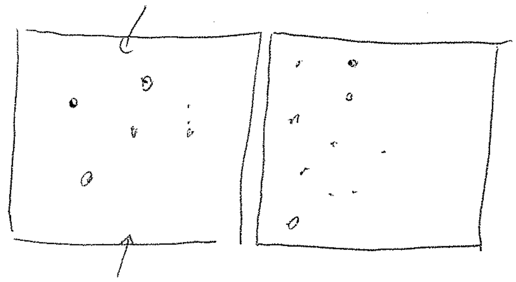
The scaling theory of localitation:

take two slabs (disordered)

make one from them



$G_{LR} = ?$

wave functions $\varphi^a_\epsilon(i) \rightarrow a^+_\epsilon$

$\varphi^b_{\epsilon'}(j) \rightarrow b^+_{\epsilon'}$

$$H_L = \sum_\epsilon \epsilon\, a^+_\epsilon a_\epsilon + \sum_{\epsilon'} \epsilon'\, b^+_{\epsilon'} b_{\epsilon'} + \hat{T}$$

l.h.s.

$$\text{tunneling} = \hat{T} = \sum_{\epsilon,\epsilon'} \left(t_{\epsilon,\epsilon'}\, a^+_\epsilon b_{\epsilon'} + h.c \right)$$

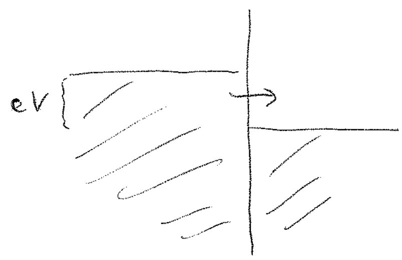
t contains information about $\varphi^{a,b}_\epsilon \sim \sum_{(i,j)\in \text{surface}} (\varphi^a_\epsilon(i))^* \varphi^b_{\epsilon'}$

extended states $t_{\epsilon,\epsilon'} \sim \frac{1}{\sqrt{V_L}} \frac{1}{\sqrt{V_R}} \cdot \sqrt{S}$

localized states $t_{\epsilon,\epsilon'} \sim e^{-L/\xi}$

conductance

L R



rate to go from left to right

$$R = \sum_{\epsilon,\epsilon'} \frac{2\pi}{\hbar} |t_{\epsilon,\epsilon'}|^2\, \delta(\epsilon - \epsilon' + eV)\, f(\epsilon)(1-f(\epsilon$$

$$= eV \frac{2\pi}{\hbar} \langle |t_{\epsilon,\epsilon'}|^2 \rangle\, N_L \cdot N_R$$

$$\sum_\epsilon \Rightarrow \int d\epsilon\, N_L(\epsilon) \ldots$$

$$I = e \cdot R = \frac{e^2}{\hbar} (2\pi)^2 N_L N_R \langle |t|^2 \rangle \cdot V$$

spin

$$\boxed{G = 2 \cdot \frac{e^2}{h} (2\pi)^2 \underbrace{\langle |t|^2 \rangle N_L N_R}_{g}}$$

gradie
$\Delta V = \frac{V}{L} \cdot a$; correct
L^{D-1}

<u>metal</u> $\langle |t^2| \rangle \sim \frac{1}{V_L} \frac{1}{V_R} \cdot S$ $\Rightarrow$ $g \propto S \propto L^{D-1}$

$N_{L,R} \sim V_L, V_R$

<u>insulator</u>: $|t|^2 \sim e^{-L/\xi}$ $\quad g \propto e^{-L/\xi}$

<u>Statement</u>: g <u>measures extension of wave function</u>

assume t is small $\rightarrow$ perturbation theory

assume $V_L = V_R$, $N_R = N_L$

$$\varphi_\epsilon^a \longrightarrow \tilde{\varphi}_\epsilon^a = \varphi_\epsilon^a + \sum_{\epsilon'} t_{\epsilon', \epsilon} \frac{1}{\epsilon - \epsilon'} \cdot \varphi_{\epsilon'}^b + \ldots$$

weight on r.h.s.: $w \sim \sum_{\epsilon'} \frac{t_{\epsilon \epsilon'}^2}{(\epsilon - \epsilon')^2} \sim$

