Disordered systems 1.

I. Glasses (Philip)

def: solids without a crystalline order that has been quenched from liquid
amorphous solids

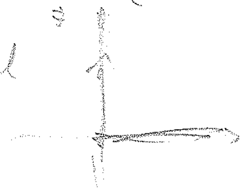
- liquid <-> amorphous solid
  - ~ indistinguishable from structure snapshot
  - ~ no regular order
  - ~ different properties

Hard spheres

N particles
D diameter
d dimension
V volume
V_d volume of sphere in d dim

packing fraction $\varphi = \frac{N \cdot V_d}{V}$

Step potential



T: irrelevant, defines time scale

$E = E_{kin}$

-> "easy" to simulate with MD

Start from $\varphi \ll 1$ compress the system with rate γ

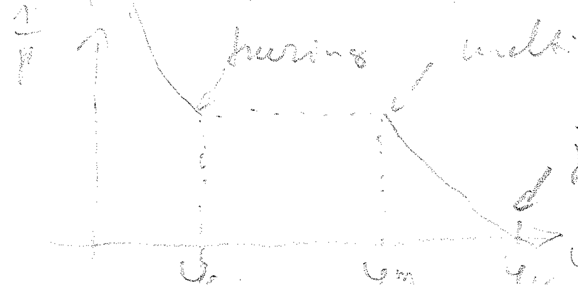
- if $\gamma \to 0$ adiabatic compression -> equilibrium equation of state

def pressure: momentum transfer -> coll / time

$P \sim T$

$p(\varphi) = \frac{P \beta V}{N}$

Start from $\varphi \ll 1$ -> first order phase transition to FCC



In the crystalline state particles oscillate around their pos

↳ here this is purely an entropy driven trans

the ordered state has higher entropy!

(what is the closest packing?

- 1d ooo
- 2d triangular lattice
- 3d FCC (HCP)
- 4d?

⋮)

model systems

- granular materials (athermal)
- colloids
- molecular glasses

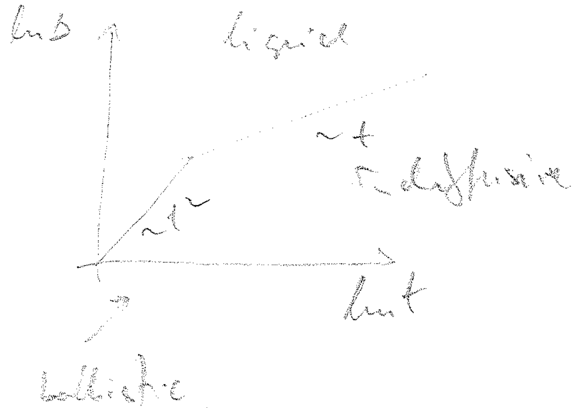
→ demo

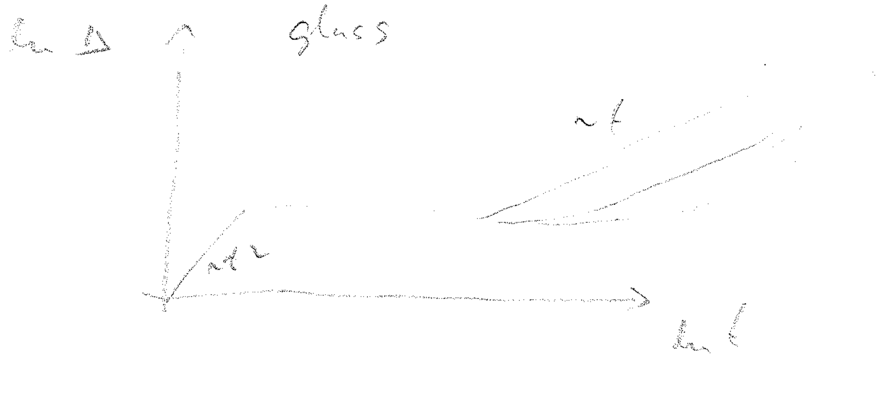
$\bar{\gamma} > 0$ finite rate compression

no crystallization → metastable state → supercooled

Mean Square Displacement (MSD)

$$\Delta(t) = \frac{1}{N}\sum_{i=1}^{N} |x_i(t) - x_i(0)|^2$$





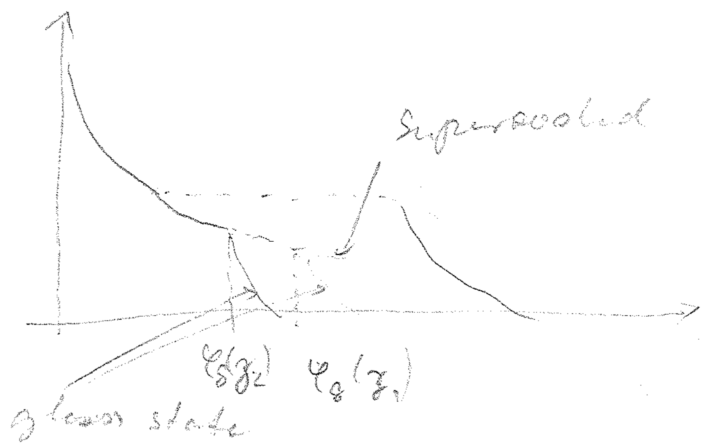
relaxation time

$\tau_\alpha \sim \exp(A/|\phi - \phi^*|^\delta)$ ϕ^*, A, δ constants

at ϕ_g relaxation time gets larger than exp. time

system does not relax to supercooled liquid but remains an amorphous solid.

