

Barkhausen effect

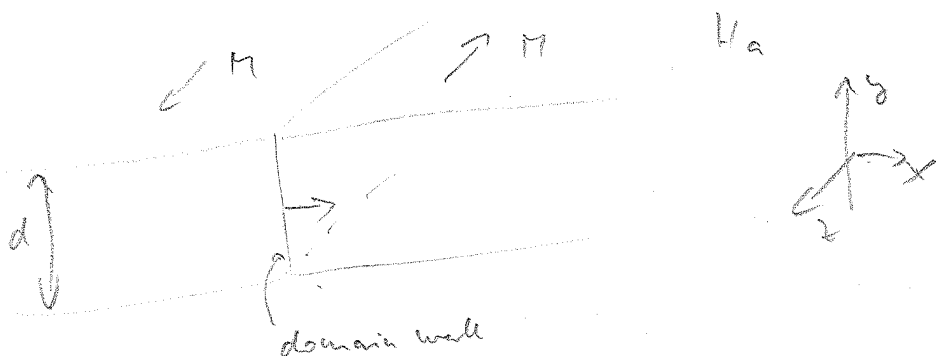
- single domain wall movement
- eddy - current damping
- stochastic domain wall dynamics
- random pinning fields
- Langevin
- Fokker-Planck

magnetic flux Φ
 magnetic field

Space average of H

$$\frac{d\Phi}{dt} = \int [M(\dots), H(\dots)] = g(\Phi, H(\dots))$$

Single domain wall



$$\frac{d\Phi}{dt} = 2 I_s d \cdot v_w = 2 \mu_0 I_s d \frac{dx_w}{dt}$$

→ eddy currents → Joule heat, small v_w limit

$$\nabla \times \mathbf{j} = 0$$

$$\nabla \cdot \mathbf{j} = 0$$

$j_x(x)$ even, $j_y(x)$ odd

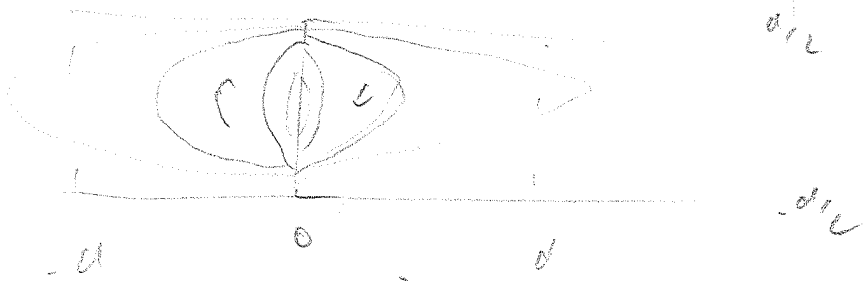
$$j_y(y = \pm d/2) = 0$$

$$j_y(x=0^-) = -j_y(x=0^+) = 6 I_s v_w = \frac{\sigma}{2d} \frac{d\Phi}{dt}$$

$$j_x = \frac{2\sigma}{d} \frac{d\Phi}{dt} \sum_{\text{odd } n} \frac{(-1)^{(n-1)/2}}{n\pi} \sin \frac{n\pi y}{d} \exp\left(-\frac{n\pi |x|}{d}\right)$$

$$j_y = \dots$$

cos

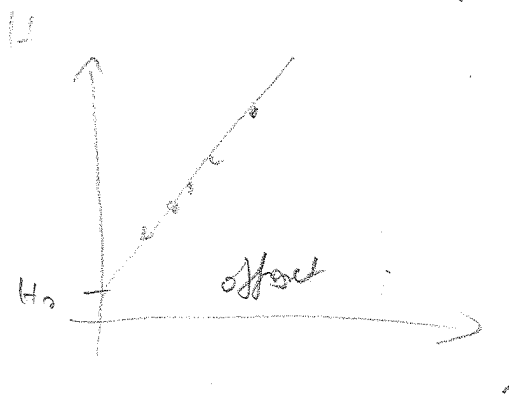


$$P_w = \sigma G \left(\frac{d\phi}{dt} \right)^2$$

$$G = \frac{4}{\pi} \sum_{\text{odd } n} \frac{1}{n^3} \approx 0,136 \dots$$

This work per unit slab and unit time by $H_a \frac{d\phi}{dt}$

$$H_a = \sigma G \frac{d\phi}{dt} = \beta \frac{dX_{ew}}{dt} \quad \beta = 2 \sigma G l_{sol}$$



linear dependence

$$\sigma G \frac{d\phi}{dt} = H_a - H_0$$

$$P_w = \left| H_0 \frac{d\phi}{dt} \right| + \sigma G \left(\frac{d\phi}{dt} \right)^2$$

