Disordered Q-magnets, strong disorder

disorder is dangereous even without frustration...

→ destabilizes (Q)-critical point

→ Griffiths phases ...

Consider random transverse field Ising model

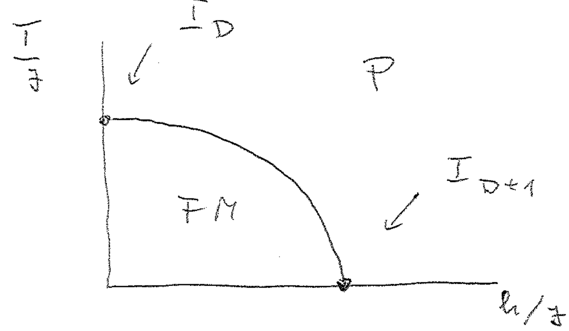
$$\boxed{H = -\sum_{(ij)} J_{ij}\, \sigma_i^z \sigma_j^z - \sum_i h_i\, \sigma_i^z}$$

h_i, J_{ij} random, ≥ 0

ordered case:

$J_{ij} = J \qquad h_i = h$

phase diagram:



Harris: do inhomogenieties influence critical (or Q-critical) behavior

answer: no if $\boxed{\nu \geq \frac{2}{D}}$ H-criterion

this inequality must be satisfied

Proof: 1) assume $\bar{J} = J_c$, but J_{ij} has small, random fluctuations $\sim \delta J$

consider region of volume L^D

$\Rightarrow \bar{J}_L - J_c \sim \delta J / L^{D/2} = \delta J_L$

$\Rightarrow$ induces correlation length $|\delta J_L|^{-\nu} \sim \xi_L$

to be harmless, we need $\xi_L > L$!

$\Rightarrow \quad \text{const}\; L^{\nu D/2} > L \quad \Rightarrow \quad \nu \geq \frac{2}{D}$ ✓ □

2) consider critical system perturbed by some scaling operator

$$\mathcal{H} = \mathcal{H}^* + \int d^D \underline{r}\; g(\underline{r})\; O(\underline{r})$$

$$\langle g(\underline{r})\, g(\underline{r}') \rangle = g_0^2\, \delta^D(\underline{r}-\underline{r}')$$

$$Z = \mathrm{Tr}\, e^{-\mathcal{H}} = \mathrm{Tr}\left\{ e^{-\mathcal{H}^*}\; e^{-\int d^D \underline{r}\; g(\underline{r})\, O(\underline{r})} \right\}$$

replica trick:

$$\langle Z^n \rangle = \mathrm{Tr}\left\{ e^{-\sum_{a=1}^{n} \mathcal{H}_a^*}\; e^{\frac{1}{2} g_0^2 \sum_{a,b} \int d^D r\; O_a(r)\, O_b(r)} \right\}$$

$$\frac{1}{2} g_0^2 \equiv \lambda$$

dimensions $\quad d^D r \sim a^D \qquad O(r) \sim \frac{1}{a^{x_0}}$ ← operator's dim.

$$\lambda = \tilde{\lambda}\; a^{-y_\lambda} \qquad y_\lambda = D - 2x_0 = 2y_0 - D$$

↑ dim less $\qquad y_0 = D - x_0$

disorder coupled to O is relevant if

$$y_\lambda = 2y_0 - D > 0$$

fluctuations in energy $\Leftrightarrow$ $O \Leftrightarrow \mathcal{H}(\underline{r}) \longrightarrow y_0 = y_t = \frac{1}{\nu}$

relevant if $\frac{2}{\nu} > D$, i.e. $\frac{2}{D} > \nu$

□

this carries over to quantum systems !

$\int d^D x \rightarrow \int d^D x \int d\tau$ etc.

