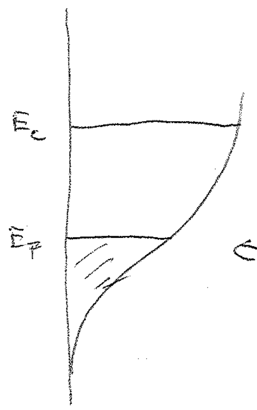
Conducting Anderson insulators:

consider Anderson insulator



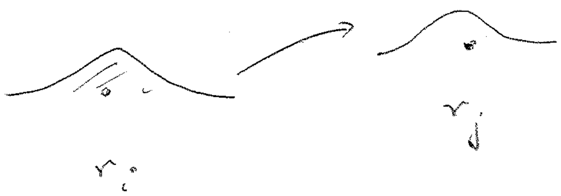
← mobility edge

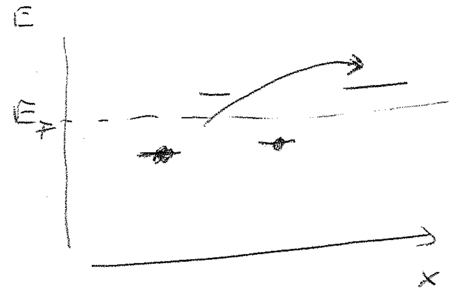
← localized states

$\sigma(T) = ?$

ξ : localization length

Variable range hopping





transition matrix element:

$$|M_{ij}|^2 \propto e^{-A\, r_{ij}/\xi}$$

Boltzmann weight

$$e^{-(\epsilon_j - \epsilon_i)/k_B T}$$

$$\text{rate} \propto W_{ij} \propto |M_{ij}|^2 e^{-(\epsilon_j - \epsilon_i)/k_B T}$$

Q: what is the most likely jump distance?

$\Delta_R \equiv$ level spacing in volume R^D

$$\Delta_R = \frac{1}{\rho(0) R^D} \sim \epsilon_j - \epsilon_i$$

$$W(R) \propto \exp\left\{ -A\frac{R}{\xi} - B\frac{1}{\rho R^D}\frac{1}{k_B T} \right]$$

minimize $\Rightarrow$

$$\frac{A}{\xi} - \frac{B\cdot D}{\varrho(0)}\frac{1}{R^{D+1}}\frac{1}{k_BT} = 0 \quad \Rightarrow$$

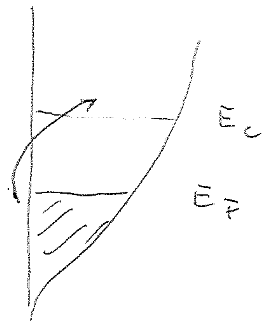
$$\tilde{R} = \left(\frac{BD}{A}\frac{\xi}{\varrho\, k_BT}\right)^{\frac{1}{D+1}}$$

$$\Rightarrow \quad W(\tilde{R}) \propto \exp\left\{-c\cdot\left(\frac{1}{\varrho(0)\,\xi^D\,k_BT}\right)^{\frac{1}{D+1}}\right\} = \exp\left\{-c\left(\frac{\Delta_\xi}{k_BT}\right)^{\frac{1}{D+1}}\right\}$$

$D=3$ dimension $\quad W \propto \boxed{\sigma_M(T) \propto e^{-T_0^{1/4}/T^{1/4}}}$

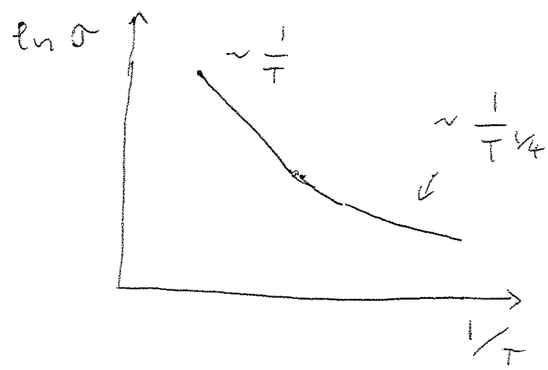
Mott's variable range formula ✓

Remark: activated behavior is also possible



$$\boxed{\sigma_a(T) \sim e^{-(E_c-E_F)/k_BT}}$$

experiment:



