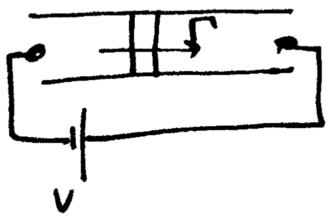
Theoretical Nanophysics, Lecture 6, 2019/03/13

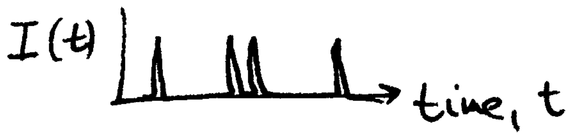
Noise in mesoscopic conductors, Part 2

Lecture 5: counting statistics. Now:

(I) Current noise spectral density of a tunnel junction

(1)





$$N(t) = \frac{1}{e}\int_0^t dt'\, I(t') \qquad I(t) = e\,\frac{dN(t)}{dt}$$

avg current: $\langle I \rangle = e\langle N(t)\rangle / t$

(2) current fluctuation: $\delta I(t) = I(t) - \langle I \rangle$

autocorrelation fn of c.f.: $S(t) = \langle \delta I(t)\,\delta I(0)\rangle$

current noise spectral density: $S(\omega) = \int_{-\infty}^{\infty} dt\, e^{i\omega t} S(t)$

[Warning: sometimes $S(t) = 2\langle \delta I(t)\,\delta I(0)\rangle$]

(II) Poissonian process is white noise

uncorrelated sequence of e-jumps: $I(t) = e\sum_i \delta(t-\tau_i)$ independent uniformly distrib.

claim: $S(t) \equiv \langle \delta I(t)\,\delta I(0)\rangle = e\langle I\rangle \delta(t)$

consequence: $S(\omega) = e\langle I \rangle$ ← freq-independent → 'white noise'.

proof: exercise

(III) Variance of transmitted charge is the zero-frequency current noise

claim: $\delta N_t^2 = \dfrac{S(\omega=0)\,t}{e^2}$ (as $t\to\infty$). proof: exercise

hint: $\langle \delta I(t)\,\delta I(0)\rangle$ has a finite width.

IV Classical and quantum noise of a harmonic oscillator

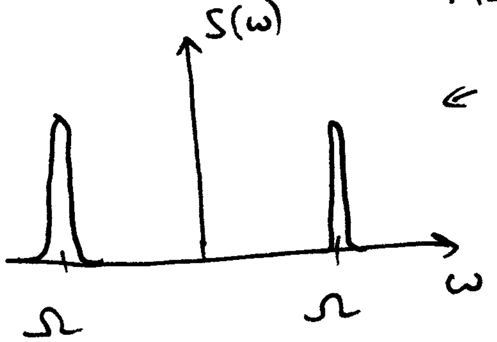
(Clerk et al. arxiv 0810.4729v2)

Clerk Appendix A

① $H = \frac{p^2}{2m} + \frac{1}{2} M \Omega^2 x^2$ $\qquad \langle x(t) \rangle_t = 0$

② Thermal eq.:

$S(t) = \langle \delta x(t) \delta x(0) \rangle = \frac{k_B T}{M\Omega^2} \cos(\Omega t) \rightarrow S(\omega) = \pi \frac{k_B T}{M\Omega^2} \left[\delta(\omega - \Omega) + \delta(\omega + \Omega) \right]$



← symmetric fn of ω (since S(t) is real)

③ Q. harm. osc: (Clerk II.A)

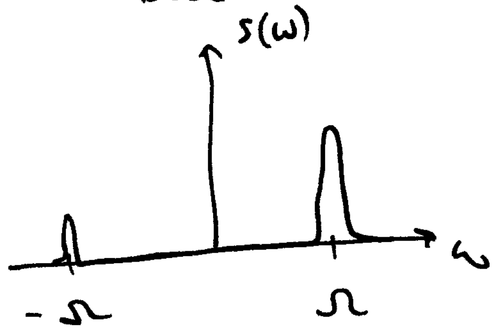
Heisenberg picture: $\hat{x}(t) = e^{i\hat{H}t/\hbar} \hat{x} e^{-i\hat{H}t/\hbar}$

thermal equilibrium

position autocorr. fn: $S(t) = \langle \hat{x}(t) \hat{x}(0) \rangle = x_{ZPF}^2 \{ n_B(\hbar\Omega) e^{i\Omega t} + (n_B(\hbar\Omega) + 1) e^{-i\Omega t} \}$

$x_{ZPF} = \sqrt{\frac{\hbar}{2M\Omega}}$

Bose-Einstein fn: $n_B(\mathcal{E}) = \frac{1}{e^{\mathcal{E}/k_B T} - 1}$



$S(\omega) = 2\pi x_{ZPF}^2 \{ n_B(\hbar\Omega) \delta(\omega + \Omega) + (n_B(\hbar\Omega) + 1) \delta(\omega - \Omega) \}$

not symmetric

④ exercise: prove that $S_{quantum}(\omega) \longrightarrow S_{cl}(\omega)$ for $k_B T \gg \hbar\Omega$

