Statistical field theory (BMETE15MF64)

Course schedule with sources

1. (Feb 12) Second order phase transitions. Critical exponents, scaling laws, universality. Field theoretic description of the Ising model in d dimensions.
   MSFT 1.1-1.2, 7.1-7.4

2. (Feb 19) Renormalisation group. Blocking, RG flow of couplings, fixed points. Linearisation near fixed points, classification of couplings.
   MSFT 8.1-8.4
   Solution of 1d classical Ising spin chain, mean field solution, transfer matrix. Blocking, RG flow. Existence of phase transition in higher D.
   SRST 3.1-3.2; SRST 2.1-2.2; MSFT 2.2;

3. (Feb 26) Landau-Ginsburg description. Criticality in Ising model and tricritical Ising model. RG in the Gaussian model. Wilson's RG in field theory, Gaussian fixed point. RG for free energy, Ising model.
   MSFT 7.7.1-7.7.3, 8.6, 8.8

4. (Mar 4) Critical exponents, beta functions. Sketch of Wilson-Fisher fixed point. fixed points and universality classes. Scale invariance at fixed points.
   MSFT 8.9-8.10
   Ising around 4d. Gauss fixed point. Wick theorem from path integral. Perurbative RG, OPE calculations using Wick theorem.
   SRST 5.1-5.4 + Appendix

5. (Mar 11) Statistical field theory and Euclidean QFT. Polyakov theorem (scale invariance implies conformal invariance). Conformal field theory in d dimensions. Conformal symmetry, energy-momentum tensor, scaling fields.
   MSFT 10.1-10.4; ACFT 1.1-1.3
   ϵ-expansion, Wilson–Fisher fixed point, critical exponents to lowest order. Free boson, energy-momentum tensor, vertex operators
   SRST 5.5; YBOOK 2.3.4, 5.3.1, 6.3.2

6. (Mar 18) **No class due to absence of lecturer**

7. (Mar 25) Conformal Ward identities. 2 point functions, 3 and 4 point functions, cross ratios. Conformal symmetry in 2 dimensions, global and local conformal transformations.
   MSFT 10.3.2, 10.5
   Primary and quasi-primary fields, energy-momentum tensor, Ward identity, operator product expansion of primary fields with energy-momentum tensor. Conformal transformation of energy-momentum tensor, Schwarzian derivative.
   MSFT 10.5-10.7; ACFT 2.1, 3.1

8. (Apr 1) **Easter Monday**

9. (Apr 8) Operator formalism. Descendant fields. Virasoro algebra, contour integral derivation of commutators. Central extansion, relation to classical conformal generators.
   ACFT 2.2-2.4; ACFT 3.3
   Free boson with canonical quantisation. Exponential map. Virasoro generators for free boson. Free fermion, Virasoro algebra for free fermion.
   ACFT 6.1-6.2; MSFT 12.2-12.3; YBOOK 6.3.1-6.3.3, 5.3.1-5.3.2

10. (Apr 15) Operator-state correspondence. In and out states. Conjugation of primary fields. Highest weight states. Descendant fields and descendant states. Conjugation of Virasoro generators, inner product. Verma module, Gram-Schmidt matrix, unitarity implies $c > 0$ and $h \geq 0$.
    ACFT 3.4-3.6; SFT 10.8
    Mode expansion on the plane. Virasoro algebra. Mode expansion of general field. Commutator of Virasoro generators with a primary field. Gram matrix. Ward-identities. Singular vectors. Differential equations for correlators.
    YBOOK 6.1; ACFT 3.3-3.6; MSFT 10.8.2, 11.2

11. (Apr 22) Virasoro representations. Singular vectors. Example: 2nd level. Kac determinant. Minimal models.
    SFT 11.1-11.4.1; ACFT 4.1-4.3

12. (Apr 29) Detour: physical meaning of central charge (Cardy). Minimal models: structure of OPE in terms of descendant fields, conformal families. Operator product expansion in minimal models, fusion rules from singular vectors.
    SFT 10.9; SFT 11.4.2-11.4.3
    Free fermion, Ising model. NS and R sectors, twist field, dimension of twist field. Relation to critical exponents, unitary minimal models. Differential equation for 4pt function in Ising model.
    MSFT 12.3.1; ACFT 5.1-5.2; (see also YBOOK 8.3.3)

13. (May 6) Modular invariance and partition functions. Operator product coefficients, CFT data.
    SFT 11.7; YBOOK 10.1, 10.5, 10.7.1-10.7.2

14. (May 13) Vicinity of critical point. C-theorem. Perturbative RG flows. Δ-theorem.
    SFT 15.1, 15.4-15.7
    Structure constants and operator algebra. Conformal blocks, crossing relations. Ising example. Brief sketch of conformal bootstrap.
    YBOOK 6.6.3-6.6.4; (see ACFT 5.2 for the Ising example)

Remark: another good source for conformal field theory (weeks 5-14) is ISFT Chapter 9

References

MSFT   G. Mussardo: Statistical Field Theory, Oxford University Press, 2010.

ACFT   P. Ginsparg: Applied Conformal Field Theory, arXiv: hep-th/9108028

ISFT   Ityzkson-Drouffe: Statistical Field Theory vol. 2, Cambridge University Press, 1989.

YBOOK   P. Di Francesco, P. Mathieu and D. Sénéchal, Conformal Field Theory

SRST   J. Cardy: Scaling and Renormalization in Statistical Physics