1. Harris criterion

- Self averaging

Consider:

$$\mathcal{H} = -\sum_{\langle ij\rangle} J_{ij}\sigma_i\sigma_j - \sum_i h_i \sigma_i \quad , \sigma \in S^{(n)}$$

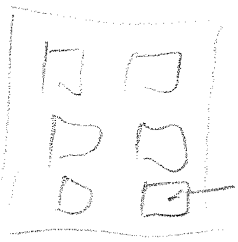
$n=1 \to$ Ising

d : dimension

- random exchange model : J_{ij} random
- random field model : h_i random
- site diluted magnets : magnitude of spin is site dependent

$J \to J_{ij}$ quenched variables

$J_{ij} = \tau_i \tau_j$ $\quad \tau \to$ annealed $\to J_{ij}$ annealed



→ average over quenched variables → how?

macroscopic

Energy on boundary is second order behind bulk

F is well defined for each subsystem but fluctuates

Average: average over averages over different subsystems.

self-averaging: distribution over subsystems is not broad, mean is equal to most probable

E.g.: free energy and observables are self-averaging for short ranged interaction e.g. $x_1 = e^{\alpha\sqrt{N}}$ $\quad p_1 = 1 - e^{-N}$

$x_2 = e^{\beta N}$ $\quad p_2 = e^{-N}$ $\quad \beta > 1$

$N \to \infty$ average $\langle x \rangle = e^{(\beta-1)N}$

most probable $x^* = x_1$

Disorder vs frustration

e.g.

$$\mathcal{H} = \sum_{\langle ij \rangle} \tau_i \tau_j \sigma_i \sigma_j \qquad (J_{ij} = \tau_i \tau_j, \quad \tau_i = \pm 1)$$

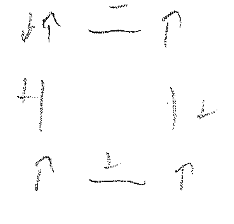
$$\sigma_i' = \tau_i \sigma_i$$

$$Z = \mathrm{Tr}_{\sigma'} \left(e^{\beta J \sum_{\langle ij \rangle} \sigma_i' \sigma_j'} \right)$$

↳ Ising for σ'

↳ disorder does not enter explicitly in the system

- Frustration



? When does the disorder change the universality

Harris criterion

- disorder (weak)
- coupling with temperature through p
- different regions different $T_c(p)$

? Local T_c varies like this → does it alter exponents?

Correlation length ξ → divide system in ξ each own T_{ci}, $\tau = T - T_c$

if $\Delta T_c < |T - T_c| = |\tau|$ all blocks are in the same phase → uniform system

Number of degrees of freedom $\sim \xi^d$

Central limit theorem $\Delta T_c \sim \xi^{-d/2}$

$\xi \sim |\tau|^{-\nu}$

$\Delta T_c < |\tau|$ in the limit of $\xi \to \infty$

$d\nu > 2$ Harris inequality
↳ asymptotically clean

Hyperscaling $\alpha = 2 - d\nu$

$\Rightarrow \alpha < 0$ disorder irrelevant
$\alpha > 0$ — " — relevant $(C \sim |\tau|^{-\alpha})$

<u>Imry-Ma argument:</u>

? Does spontaneous symmetry breaking survive in the presence of a noise?

Suppose all spins are aligned (↑)

→ cube size L ↓

Energy $\sim L^{d-1}$ $(n=1)$ sharp DW

L^{d-2} $(n \geq 2)$ cont. DW

Quenched disorder

$$\mathcal{H} = -J \sum_{\langle x x' \rangle} \sigma_x \sigma_{x'} - \varepsilon \sum_x h_x \sigma_x$$

h_x : independent identically distributed random variable with zero mean. e.g. Gaussian

$\varepsilon > 0$ small constant, disorder density

? can a zero centered random noise have any effect?

