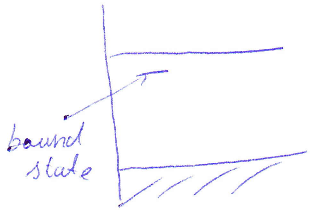
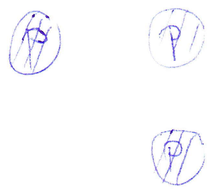
Anderson's localization theory:

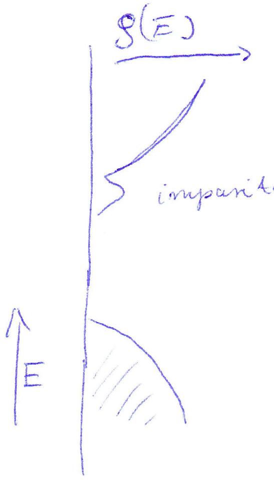
disordered metals
↑(strongly?)

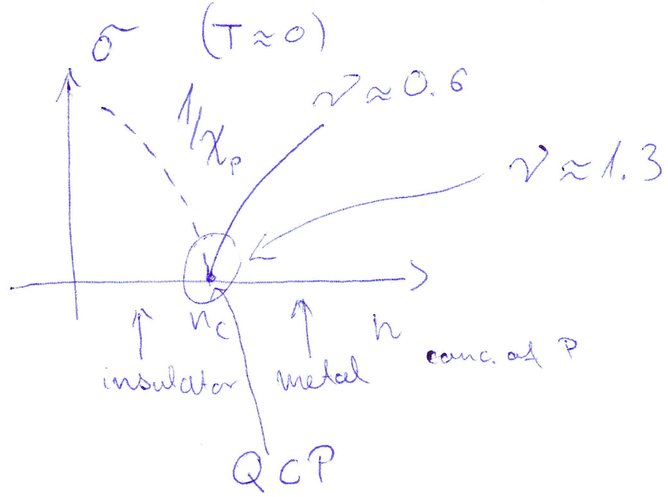
Si:P





increasing P conc.
→ ~~in~~ overlap





$$\sigma_{T=0}(n) \sim (n - n_c)^{\nu}$$

$$\chi_p \sim (n - n_c)^{-\nu} \quad \text{polarizability}$$

Anderson model:

random positions → $\underline{R}_i$

$$\hat{H} = \sum_{\substack{(i,j)\\ \sigma}} \underbrace{t_{ij}}_{\text{hopping}} c^{+}_{i\sigma} c_{j\sigma} + \sum_i \underbrace{\epsilon_i}_{\text{local energy shift}} c^{+}_{i\sigma} c_{i\sigma} + \text{interactions ?}$$

$$\epsilon_i = \underbrace{E_b}_{\text{binding}} + \underbrace{\xi_i}_{\text{random}}$$

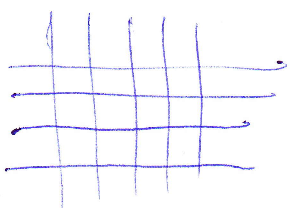
interaction:

① let's neglect long range ia.

$U \sum_i n_{i\uparrow} n_{i\downarrow}$ too high shift

long ranged ia should be considered (in an insulator there's no screening)

Anderson:
- no ia. just disorder
- consider reg. lattice + rnd ~~onsi~~ on-site energies



$$P(\xi) = \begin{cases} \frac{1}{2W} & \text{if } \xi \in [-W, W] \\ 0 & \text{otherwise} \end{cases}$$

W: strength of disorder

simplifications ~ universality

Anderson: all eigenstates of

$$\hat{H} = \sum_{(i,j)} t_{ij} c_i^+ c_j + \sum_i \xi_i c_i^+ c_i \quad \xi_i \leftarrow P(\xi)$$

are localized if $\frac{W}{t} > \left(\frac{W}{t}\right)_c$

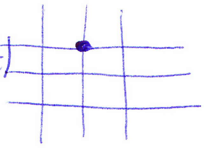
[$t_{ij} = t$ for nearest neighbors, 0 otherwise]

- diagonalization $\Rightarrow c_E = \sum_i \psi_E(i) c_i$ (1)

$$\hat{H} = \sum_E E\, c_E^+ c_E$$

$\uparrow$ wave functions

$i = 0$: $\langle 0 | c_0(t) c_0^+ | 0 \rangle \equiv i G_{00}(t)$



$$P_0(t) = |\langle 0 | c_0(t) c_0^+ | 0 \rangle|^2$$

$\uparrow$ prob of state ~~...~~ remaining at site 0 at time t

$$\rightarrow P_0(t) = |G_{00}(t)|^2 \qquad P_{00} \equiv \lim_{T\to\infty} \frac{1}{T} \int_0^T dt\, P_0(t)$$