<u>Coulomb interaction</u>, <u>Coulomb gap</u>:

simple model:

$$E = \sum_i \epsilon_i n_i + \sum_{ij}\frac{e^2}{\epsilon r_{ij}} n_i n_j$$

$n_i = 0,1$ $\sim$ spin model

ϵ_i random

stability of ground state:

a) $\Delta E \geq 0$ upon adding / removing e^-

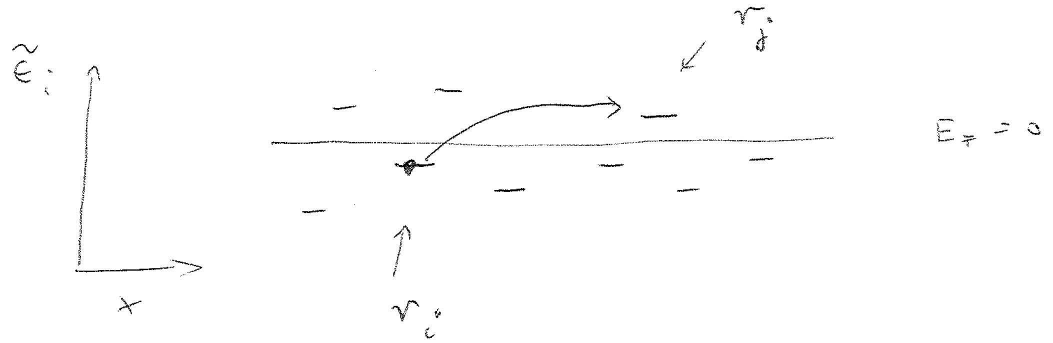
$$\tilde{\epsilon}_i \equiv E(n_i = 1, n_{k\neq i} \text{ fixed}) - E(n_i = 0, n_{k\neq i} \text{ fixed})$$

$$= \epsilon_i + \sum_{k\neq i} \frac{e^2}{\epsilon \cdot r_{ik}} n_k$$

in the ground state: $\tilde{\epsilon}_i > 0 \Rightarrow n_i = 0$

$\tilde{\epsilon}_i < 0 \Rightarrow n_i = 1$

b) stability against e^--hole excitations



$\Delta E_{i\to j} = ?$

initial occupancies: $n^0_{k\neq i,j}$, $n^0_i = 1$, $n^0_j = 0$

final: $\{n^0_{k\neq i,j}\}$, $n_i = 0$, $n_j = 1$

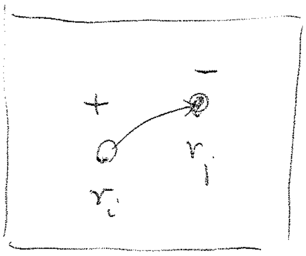
$$E = \sum_{k\neq i,j} \epsilon_k n_k + \epsilon_i n_i + \epsilon_j n_j + \sum_{\substack{(k,l)\\ k,l\neq i,j}} \frac{e^2}{\epsilon r_{kl}} n_k n_l$$

$$+ \sum_{k\neq i,j} \left(\frac{e^2}{\epsilon r_{ik}} n_i n_k + \frac{e^2}{\epsilon r_{jk}} n_j n_k \right) + \underbrace{\frac{e^2}{\epsilon r_{ij}} n_i n_j}_{= 0}$$