Transverse field Ising, QCP:

$D = 1 \rightarrow \nu = 1 < \frac{2}{D} = 2$

$D = 2 \rightarrow \nu \approx 0{,}67 < \frac{2}{D} = 1$

$D = 3 \rightarrow \nu = \frac{1}{2} < \frac{2}{3}$

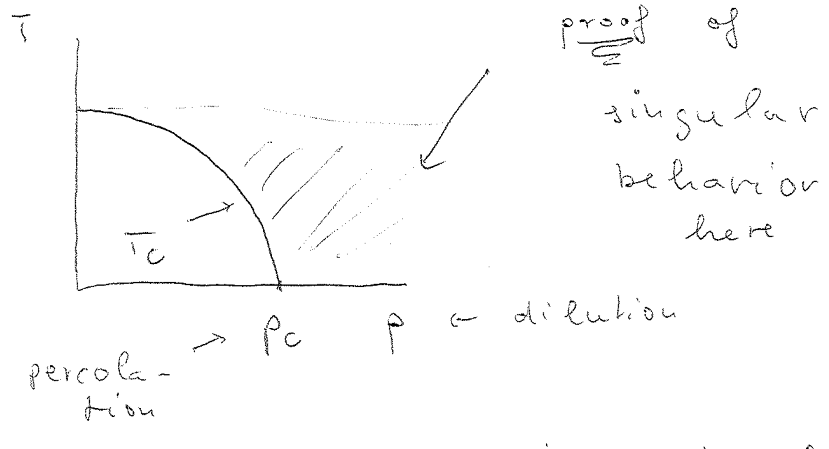
} disorder changes critical behavior! even mean field!

Remark: situation is better for classical critical points ...

new methods needed => FRG = functional RG
RSB = replica symmetry breaking

Griffiths singularities:

consider classical, diluted Ising model

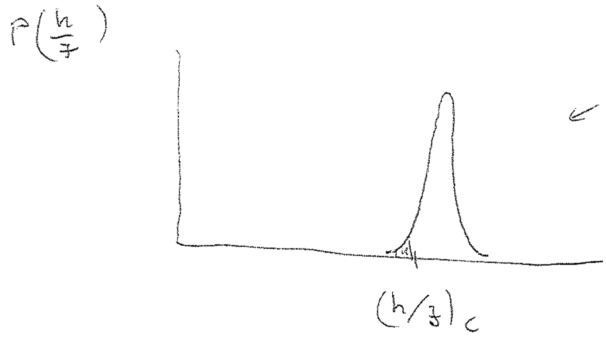


idea: rare big clusters give singular contributions...

=> power laws with non-universal power!

take random J_{ij} TFIM

$$h/J \gtrsim (h/J)_c \rightarrow \text{paramagnetic}$$

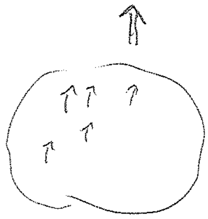


← with a small probability $p < 1/2$ bond is ferromagn.

probability of ferromagnetic cluster $\sim L^D$

$$P(L) \sim p^{L^D} \sim e^{-a \cdot L^D}$$

ground states are $|\Uparrow\rangle$ and $|\Downarrow\rangle$



h induces tunneling $|\Uparrow\rangle \rightarrow |\Downarrow\rangle \sim e^{-b L^D}$

$$\Rightarrow \text{splitting} \quad \omega \sim \omega_0 e^{-b L^D}$$

$$\Rightarrow \quad \chi''_{zz}(\omega) \sim \sum_L P(L)\, \delta\left(\omega - \omega_0 e^{-b L^D}\right)$$

$$x \equiv \omega_0 e^{-b L^D} \quad \Rightarrow L^D = \frac{1}{b} \ln \frac{\omega_0}{x}$$

$$\sim \int dx \, \frac{1}{x} \, e^{-\frac{a}{b} \ln \frac{\omega_0}{x}} \, \delta(\omega - x) \sim \frac{1}{\omega^{1-a/b}}$$

$$\boxed{\chi''_{zz}(\omega) \sim \frac{1}{\omega^{1-a/b}}}$$

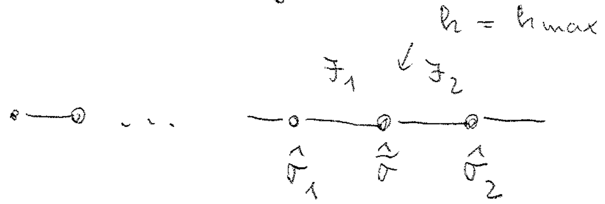
↑ non-universal! changes continuously!