$\rightsquigarrow$ level spacing on r.h.s. $\delta\epsilon = \frac{1}{N}$

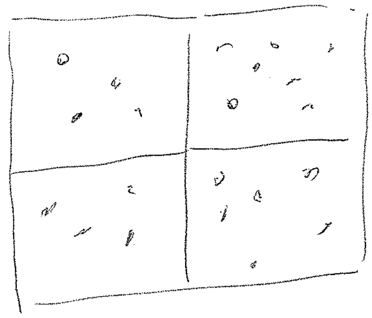
$$\Rightarrow w \sim \frac{1}{\delta\epsilon^2} \langle t_{LR}^2 \rangle = N^2 \langle |t|^2 \rangle = \frac{1}{(2\pi)^2} g$$

- states remain localized on one side, if

$g \lesssim 1$

get extended if $g \gtrsim 1$

one parameter scaling:



join slabs

$g(L) \Leftrightarrow t$ typical

$\Leftrightarrow$ structure of φ $\Rightarrow g(2$

$\sim \varphi(2$

if I know $g(L)$ $\Rightarrow$ determines $g(2L)$

$$g(\overbrace{\alpha \cdot L}^{L'}) = f(g(L), \alpha) \quad \text{(assumption)}$$

$\uparrow \ln \frac{L'}{L}$

$$\Rightarrow \boxed{\frac{\partial \ln g}{\partial \ln L} = \frac{1}{g} \partial_\alpha f(g;\alpha)\Big|_{\alpha=0} = \beta(g)}$$

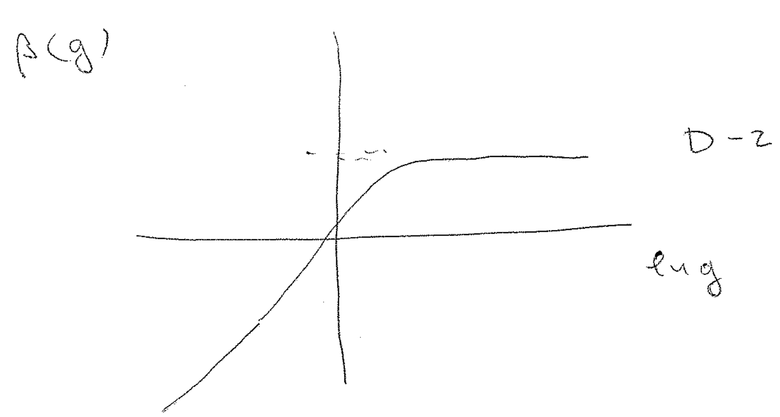
verify:

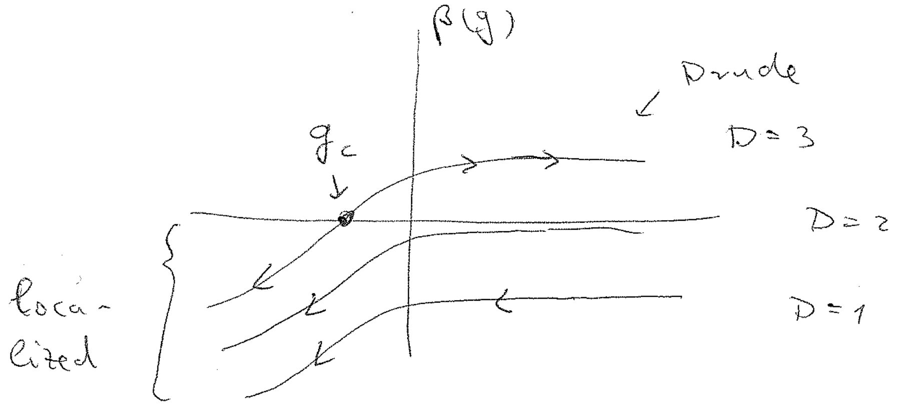
- $g \sim L^{D-2}$ metal

$\ln g = (D-1) \cdot \ln L \to \frac{\partial \ln g}{\partial \ln L} = D-1$ ✓

- $g \sim e^{-L/\xi} \Rightarrow \ln g = -\frac{L}{\xi}$

$\frac{\partial \ln g}{\partial \ln L} = L \frac{\partial}{\partial L}\left(-\frac{L}{\xi}\right) = -\frac{L}{\xi} = \ln g$ ✓





1 and 2D : states are localized !