$$\Delta E_{i\to j} = \epsilon_j + \underbrace{\sum_{k\neq i,j} \frac{e^2}{\epsilon r_{kj}} n_k^0}_{\substack{\sum_{k\neq j} \frac{e^2}{\epsilon r_{kj}} n_k^0 - \frac{e^2}{\epsilon r_{ij}} \\ n_i^0 = 1\ !}} - \epsilon_i - \underbrace{\sum_{k\neq i,j} \frac{e^2}{\epsilon r_{ik}} n_k^0}_{\substack{\sum_{k\neq i} \frac{e^2}{\epsilon r_{ik}} n_k^0 \\ n_j^0 = 0}}$$

$$\boxed{\Delta \tilde{E}_{i\to j} = \tilde{\epsilon}_j - \tilde{\epsilon}_i - \frac{e^2}{\epsilon r_{ij}}}$$

interaction of e^- - hole !



$$(*) \quad \boxed{\Delta E_{i\to j} = \tilde{\epsilon}_j - \tilde{\epsilon}_i - \frac{e^2}{\epsilon r_{ij}} \geq 0} \quad ! \quad \forall (i,j)$$

Now take $\tilde{\rho}(\tilde{\epsilon})$ ← DOS of M.F. energies

measured by STM

Statement: $\boxed{\tilde{\rho}(0) = \emptyset}$

<u>Proof</u>: assume $\tilde{\rho}(0)$ = finite

consider volume R^D → level spacing

$$\sim \delta \tilde{E} \approx \frac{1}{\tilde{\rho}(0) \cdot R^D}$$

For R sufficiently large

$$\delta\epsilon \sim \frac{1}{R^D} < \frac{e^2}{\epsilon R} \quad \Rightarrow \quad \exists \quad (i,j)$$

such that $\quad \Delta E_{i \to j} < 0 \quad \ldots$

Assume now : $\boxed{\tilde{\rho}(\tilde{\epsilon}) = C\,|\tilde{\epsilon}|^{\alpha}}$

α is such that $(*)$ is saturated !

consider volume R^D $\rightarrow$ compute probability

that $\exists$ $\tilde{\epsilon}_i, \tilde{\epsilon}_j$ pair with $\underset{\tilde{\epsilon}'}{\tilde{\epsilon}_j} - \underset{\tilde{\epsilon}}{\tilde{\epsilon}_i} < \Delta$

$$P(\tilde{\epsilon}' - \tilde{\epsilon} < \Delta) = \underset{\tilde{\epsilon}' - \tilde{\epsilon} < \Delta}{\int_0^{\infty} d\tilde{\epsilon}' \int_{-\infty}^{0} d\tilde{\epsilon}} \; C^2 R^{2D} |\tilde{\epsilon}|^{\alpha} |\tilde{\epsilon}'|^{\alpha} =$$

$$= \text{cst} \cdot R^{2D} \cdot \Delta^{2(\alpha+1)}$$

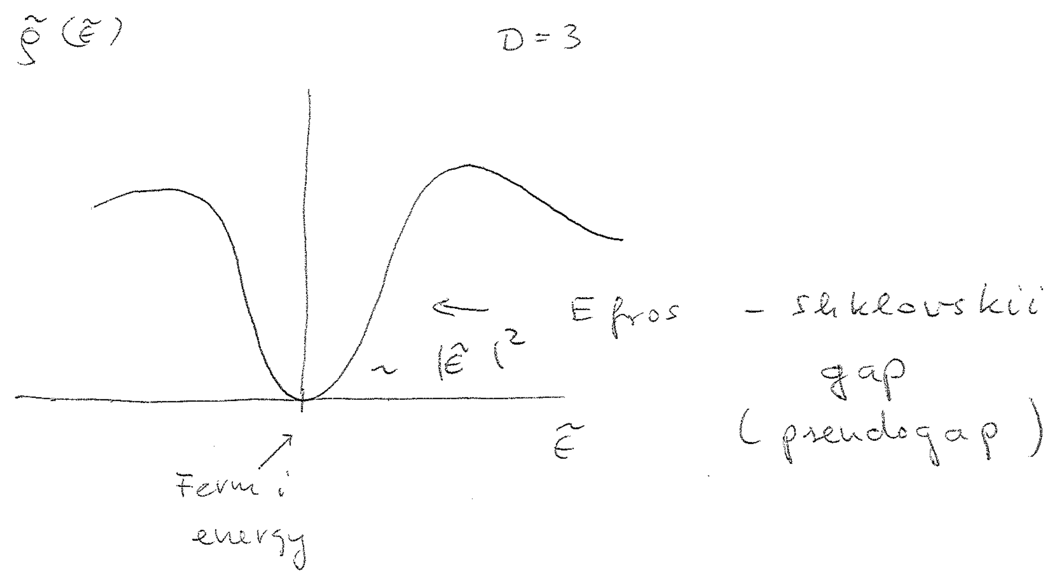
typical excitation energy in volume R :