(1) $\rightarrow$

$$c_0 = \sum_E \psi_E^*(i=0) c_E$$

$$\rightarrow c_0(t) = \sum_E \psi_E^*(0) c_E e^{-iEt}$$

(time evolution generated by the hamiltonian)

$$\Rightarrow G_{00}(t) = -i \sum |\psi_E(0)|^2 e^{-iEt}$$

$$P_0 = \lim_{T\to\infty} \frac{1}{T} \int_0^T dt\, |G_{00}(t)|^2 = \lim_{T\to\infty} \frac{1}{T} \int_0^T dt \sum_{EE'} |\psi_E(0)|^2 |\psi_{E'}(0)|^2 e^{i(E-E')t}$$

$$\stackrel{?}{=} \sum_E |\psi_E(0)|^4$$

claim: P_0 picks up contributions from localized eigenstates ψ_E

extended state $\psi_E(i) \sim \frac{1}{\sqrt{vol}}$

$$|\psi_E(0)|^4 \sim \frac{1}{|volume|^2}$$

# of states in same energy window $\sim$ Volume

Maximum # of extended states $\sim$ Volume

their contribution $\lesssim Vol \frac{1}{Vol^2} \xrightarrow[\infty]{Vol} 0$

localized states are such that they're not extended

$$|\psi_E(0)|^4 \xrightarrow[\infty]{Vol} \text{finite}$$

added energy and site resolution

$$P_i(E) = \sum_{E'} |\psi_{E'}(i)|^4 \delta(E-E')$$

probability e^- at site „i" stays there and has energy E

$$\int P(E)\, dE = P_i$$

$P(E) \equiv \langle P_i(E) \rangle_i$ Inverse Participation ratio

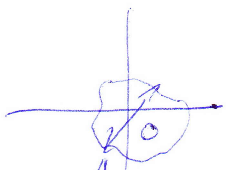
$$P(E) = \sum_{E'} \langle |\psi_{E'}(i)|^4 \rangle_i \, \delta(E-E')$$

claim $P(E) = \langle \sum_i |\psi_E(i)|^4 \rangle_E$ ← in a small window

extended state:

$$\sum_i |\psi(i)|^4 \sim Vol \frac{1}{Vol^2} \longrightarrow 0$$

localized state of size ξ →



$\psi(i) \sim \frac{1}{\sqrt{\xi^D}}$ in a volume of ξ^D

ξ: typical size

$$\Rightarrow \sum_i |\psi(i)|^4 \sim \xi^D \frac{1}{(\xi^{D/2})^4} \sim \frac{1}{\xi^D}$$

inverse participation ratio

$0 \leq IPR \leq 1$

and $IPR \sim 1/\xi^D$

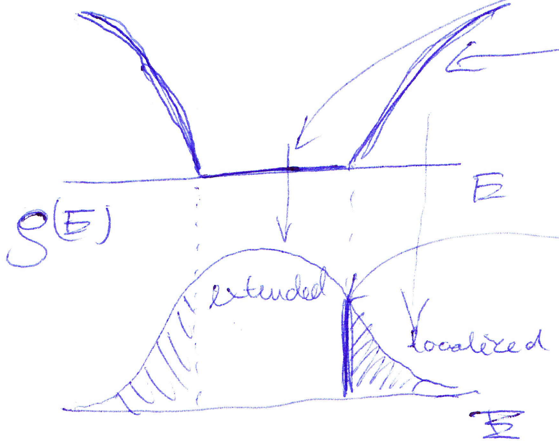
ξ: size of localized states

$\frac{W}{t}$ relatively small (≈ 4)

$IPR(E) \sim \xi^{-3}$

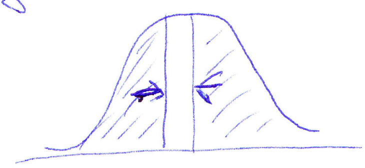
IPR $= 0$ $\rightarrow$ states are delocalized