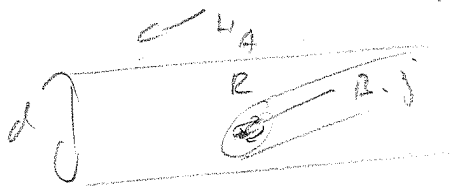
↑ rate dependent
 ↑ independent of the wall velocity
 rate independent hypothesis

loss separation

- ↳ internal degrees of freedom
- ↳ structural disorder

Internal degrees of freedom

Bardeen-Louisen jump



$$\frac{d\phi}{dt} = v \Delta \phi$$

flux change (inside R)

average number of jumps per unit time

coordinates of the jump

Dissipation: $f(x, t, x_i(t))$

$$j = \sum_i j_i$$

v number of jumps in T

$$|j|^2 = \sum_i |j_i|^2 + \sum_{i \neq j} j_i \cdot j_j = v \left(T \langle |j_i|^2 \rangle \right) + v^2 \left(T \langle j \rangle \right)^2$$

$$\gamma \sim \frac{d\phi}{dt}$$

$$P_2 = \left| H_0 \frac{d\phi}{dt} \right|^2 + \sigma G' \left(\frac{d\phi}{dt} \right)^2$$

H_0, G' const. funct of $\Delta \phi, j, P(x)$

same structure as before

Stochastic domain wall dynamics

$$\left(\gamma \frac{dx}{dt} = H(t) - \frac{\partial F}{\partial x} = H(t) - H_T(x) \right)$$

(1) $\gamma G \frac{d\phi}{dt} = H_a(t) - H_T(\phi)$

(2) $H_T(\phi) = \frac{\phi}{S\mu} = H_P(\phi)$ random pinning field

large scale stability

macroscopic magnetization constant slope

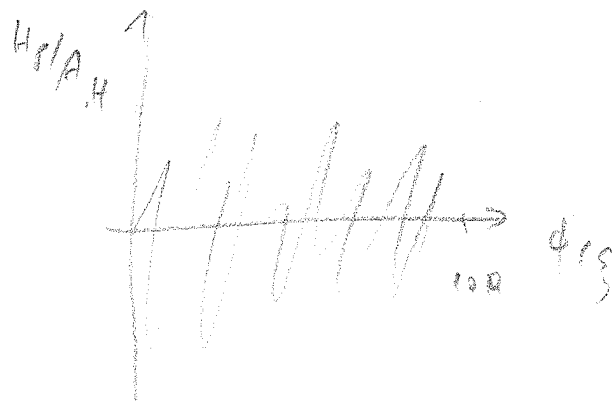
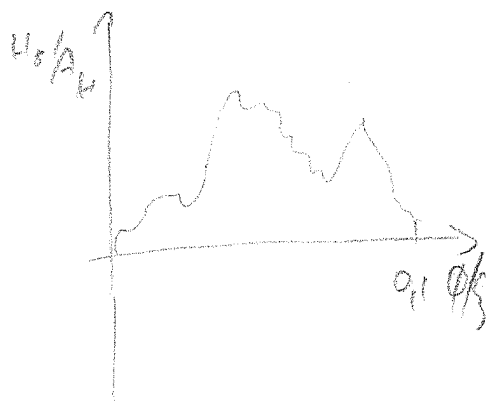
correlation length } correlations destroyed beyond by randomness

Ornstain-Uhlenbeck process

(3) $\frac{dH_T}{d\phi} + \frac{H_P}{\gamma} = \frac{dW}{d\phi}$ $\langle dW \rangle = 0$
 $\langle dW \rangle^2 = \frac{2A_H^2}{\gamma} d\phi$

dW Wiener process Gaussian white noise

$P_0(H_T) = \frac{1}{\sqrt{20A_H^2}} \exp\left(-\frac{H_T^2}{2A_H^2}\right)$ Gy. dist.