Strong disorder RG (D.S. Fisher)

assume. $P(J)$ $P(h)$ wide

R.G. eliminate strongest fields / bonds

elimination of fields:



$h \gg J_1, J_2, \ldots$

$\Rightarrow$ perturbation theory in J_1, J_2

h polarizes $\tilde\sigma$ along x : $|\Rightarrow\rangle$

$J_1 = J_2 = 0$ $\rightarrow$ cluster states $|\sigma_1 \Rightarrow \sigma_2\rangle$

energy $= -h$

degenerate perturbation theory

$$\langle \sigma_1 \sigma_2 | H_{eff} | \sigma_1' \sigma_2' \rangle = \langle \sigma_1 \Rightarrow \sigma_2 | (\hat J_1 + \hat J_2) \underbrace{\frac{1}{E_0 - \hat H_0}}_{\simeq -\frac{1}{2h}} (\hat J_1 + \hat J_2) | \sigma_1' \Rightarrow \sigma_2' \rangle$$

flips $\tilde\sigma_x \to -\tilde\sigma_x$

$$\cong -\frac{1}{2h} \langle \sigma_1 \Rightarrow \sigma_2 | \underbrace{(\hat J_1 + \hat J_2)^2}_{\text{cst} + 2 J_1 J_2 \sigma_1^z \sigma_2^z} | \sigma_1' \Rightarrow \sigma_2' \rangle$$

$$H_{eff}^{12} = - J_{12} \sigma_1^z \sigma_2^z \qquad J_{12} = \frac{J_1 J_2}{h} \ll h, J_1, J_2$$

$$\boxed{\begin{array}{l} \text{spin elimination} \Rightarrow \tilde J = \dfrac{J_1 J_2}{h} \\ \text{bond elimination} \Rightarrow \tilde h = \dfrac{h_1 h_2}{J} \end{array}}$$

Remark: duality!

$$\sigma^z_e = \prod_{i\le e} \tau^x_i \quad \Rightarrow \quad \sigma^z_{e-1}\sigma^z_e = \tau^x_e$$

$$\sigma^x_e = \tau^z_e \, \tau^z_{e+1}$$

$\{J_e\} \longleftrightarrow \{h_e\}$, transformation is the same ...

R.G. variables:

- energy cut-off $\Omega_0 \to \Omega$
- scaling variable $b \equiv \ln \frac{\Omega_0}{\Omega}$
- logarithms of couplings

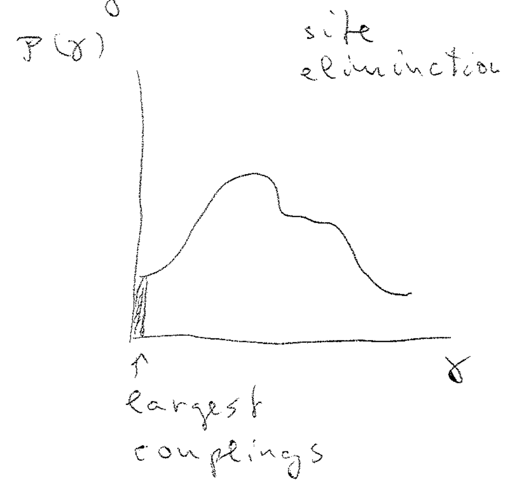
$$\gamma = \ln \Omega/h \qquad \mathcal{H} \equiv \ln \Omega/J$$

$$P_0(J) \sim Q_0(\mathcal{H}) \quad \overset{R.G.}{\Longrightarrow} \quad Q(\mathcal{H}, b) = Q(\mathcal{H}) \quad \longleftrightarrow \quad P(J, b)$$

$$P_0(h) \sim P_0(\gamma) \quad \Longrightarrow \quad \mathcal{P}(\gamma) = \mathcal{P}(\gamma, b) \quad \ldots$$

distribution of effective couplings

number of sites reduced!