Energy due to flip in the domain (Q_L spins)

$$2 \varepsilon \sum_{x \in Q_L} h_x \, .$$

$\Rightarrow \sum_{x \in Q_L} h_x \sim$ Gaussian with variance L^d

fluctuations $\sim L^{d/2}$

If :

$d/2 \geq d-1$ ($n=1$) or $d/2 \geq d-2$ ($n \geq 2$)

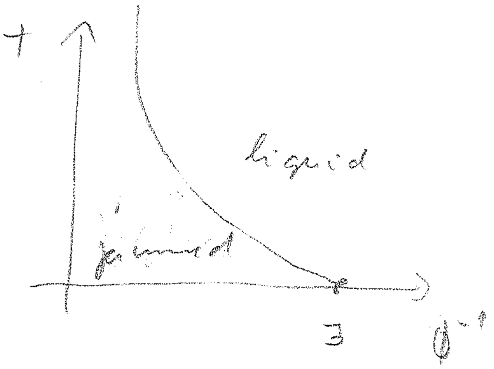
there are arbitrary large domains where we can decrease the energy by flipping the spins due to random fluctuations. This is <u>independent</u> of ε.

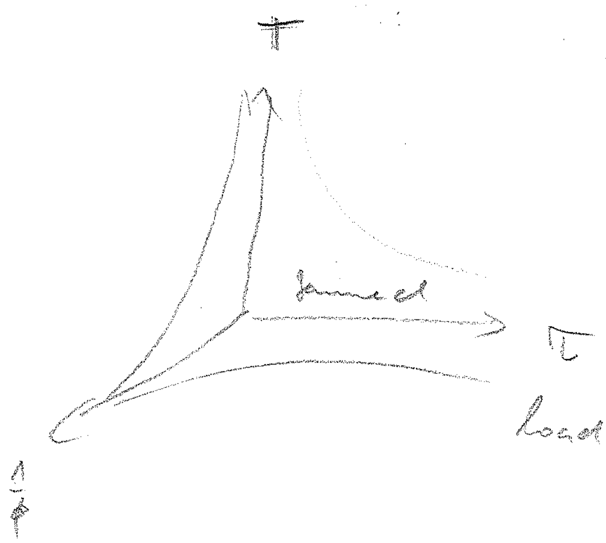
Symmetry breaking unstable when

$d \leq 2$ ($n=1$)

$d \leq 4$ ($n \geq 2$)

Physics of jammed states





Statistics of the jammed state

Volume function $W(q)$ system volume as function of their position

- granular entropy (configurational)

$$S(V) = \lambda \ln \Omega(V)$$

$$\Omega(V) = \int dq \; \delta(V - W(q)) \; \Theta_{jam}$$

Microcan. $V = W(q)$

Jammed Θ_{jam} → constraint

$$\Theta_{jam} = \prod_{ij=1}^{N} \Theta(|r_i - r_j| - 2R) \times \delta(\text{force balance})$$

$$P_{mic}(q) = \Omega(V)^{-1} \, \delta(V - W(q)) \, \Theta_{jam}$$

$$X^{-1} = \frac{\partial S(V)}{\partial V} \qquad \frac{1}{T} = \frac{\partial S(E)}{\partial E}$$

compactivity

X large fluffy

small hard to shrink

$$P_{can}(q) = \frac{1}{Z} e^{-W(q)/X} \cdot \Theta_{jam}$$

$$Z = \int dq \; e^{-W(q)/X} \; \Theta_{jam}$$

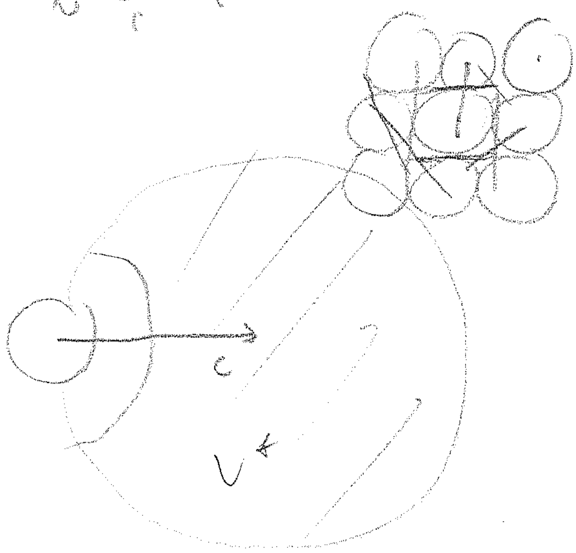
	Gran. Mat.	Hard-Sphere	Spin-Glass
Thermo. desc.	$W(q)$ volume function	$S(\rho(v))$ density functional	$\mathcal{H}(\sigma)$ Hamiltonian
Lagrange mult.	X compactivity	P pressure	T temperature
Entropy	$S(V)$ Edwards ent.	Σ config. ent.	Σ complexity
Metastable States	Minima of $W(q)$ + jam.	Minima of $S(\rho(v))$	Minima of $\mathcal{H}(\sigma)$ at $T=0$