$\Delta(R)$: $\quad P(\tilde{\epsilon}' - \tilde{\epsilon} < \Delta) \approx 1$

$$\Rightarrow \quad \Delta \sim R^{-\frac{D}{\alpha+1}}$$

must balance Coulomb gain $\sim -\frac{1}{R}$

$$\Rightarrow \quad \alpha + 1 = D \quad \boxed{\alpha = D - 1}$$



surprise: not visible in $\sigma(T)$!

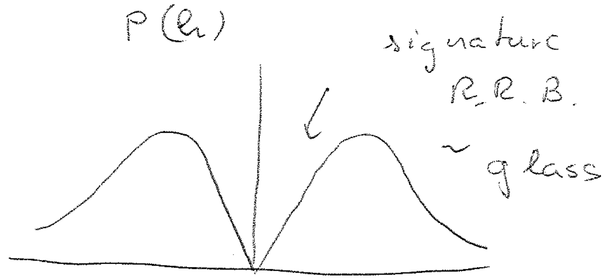
reason $\Delta E = \tilde{\epsilon}_j - \tilde{\epsilon}_i - \frac{e^2}{\epsilon r_{ij}} \approx 0$

reduces the energy of e-h excitations

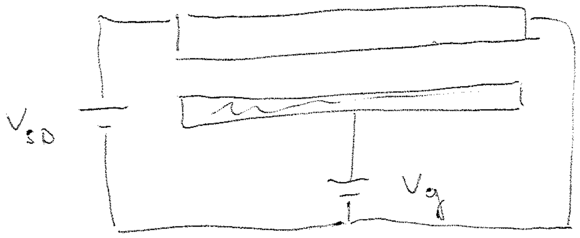
density of states of e-h excitations is unaltered ... $\Leftarrow$

spin language: $\tilde{\epsilon}_i \rightarrow h_i$ local field!

SK model:



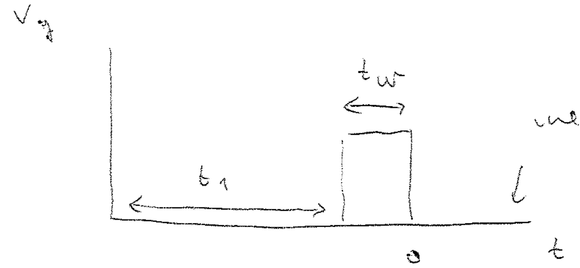
Coulomb glass, memory effects



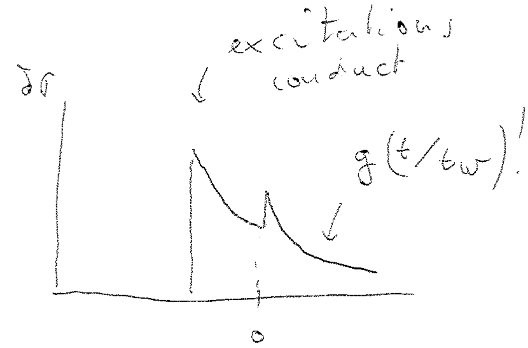
$\Rightarrow \quad V_g = V_g(t)$

$\Rightarrow \quad \delta\sigma(t) = \, ?$

memory effect:



measure $\Rightarrow$



data collapse:

simple minded explanation:

assume response $e^{-\lambda t}$ with $P(\lambda)$

$$\Rightarrow \quad P(t) = \int d\lambda \, e^{-\lambda t} \, P(\lambda)$$

if $\boxed{P(\lambda) = \frac{c}{\lambda}}$ (~1/f noise!!)