3D : ∃ critical conductance g

take μ-scopic slab of size L_0 , $g_0 = g(L_0)$

if $g(L_0) = g_0 < g_c \Rightarrow$ insulator

$g_0 > g_c \Rightarrow$ metal

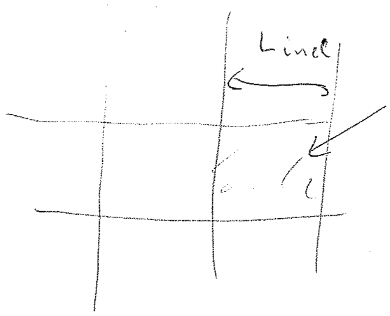
$$g_c \sim 1 \quad \Rightarrow \quad G_c \sim \frac{2e^2}{h} = G_Q \rightarrow \quad R_c \sim \frac{1}{G_Q} \sim 25$$

<u>remarks:</u> this is the conductance of a <u>coheren</u>

piece of metal (no inelastic processes) , T = 0 temperat

finite T → ??

T ≠ 0 inel. processes → τ_{inel} → $L_{inel}(T)$



Q-effects ~ interference inside

$$g(L) \approx \left(\frac{L}{L_{inel}}\right)^{D-1} \cdot \frac{1}{(L/L_{inel})}\, g(L_{inel})$$

$$= L^{D-2} \underbrace{\frac{1}{L_{in}^{D-2}}\, g(L_{inel})}_{\sim\, \sigma(T)}$$

vanishing of critical conductivity :

$D = 3$ $\quad g(0) \leftarrow$ is a function of pressure, concentration etc.

$\quad\quad\quad\quad \shortparallel$

$\quad\quad\quad\quad g_c$

$g(L) = ?$ $\quad \beta(g) \approx \begin{cases} s\cdot(\ln g - \ln g_c) & g < \\ D - 2 = 1 & g > \end{cases}$

$$\ln \frac{L}{L_0} = \int_{\ln g_0}^{\ln g} d(\ln g) \, \frac{1}{\beta(g)}$$

$$\approx \ln \frac{g}{\tilde{g}} + \underbrace{\int_{\ln g_0}^{\ln \tilde{g}} \frac{1}{\ln g - \ln g_0}}_{\propto \ln\left(\frac{\ln(\tilde{g}/g_c)}{\ln(g_0/g_c)}\right)} \approx s \ln$$

$$\Rightarrow \quad g = \frac{L}{L_0} \tilde{g} \, (g_0 - g_c)^s$$

$\approx 1{,}3$

$$\boxed{\sigma \approx \frac{e^2}{h} \frac{1}{L_0} \tilde{g} \, (g_0 - g_c)^s}$$

weak localization corrections:

$g \gg 1 \quad \sim$ weak scattering $\quad \to$ perturbation theory in scattering