$b \to b + db$ shifts

$$\gamma \to \gamma - db$$

$$N_{sites} \to N_{sites}\left(1 - db\, \mathcal{P}(0)\right)$$

bond elimination works same way:

$$N \to N\left(1 - db\left(\mathcal{P}(0) + Q(0)\right)\right) = N'$$

site elimination:

$$\tilde{f} = \frac{f_1 \cdot f_2}{h} \Rightarrow \tilde{\mathcal{H}} = \mathcal{H}_1 + \mathcal{H}_2$$

$$= \Omega \; !$$

bonds: $\tilde{\gamma} = \gamma_1 + \gamma_2$

introduce $\mathcal{N}_b(\mathcal{H}) \equiv \mathcal{N} \cdot Q(\mathcal{H})$

$$\mathcal{N}_b'(\mathcal{H}) = \underset{\text{shift}}{\mathcal{N}_b(\mathcal{H}+db, b)} + \underbrace{db\, \mathcal{N}_{site}(\gamma=0)}_{\#\text{ of eliminated sites}} \iint d\mathcal{H}_1\, d\mathcal{H}_2 \,\cdot$$

$$\cdot \Big\{ \underset{\text{added bond}}{\delta(\mathcal{H}-\mathcal{H}_1-\mathcal{H}_2)} - \underset{\text{removed}}{\delta(\mathcal{H}-\mathcal{H}_1)} - \delta(\mathcal{H}-\mathcal{H}_2) \Big\} \; Q(\mathcal{H}_1)\, Q(\mathcal{H}_2)$$

$$= \mathcal{N} \cdot \Big(Q(\mathcal{H}) + db \cdot \partial_{\mathcal{H}} Q(\mathcal{H}, b) \Big)$$

$$+ db \cdot \mathcal{N} \cdot P(\gamma=0) \Big[(Q * Q)(\mathcal{H}) - 2\, Q(\mathcal{H}) \Big]$$

$$Q(\mathcal{H}, b+db) = \frac{\mathcal{N}_b'(\mathcal{H})}{\mathcal{N}'} = \frac{Q(\mathcal{H})}{1 - db\,(P(0)+Q(0))} +$$

$$+ db \Big\{ \partial_{\mathcal{H}} Q(\mathcal{H}) + P(0)\, (Q*Q)(\mathcal{H}) - 2\, P(0)\, Q(\mathcal{H}) \Big\}$$

$$\boxed{\begin{aligned} \partial_b Q(\mathcal{H}, b) &= \partial_{\mathcal{H}} Q(\mathcal{H}, b) + (Q(0) - P(0))\, Q(\mathcal{H}, b) \\ &\quad + P(0) \int_0^{\mathcal{H}} d\tilde{\mathcal{H}}\; Q(\tilde{\mathcal{H}})\, Q(\mathcal{H}-\tilde{\mathcal{H}}) \end{aligned}}$$

same for fields

$$\boxed{\partial_b P(\gamma, b) = \text{" } Q \leftrightarrow P \text{ "}}$$

criticality:

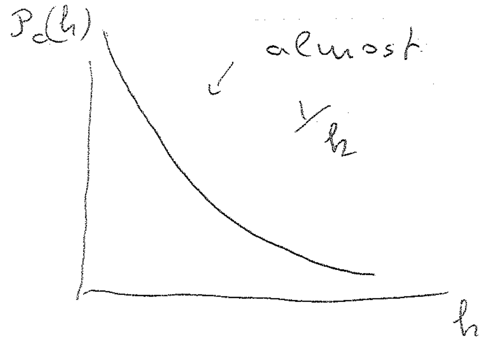
$$P(\gamma) = Y(\gamma)$$
$$Q(\kappa) = Y(\kappa) \quad \leftarrow \text{identical! (duality!)}$$

asymptotic solution:

$$\boxed{P_c(\gamma) = \frac{1}{b} e^{-\gamma/b}}$$

$b \to \infty$: gets broader and broader !?

$$P_c(h)\,dh \sim dh \frac{1}{h^{(1-1/b)}}$$
$$\gamma = \ln \frac{\Omega}{h}$$



Off criticality:

$$\boxed{P(\gamma) = \frac{1}{b} \tilde{Y}\left(\frac{\gamma}{b}, r \cdot b\right)} \qquad \boxed{Q(\kappa) = \frac{1}{b} \tilde{Y}\left(\frac{\kappa}{b}, -r \cdot b\right)}$$

