf plot

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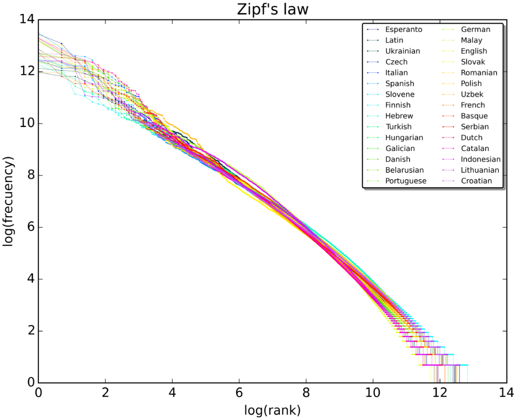

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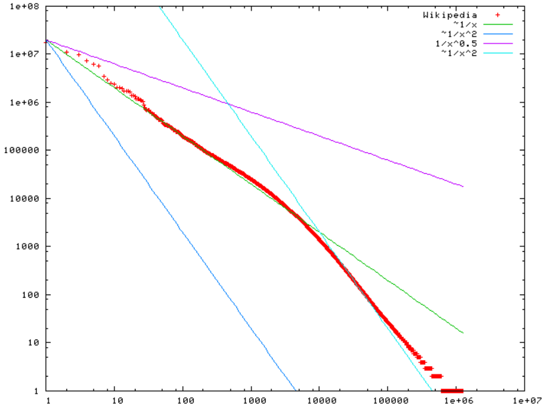
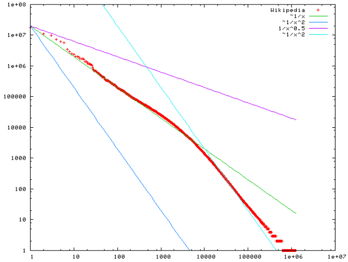
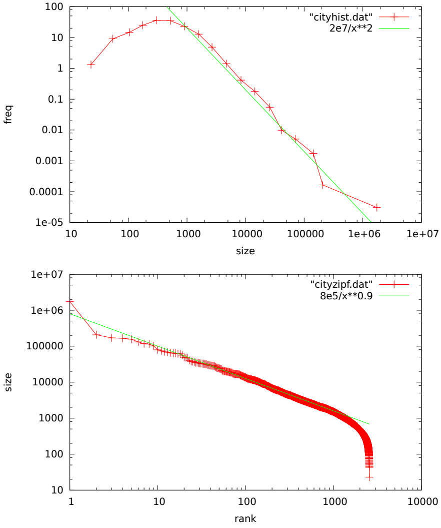


Zipf plots

- Meaning of Zipf plot
- Rank n with frequency $f(n) = n^{-\beta}$
- There are n more frequent words than $f^{-1}(n)$
- In other words $f^{-1}(n)$ is equivalent to the cumulative frequency distribution $\beta = 1/(\gamma - 1)$



Hungarian cities



Inhomogeneities in networks

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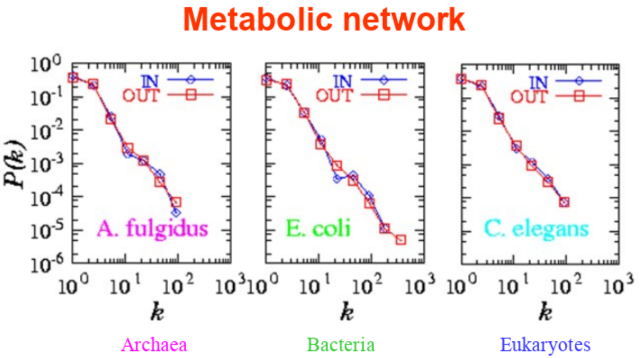
Scale free networks



Organisms from all three domains of life are
scale-free networks!

H. Jeong, B. Tombor, R. Albert, Z.N. Oltvai, and A.L. Barabasi, Nature, **407** 651 (2000)

Scale free networks

Network	*N*	*L*	⟨*k*⟩	⟨*kin2*⟩	⟨*kout2*⟩	⟨*k2*⟩	*γin*	*γout*	*γ*
Internet	192244	609066	6.34	-	-	240.1	-	-	3.42*
WWW	325729	1497134	4.6	1546	482.4	-	2	2.31	-
Power Grid	4941	6594	2.67	-	-	10.3	-	-	Exp.
Mobile-Phone Calls	36595	91826	2.51	12	11.7	-	4.69*	5.01*	-
Email	57194	103731	1.81	94.7	1163.9	-	3.43*	2.03*	-
Science Collaboration	23133	93437	8.08	-	-	178.2	-	-	3.35*
Actor Network	702388	29397908	83.71	-	-	47353.7	-	-	2.12*
Citation Network	449673	4689479	10.43	971.5	198.8	-	3.03*	4.00*	-
E. Coli Metabolism	1039	5802	5.58	535.7	396.7	-	2.43*	2.90*	-
Protein Interactions	2018	2930	2.9	-	-	32.3	-	-	2.89*-

Scale free networks: moments

- Moments of power law distribution

$$\langle k^m \rangle = \int_{k_{min}}^{\infty} k^m P(k) dk$$

- Normalization ($\gamma > 1$)

$$P(k) = (\gamma - 1) k_{min}^{\gamma-1} k^{-\gamma}$$

- Moments, if $1 + m < \gamma$:

$$\langle k^m \rangle = \frac{k_{min}^m (\gamma - 1)}{\gamma - 1 - m}$$

- If $m \geq \gamma - 1$ the moment diverges

Scale free networks: moments

- Moments diverge for $m \geq \gamma - 1$
- $\gamma \leq 2 \rightarrow$ No average
- $\gamma \leq 3 \rightarrow$ No variance
- Many networks fall in this category

Network	***N***	***L***	**⟨*k*⟩**	**⟨*kin2*⟩**	**⟨*kout2*⟩**	**⟨*k2*⟩**	***γin***	***γout***	***γ***
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Distances in scale free networks

Average distance scale with node number N as

- $\langle l \rangle \sim const.$ for $\gamma = 2$ Size of the biggest hub is of order $\mathcal{O}(N)$
- $\langle l \rangle \sim \frac{1}{\log(\gamma - 1)} \log \log N$ for $2 < \gamma < 3$. Path length increases slower than logarithmically, ultra-small world
- $\langle l \rangle \sim \log N / \log \log N$ for $\gamma = 3$. Some key models produce $\gamma = 3$
- $\langle l \rangle \sim \log N$ for $\gamma > 3$. The second moment of the degree distribution is finite, similar to random network. Small world.

Network	***N***	***L***	**⟨*k*⟩**	**⟨*kin2*⟩**	**⟨*kout2*⟩**	**⟨*k2*⟩**	***γin***	***γout***	***γ***
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