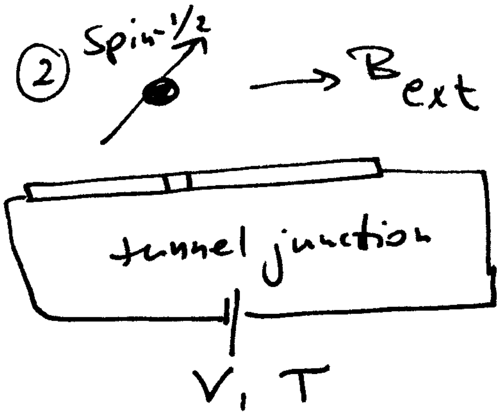
⑤ interpretation: for $\omega < 0$, $S(\omega)$ is related to emission capability

— " — $\omega > 0$, $S(\omega)$ — " — absorption — " —

⑥ often-used model: continuum bath of harmonic oscillators

V Physical relevance of q-noise

① Spin qubit (or any two-level system, TLS) serves as a quantum noise spectral analyzer (Clerk II.B)

② 

spin: $\hat{H}_0 = \frac{\hbar\omega_{01}}{2}\hat{\sigma}_z$ $\updownarrow \hbar\omega_{01}$ $(\omega_{01} \propto B_{ext})$

effect of wire current on spin

$\hat{H} = A\hat{F}\sigma_x$ here: $\hat{F} = \delta\hat{I}$

A depends on geometry

③ claim: (lengthy derivation with approximations):

$$\begin{cases} \dot{p}_0(t) = -\Gamma_\uparrow p_0(t) + \Gamma_\downarrow p_1(t) \\ \dot{p}_1(t) = -\Gamma_\downarrow p_1(t) + \Gamma_\uparrow p_0(t) \end{cases}$$

where

$$\Gamma_\uparrow = \frac{A^2}{\hbar^2} S(-\omega_{01})$$

$$\Gamma_\downarrow = \frac{A^2}{\hbar^2} S(+\omega_{01})$$

That is, q-noise of conductor governs spin dynamics.

Reason to study q-noise.

④ results of master equation:

(i) steady state: $p_0 = \frac{\Gamma_\downarrow}{\Gamma_\downarrow + \Gamma_\uparrow}$, $p_1 = \frac{\Gamma_\uparrow}{\Gamma_\downarrow + \Gamma_\uparrow}$

(ii) inverse time scale (= rate) of reaching steady state:

$$\Gamma_{relax} = \Gamma_\downarrow + \Gamma_\uparrow = \frac{A^2}{\hbar^2}\left[S(\omega_{01}) + S(-\omega_{01})\right]$$

exercise: prove these.

(5) Conjecture: if the noise source is in thermal equilibrium at T, then $p_1/p_0 = e^{-\hbar\omega_{01}/k_B T}$ (Boltzmann)

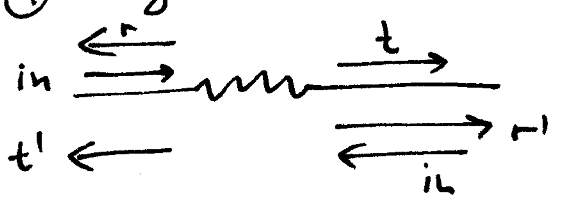
Exercise: check this for the harm. osc. above.

(6) If noise source is not in th. equilibrium, then (e.g. voltage-biased tunnel junction) the spin level populations define an effective T:

$$T_{eff} = \frac{\hbar\omega}{k_B \log \frac{S(\omega_{01})}{S(-\omega_{01})}}$$

(VI) Current noise spectral density of a mesoscopic conductor

(1) Single-channel leads



$$S = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix} \quad \text{(unitary)}$$

$$1-\underbrace{|t|^2}_{\equiv \tau} = 1-|t'|^2 = |r|^2 = |r'|^2$$

Task: $\langle \hat{I} \rangle = \langle I \rangle (V, T, \tau) = ?$

$\langle \delta\hat{I}(t)\,\delta\hat{I}(0)\rangle (V, T, \tau) = ? \longrightarrow S(\omega; V, T, \tau) = ?$

(2) procedure:

- single-e current operator (1D): $\hat{j}(x) = e\frac{1}{2}\{\delta(\hat{x}), \frac{\hat{p}}{m}\}$
- Fock-space current operator $\hat{I}(x)$
- use basis of incoming states
- use Heisenberg picture $\hat{I}(x,t)$
- specify x appropriately
- use 'voltage-biased equilibrium' steady-state
- use Wick's theorem for noise

$\longrightarrow$ + approximations (see: Heikkilä; Blanter & Büttiker)

Noise in mesoscopic circuits, Part 2

Theoretical Nanophysics
BME, 2019 Spring
Lecture 6, 2019/03/13

Literature

Aash Clerk et al.
Quantum Noise, Measurement and Amplification
Rev. Mod. Phys 2010

Tero Heikkila
The Physics of Nanoelectronics
Oxford University Press, 2013
(chapters 3, 6)

Yaroslav Blanter and Markus Buttiker
Shot Noise in Mesoscopic Conductors
Phys. Rep. 336, 1 (2000)

see also:
BME course *Fundamentals of Nanophysics*
on fizipedia
by Andras Halbritter, Szabolcs Csonka, Peter Makk