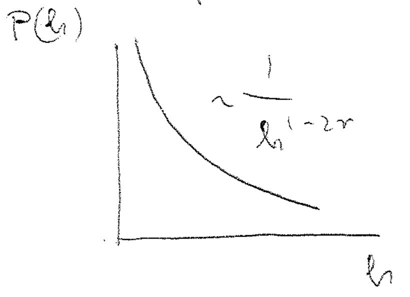
$$\tilde{Y}(x, y) = \frac{2y}{1-e^{-2y}} \exp\left(-x \frac{2y}{1-e^{-2y}}\right)$$

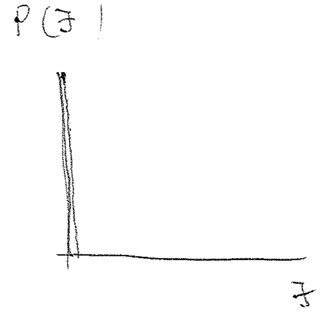
r controls phase transition !

$r > 0$ $\quad b \to \infty$

$$P(\gamma) \to 2r\, e^{-2r \cdot \gamma} \Rightarrow \boxed{P(h) \sim \frac{1}{h^{1-2r}}} \; !$$
$$Q(\kappa) \to \epsilon\, e^{-x \cdot \epsilon} \Rightarrow Q(J) \sim \delta(J)$$

random paramagnet:





$\Rightarrow$ classical phase !

Length scales!

$$\frac{dN}{db} = -N(P(0)+Q(0)) = -\frac{2N}{b} \quad \Rightarrow \quad N \sim \frac{1}{b^2}$$

at criticality

separation of spins: $a \sim \frac{N_0}{N} \sim b^2 = \ln^2 \frac{\Omega_0}{\Omega}$

cross-over scale:

$$r \cdot b \simeq 1 \quad \Rightarrow \quad b \simeq \frac{1}{r} \quad \Rightarrow$$

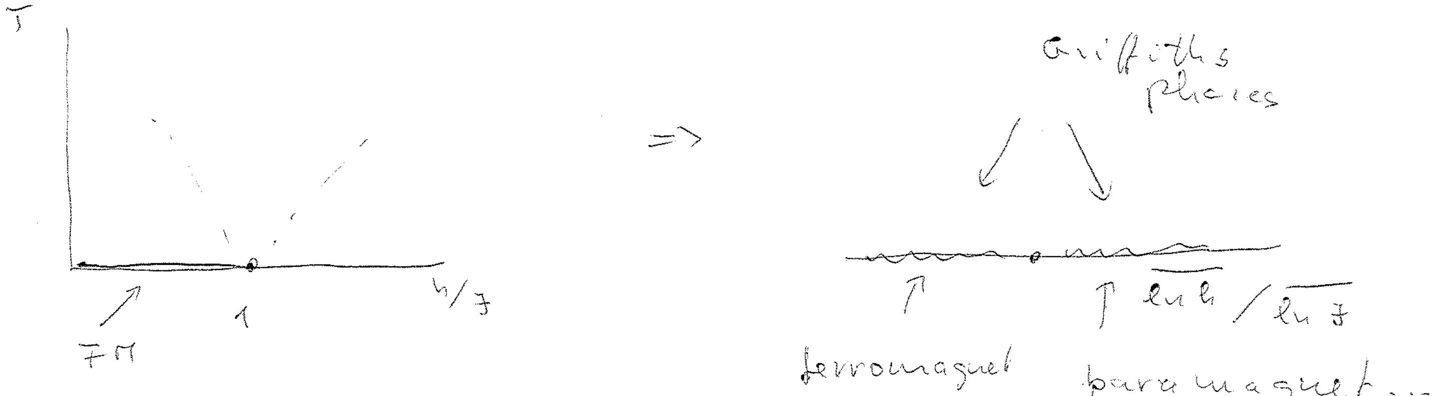
$$\boxed{\xi \sim \frac{1}{r^2}} \quad \Rightarrow \quad \boxed{\nu = 2}$$

saturates Harris!

Thermodynamics $\rightsquigarrow$ power law with non-univ. powers!

$\sim$ "Griffiths" phases

orig diagram



universal powers at criticality:

e.g. $\boxed{M \sim |r|^{\beta}} \leftarrow \quad \beta = 2-\mu, \quad \mu = \frac{\sqrt{5}+1}{2}$