Lie groups in physics

May 30, 2019

1. Definitions and basic topological properties: groups, manifolds, and definition of Lie groups.
2. Examples. definitions, and simple properties of: $Gl(N)$, $Sl(N)$, $SO(N)$, $SU(N)$.
3. The story of $SU(2)$ and $SO(3)$, with the quaternions (definitions, properties), including the topological properties (universal covering group).
4. The story of $SO(4)$, $SU(2) \times SU(2)$, and $SO(3,1)$. (only on the level of the group, not the Lie algebra)
5. Lie algebras, Lie bracket of vector fields, left-invariant fields, and going back to Lie bracket of matrices.
6. Baker-Campbell-Hausdorff series, the story of global vs. local properties.
7. Examples for Lie algebras.
8. Complexification of real Lie algebras, and the main examples in detail.
9. Definition of ideals of Lie algebras.
10. Representation theory of Lie groups, definitions, faithful, real, unitary (main statements, finite groups, compact Lie groups). Invariant subspaces, definition, examples and counter-examples (examples for a reducible but not completely reducible representation). Connections between unitarity and complete reducibility. Adjoint representation.
11. Lie algebra representations, Ado's theorem. The story between local and global properties. Adjoint representation.
12. All irreducible representations of $SU(2)$ in detail. Unitary infinite dimensional of $E(3)$.
13. Killing form, invariance property. Cartan sub-algebra, roots. The $Sl(2,\mathbb{C})$ subgroups corresponding to the roots, and their connection. Positive and simple roots.
14. The structure of $SU(3)$ in detail. Cartan sub-algebra, roots, $SU(2)$ subgroups, representations of the subgroups within the adjoint rep.
15. Dynkin diagrams. Root generators and vectors (explicit representation) for $SU(N)$, $SO(2N)$ and $SO(2N+1)$. Low dimensional coincidences.
16. The highest weight method. Irreps of $SU(3)$ in detail. (only highest weight method, and explicit representation of the highest weight vector in the tensor method, but not the Young diagrams)
17. Young diagrams: their purpose, Young symmetrizers, connection with the highest weight method, explicit examples in $SU(3)$.
18. The story of $SO(4)$ and $SO(3,1)$ on the Lie algebra level.