φ_g depends on γ $\quad \gamma_1 < \gamma_2 \quad \varphi_g(\gamma_1) > \varphi_g(\gamma_2)$



- glass transition → it is not any more supercooled liquid
- at $\varphi_J(\gamma)$ jamming transition, pressure diverges rigid system
  average connectivity $\bar{z} = 2d$ (not for granular)

  $\bar{z} = 2d \quad \mu = 0$

  $\bar{z} = \begin{cases} 3 & d=2 \\ 4 & d=3 \end{cases} \quad \mu = \infty$

  $P(f) \sim f^{\theta} \quad f \to 0$

  $h_{ij} = |x_i - x_j| - D \qquad g(h) \sim h^{-\delta_g}$ $(h \to 0^+)$

  amorphous jammed packings are marginally stable

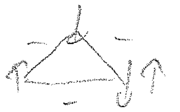
? What is the drive behind freezing
- no symmetry breaking
- no change in the thermodynamic potentials

Random first order theory

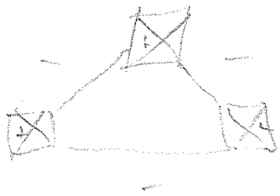
Picture
- metastable states → glasses
- relaxation to one state
- occasional jumps to other one (separated in time)
- cooling → increasing lifetime of metastable state diverging in the thermodynamic limit Kauzmann T_K

In infinite dimension → infinite lifetime → static
→ broken ergodicity

Example : spin glass



6 ground states

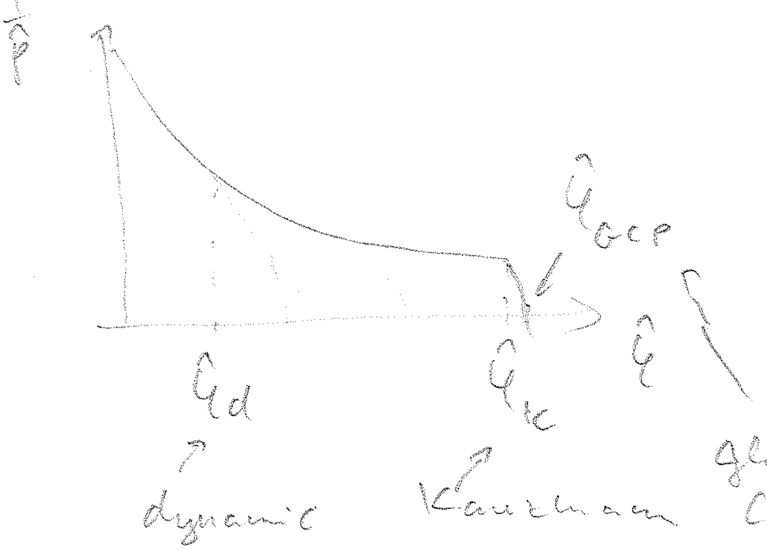


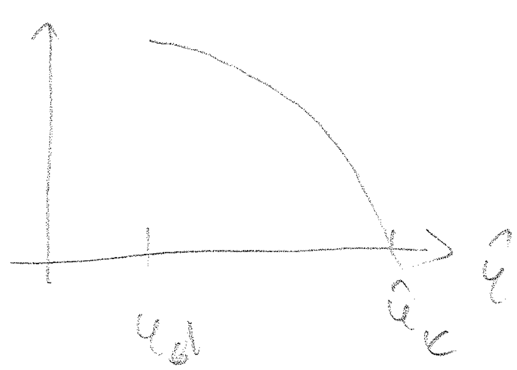
- 6 ground states
- non ergodic at $T \ll J$
system is trapped in one metastable state
- using Metropolis one cannot reach the others

- Theory of hard spheres in infinite dim
Solution!
- there is no first order transition to any crystal

Scaled packing fraction $\hat{\varphi} = \frac{2^d \varphi}{d}$

Scaled reduced pressure $\hat{p} = \frac{p}{d}$





Relaxation time τ_α diverges at $\hat{\varphi}_{\alpha}$

$\tau_\alpha \sim |\varphi - \hat{\varphi}_{\alpha}|^{-\gamma_{MCT}}$ ← analytical

↳ infinite number of glassy states

($N \to \infty$ $t \to \infty$ trapped , $t \to \infty$, $N \to \infty$ hop)