IPR finite $\rightarrow$ localized states



mobility edge

$\xrightarrow[W/t]{\text{increase}}$

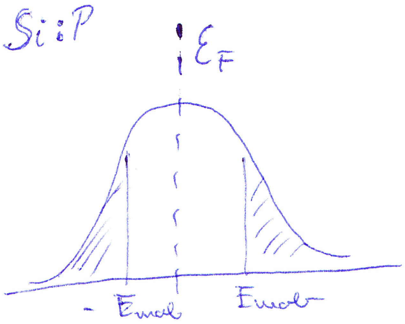


at $(W/t)_c$ they touch

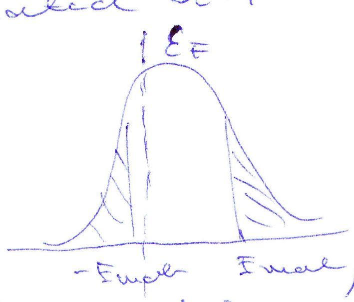
if $(W/t) > (W/t)_c \Longrightarrow$ there is no delocalized state

$\longrightarrow$ we have an insulator

Si:P



compensated Si:P



in reality M-I transition happens when $E_F = E_{mob}$

$$\hat{H} = \overbrace{\sum_{i,j} t_{ij} c_i^+ c_j}^{\hat{T} = \hat{V}} + \overbrace{\sum_i \xi_i c_i^+ c_i}^{\hat{H}_0}$$

$t_{ij} \sim t$ is small $\Rightarrow$ do perturbation theory in t

$t = 0 \Rightarrow \psi_k^0(i) = \delta_{ki}$ energy ξ_k $\qquad |\psi_k^0\rangle = c_k^+ |0\rangle$

$\uparrow$ loc at site k

$$|\psi_k^{(1)}\rangle = |\psi_k^{(0)}\rangle + \frac{1}{\xi_k - \hat{H}_0} \hat{T} |\psi_k^{(0)}\rangle$$

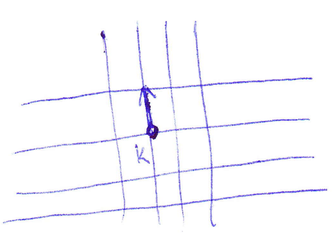
$\Downarrow$

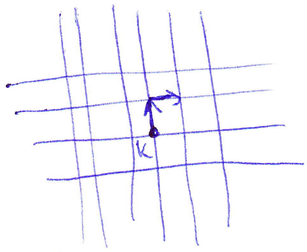
$$\psi_k^{(1)}(i) = \delta_{k,i} + \frac{1}{\xi_k - \xi_i} t_{ik}$$

(no normalization yet!)

$(\dots)$

$$\psi_k^{(2)}(i) = \delta_{k,i} + \frac{1}{\xi_k - \xi_i} t_{ik} + \sum_j \frac{1}{\xi_k - \xi_i} t_{ij} \frac{1}{\xi_k - \xi_j} t_{jk} + \dots$$





denominators $\sim W$
nominators $= t$
amplitude at hopping distance "l"

$$\sim\left(\frac{t}{W}\right)^{l} \sim \exp\left(-\ln\left(\frac{W}{t}\right)\cdot l\right)$$

exponential suppression

• perturbation theory if t converges
$\Longrightarrow$ states are localized $\rightarrow$ $P(E)$ finite

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$$G^{\infty}_{kk}(E+i\delta) = \frac{1}{E+i\delta-\xi^{0}_{k}-\Delta^{\infty}_{kk}(E+i\delta)}$$

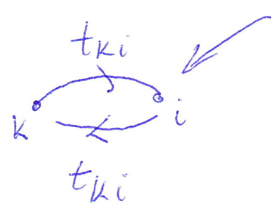
"self-energy"

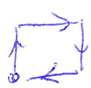
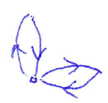
$\left(\cancel{\Delta_{\infty} = \Delta_{\infty}}\right)$

δ : infinitesimal

$$P_k(E) \Rightarrow \frac{G_{kk}(E+i\delta)\,G_{kk}(E-i\delta)}{\pi\,\delta}$$

$$\langle P_k(E)\rangle_k \sim \langle \rho_k(E)\rangle_k \left\langle \frac{1}{1-\frac{\mathrm{Im}\,\Delta_{kk}(E+i\delta)}{\delta}} \right\rangle$$

$\Delta_{kk} =$ 

$\frac{1}{E+i\delta-\xi_i}$ $+$  $+$  $+\dots$

if Pert. th. converges $\Longrightarrow$ $\mathrm{Im}\,\Delta\sim\delta$ $\Longrightarrow$ $\langle P_k\rangle$ = finite

if doesn't : metals $\mathrm{Im}\,\Delta(E)$ – finite $\rightarrow$ $P_k(E)\rightarrow 0$