$$V_0 = \frac{4\pi}{3} R_0^3$$

$$\phi = \frac{V_0}{\frac{1}{N}\sum_i W_i}$$

$W_i = ?$ Voronoi cell z!!!

$P(c,z)$ Voronoi boundary at c



$P_>(c,z)$ Voronoi boundary larger than c in dir. c

$$P(c,z) = -\frac{d}{dc} P_>(c,z)$$

cumulative probability distr.

$$V^* = V_0\left(\left(\frac{c}{R}\right)^3 - 1 + 3\frac{R}{c}\right)$$ ← all $N-1$ particles outside this

$$\bar{\rho} = \frac{1}{\bar{W} - V_0}$$

$$\bar{W} = V_0 + 4\pi \int_R^\infty dc\, c^2 P_>(c,z)$$

(1)

RR 2

$P_>(c,t) = P_B(c) - P_c(c,t)$

$\downarrow$ V^* empty

$\uparrow$ contacting particles outside excursion

$P_B(c) = e^{-\bar{s} V^*(c)}$ $\uparrow$ vol dens

$\bar{s} = \frac{1}{\bar{w} - V_0}$ $\quad V^* = V_0\left(\left(\frac{c}{R}\right)^3 - 4 + 3\frac{R}{c}\right)$

$P_c(c) = e^{-\sigma(z) S^*(c)}$ $\uparrow$ surface dens

$S^*(c) = 2S_0\left(1 - \frac{R}{c}\right)$

$S_0 = 4\pi R^2$

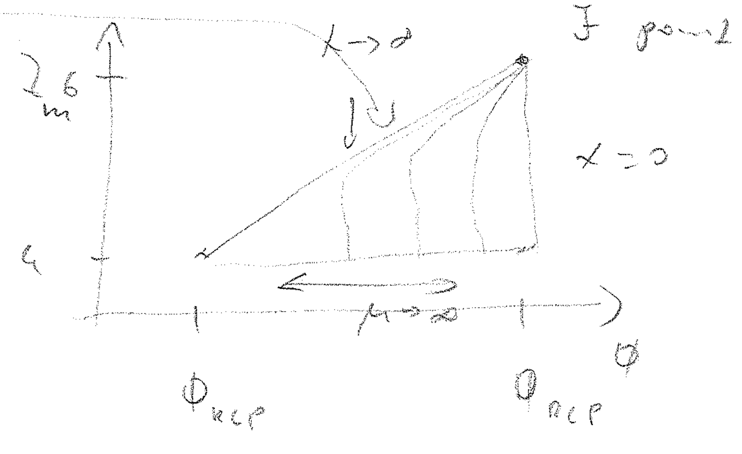
$P_>(c,z) = \exp\left(-\frac{V^*(z)}{\bar{w} - V_0} - \sigma(z) S^*(z)\right)$ $\quad \sigma(z) \approx \frac{z}{4S_0}\sqrt{3}$ $\quad z > 1$ $\quad (2)$

$\left[Z = \int g(z) e^{-\frac{2\sqrt{3}}{zX}} dz \right]$

(1) + (2) $\Rightarrow$ $\boxed{\phi(z) = \frac{V_0}{\bar{w}} = \frac{z}{z + 2\sqrt{3}}}$ equation of state

$X \to 0$ $\quad \bar{Z} = 6$ $\quad$ RCP $\quad \phi_{RCP} = \frac{1}{1 + 1/\sqrt{3}} \approx 0.634$

$X \to \infty$ $\quad \bar{Z} = 4 - 6$ RLP $\quad \Phi_{RLP}^{(z=4)} = \frac{1}{1 + \sqrt{3}/2} \approx 0.536$



$4 \le z_m \le z \le 6$

$\uparrow$ force bearing contacts