MSD is finite

$\lim_{t \to \infty} \Delta(t) = \Delta_{EA} < \infty$

↳ spheres are blocked in cages

- dynamical transition

(mode coupling for structural glasses)

At $\hat{\varphi}_{d}$

below : particles diffuse , system is ergodic

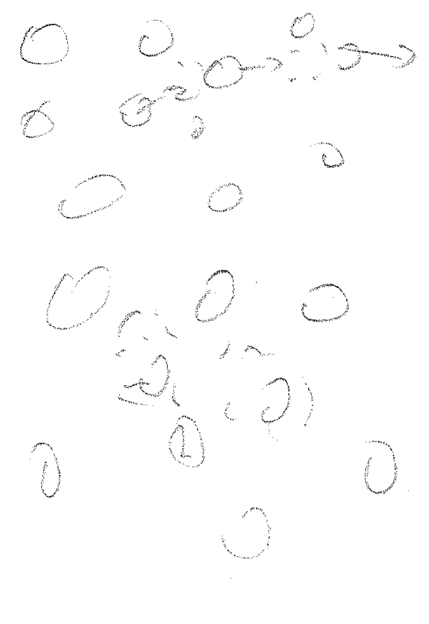
above : exponential number of glassy states in N

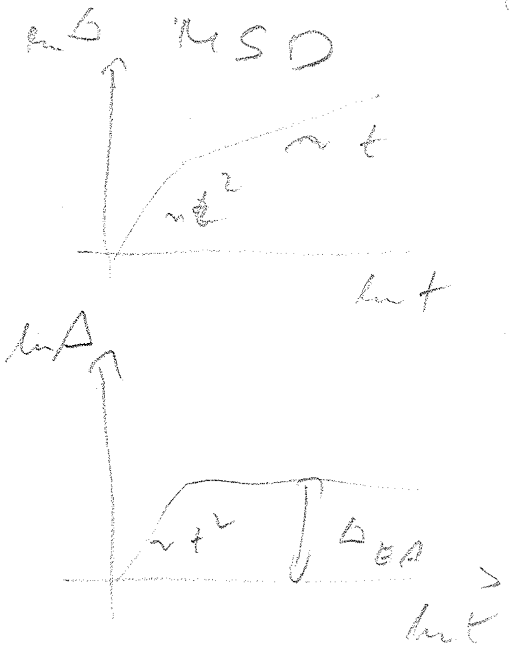
free energy is analytic → no thermodyn. phase transition

Phase space

Real space

Liquid





glass

$\mathcal{N}$ number of glassy states

$$\Sigma = \lim_{N\to\infty} \frac{1}{N} \ln \mathcal{N}$$

Σ configurational entropy

Σ is maximal at $\hat{\varphi}_d$ and for $\hat{\varphi} \to \hat{\varphi}_k$ $\Sigma \to 0$

this is the Kauzmann transition point

for $d \to \infty$ $\hat{\varphi}_k \sim \ln d$ Kauzmann is a real phase transition of Random first order type

Other glassy system: Random coloring problem

(constraint satisfaction problem)

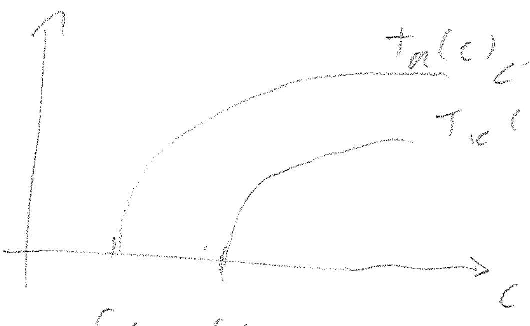
- graph
- q color
- nodes are colored
- no links connecting similar colored nodes

$q > 2$ NP

c is connectivity of the graph (RR, ER)

? in average can it be satisfied?

T



q fixed

$$N \to \infty \quad P(c) = \begin{cases} 1 & c < c_s \\ 0 & c > c_s \end{cases}$$

$$E(s) = \sum_{\langle ij \rangle} \delta_{s_i s_j}$$

Potts model

RR 1

$E(x) = \sum_{\langle ij \rangle} \delta_{x_i x_j}$ Potts model

$c < c_a$ no local search algorithm

- graph partitioning problem

  $N \Rightarrow 2 \times N/2$ K links between them

  minimize K

- oral exam
- solving problem sets } presence
- writing paper

Structural disorder

- polimers, fractals, glasses, quasicristals, amorphous mat. gran. mat., percolation

Disordered magnetic systems:

- Hysteresis, memory effects, Preisach model, domain wall in motion, Barkhausen noise
- Disordered ferromagnets, Griffith phase, spinglasses. Sherrington - Kirkpatrick model, TAP model, replicas, replica symmetry breaking, dropplet
- Localization theory
  - diso. semicond. Anderson's theory
  - Coulomb glass
  - Quantum Hall effect
- Quantum glasses
  - Bose gas, Fischer scaling, strong disorder fixed point

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