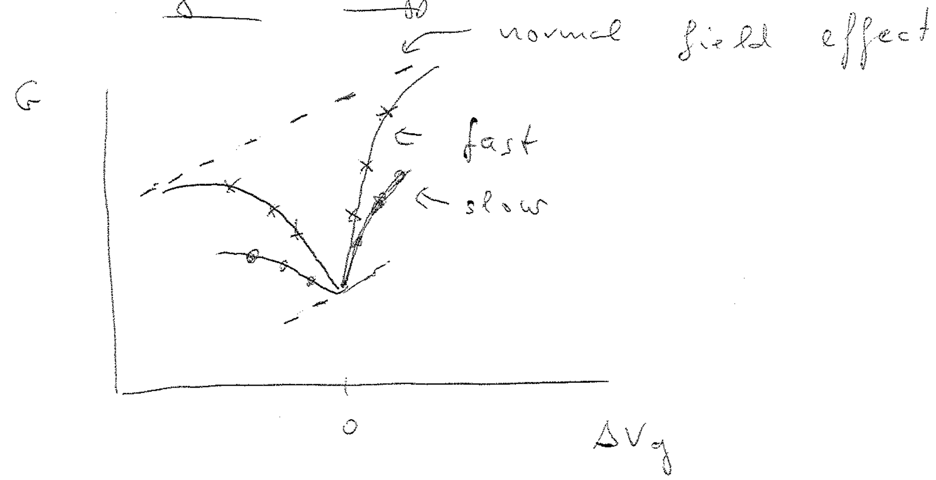
$\lambda \in [\lambda_{min}, \lambda_{max}]$

$$\Rightarrow \quad \boxed{P(t) \approx -\gamma_E - \ln(t \cdot \lambda_{min})}$$

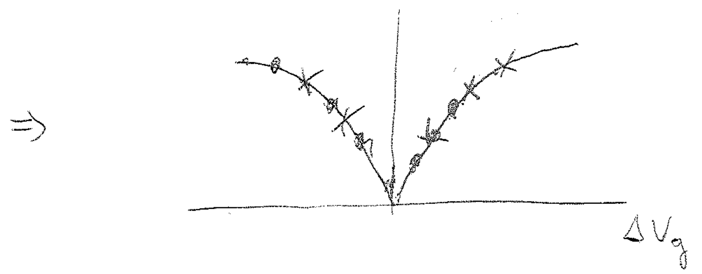
if $t > 1/\lambda_{max}$

$$\boxed{\lambda_{min} \Longleftrightarrow \frac{1}{t_w}}$$

Anomalous field effect

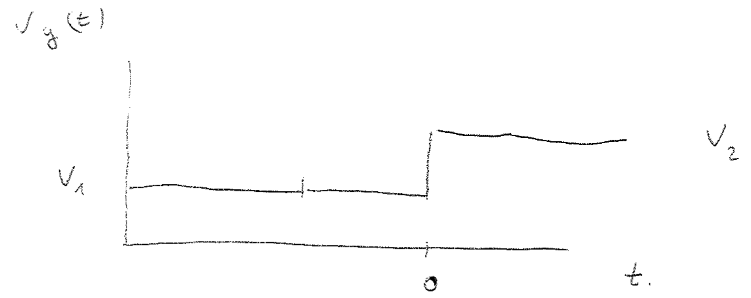


anomalous part can be scaled together



Glass:

- arbitrarily ~~slow~~ relaxations
- 1/f noise
- memory effects



conductance scan at

$t = 0$ $\qquad$ $t \approx 1h$