(a) & (b) & (c) dynamics of domain wall
 time space

→ too complicated → approx

→ limit

$$\xi \rightarrow \infty \quad A = \frac{A_H^2}{\xi}$$

$\frac{H}{\xi} \rightarrow 0 \Rightarrow H_p$ Wiener process with $\langle dH_p|^2 \rangle = 2A d\phi$

$$\Rightarrow \sqrt{G} \frac{d\phi}{dt} = H_a(t) - \frac{\phi}{\xi \mu} - H_p(\phi)$$

$$\langle dH_p \rangle = 0 \quad \langle |dH_p|^2 \rangle = 2A d\phi$$

Decomposition

const:

$$\Sigma = G G S \mu \quad \Phi_0 = A S^2 \mu^2 \quad H_0 = A S \mu$$

$$c = \frac{\Sigma}{H_0} \frac{dH_a}{dt} \rightarrow \text{dimensionless parameter}$$

$$h = \frac{H}{c} \quad x = \frac{\phi}{\Phi_0} \quad \sigma = \frac{d\phi}{d\mu} \quad h_p = \frac{H_p}{H_0}$$

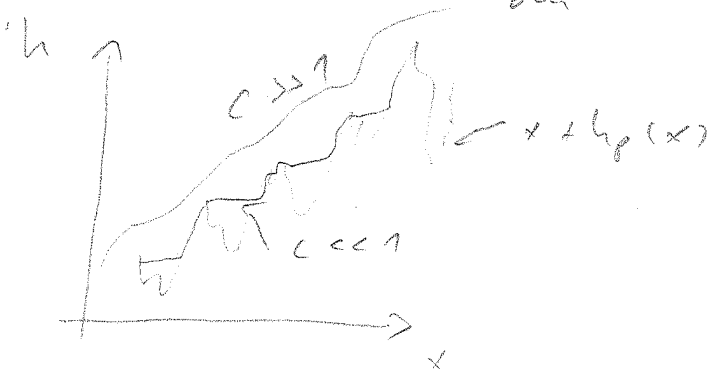
$$\frac{dx}{d\mu} = h_a - x - h_p(x)$$

$$\frac{dh_a}{d\mu} = c \quad \langle dh_p \rangle = 0 \quad \langle |dh_p|^2 \rangle = 2dc$$

Langevin approach

$$\frac{d\mu}{d\mu} + (b-c) = - \frac{dh_p}{d\mu}$$

$$\langle dh_p \rangle = 0 \quad \langle |dh_p|^2 \rangle = 2\sigma d\mu$$



$$\langle v \rangle = c \Leftrightarrow \frac{1}{S} \frac{d\langle \phi \rangle}{dt} = \mu \frac{dH_a}{dt}$$

macroscopic law of linear magnetization

Noise term

(Bose-Einstein fluctuation at thermal equilibrium)

Here zero temperature, random driving force
rate of change from disorder

$$\frac{dh_p}{dt} = ?$$

$$dh_p \text{ vs. } dx \rightarrow \langle |dh_p|^2 \rangle = 2 dx$$

? variance for dx

$$dx = v \cdot dt \Rightarrow \langle |dh_p|^2 \rangle = 2 dx = 2v dt$$

$\Rightarrow \frac{dh_p}{dt}$ white noise term of intensity from time to time in proportion to v

It's stochastic diff. eq.

$$dv + (v-c) dt = \sqrt{2v} dw \quad \begin{matrix} \text{Wiener process} \\ \langle dw dt \rangle = dt \end{matrix}$$

Fokker-Planck equation

$$\frac{\partial P}{\partial t} + \frac{\partial}{\partial v} (c-v)P - \frac{\partial^2}{\partial v^2} vP = 0$$

$$\frac{\partial P}{\partial t} + \frac{\partial j_P}{\partial v} = 0$$

$$j_P = (c-v)P - \frac{\partial}{\partial v} vP$$

probability current

$P(u, u=0 | u_0) = \delta(u - u_0)$ decays exponentially for $u \rightarrow \infty$

$f_r(u=0^+) = 0$ $u=0$ is a reflecting boundary

$$P_x(u, u) = C_x e^{-\lambda u} e^{-u} f_x(u) \quad \lambda > 0$$

$$P_0(u) = \frac{1}{\Gamma(c)} u^{c-1} e^{-u}$$



Self similarity

$$f_P = (c-1-u)P - u \frac{\partial P}{\partial u}$$

$0 < c < 1$ neglected

scale invariance : $u \rightarrow bu, v \rightarrow bv$



$$P_0(u) \sim u^{c-1}$$

$$P_u(\Delta u) \sim (\Delta u)^{-\alpha}$$

$$P_x(\Delta x) \sim (\Delta x)^{-\beta}$$

$$\alpha = 2 - c$$

$$\beta = 3/2 - c/2$$

