A13

The Lagrangian of a one-dimensional continuum is the following:

$$\mathcal{L} = \frac{1}{2}\partial_t u \partial_x u + \frac{\alpha}{6}(\partial_x u)^3 - \frac{\nu}{2}(\partial_x^2 u)^2 \tag{1}$$

(a) Give the S action of the above system!

(b) Express the δS variation of the action (Be careful with the secodn derivative term)!

(c) Bring the variation of the action to the following form $\delta S = \int \mathrm{d}t\, \mathrm{d}x\, M(x,t)\delta u(x,t)$ and express $M(x,t)$ with the field u and its derivatives!

(d) Give the energy density of the system!

B10

An elastic chain of length L is fastened to the ceiling. Its linear density is denoted by λ and the gravitational force points downwards. The aim of this exercise is to describe transverse waves in the rod. Let the displacement of the chain at height z be $u(z,t)$.

(a) Consider a dz segment of the chain at z and show that its vertical position is given by

$$h(z,t) = \int_0^z \mathrm{d}z' \left[1 - \sqrt{1 - \left(\frac{\partial u(z',t)}{\partial z'}\right)^2}\right] \tag{2}$$

According to this show that the action can be expressed as

$$S = \int \mathrm{d}t \int_0^L \mathrm{d}z \left[\frac{\lambda}{2}(\partial_t u)^2 - \int_0^z \mathrm{d}z'\, \lambda g \left(1 - \sqrt{1 - (\partial_{z'} u(z',t))^2}\right)\right] \tag{3}$$

(b) Change the order of the integrations and show that the action can be written as

$$S = \int \mathrm{d}t \int_0^L \mathrm{d}z \left[\frac{\lambda}{2}(\partial_t u)^2 - \lambda g (L-z) \left(1 - \sqrt{1 - (\partial_z u(z,t))^2}\right)\right] \tag{4}$$

(c) Approximate the second term in the action for small displacements and write down the Euler-Lagrange equations.

(d) Look for the solution in form of $u(z,t) = \varphi(z)e^{i\omega t}$! What equation do you get for $\varphi(z)$?

(e) Solve it nuemrically and plot it for different ω-s (Such questions will not arise in the large test)!