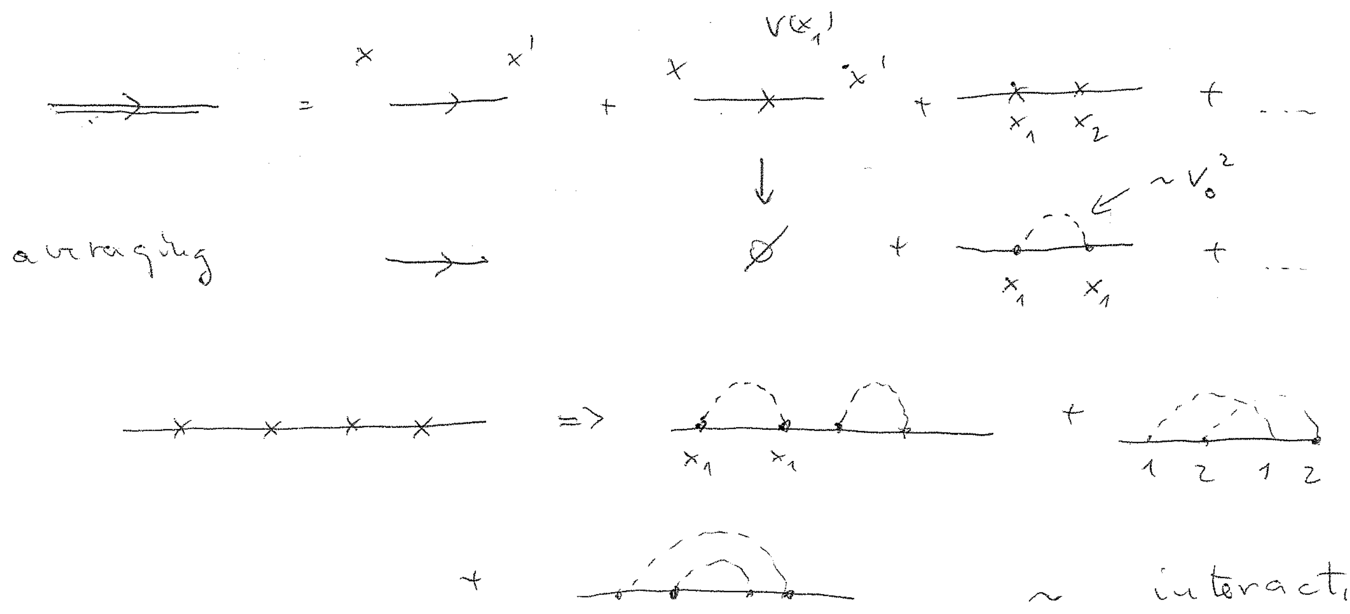
$$H = \sum_\sigma \int d^Dx \left\{ \psi_\sigma^+(x) \left(-\frac{\Delta}{2m} - \mu \right) \psi_\sigma(x) + V(x)\, \psi_\sigma^+(x)\, \psi_\sigma(x) \right\}$$

random potential

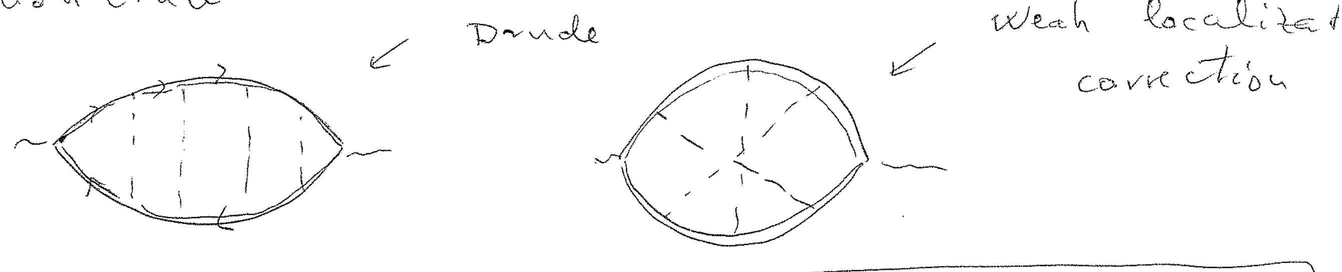
for simplicity, Gaussian $\Rightarrow \langle V(x)\, V(x') \rangle$ disorder average

$$= v_0^2\, \delta^D(x-x') \qquad \text{but } \langle V(x) \rangle = 0 \quad + \text{Wick}$$

real space



conductance



$$\Rightarrow \boxed{\beta(g) = D - 2 - \frac{c_D}{g} + \mathcal{O}\left(\frac{1}{g^2}\right)}$$