Problem 1

In the upper athmosphere μ particles or muons are produced by cosmic rays colliding with molecules, and then these unstable particles are moving with almost constant velocity towards the Earth's surface. The half time of decay for resting μ is $T_{1/2} = 2.2\mu\text{s} = 2.2 \cdot 10^{-6} s$.

a.) Assuming Newtonian mechanics to be correct, what distance would a μ travel (with having velocity $V_\mu \approx c$) until it is expected to decay?

b.) Assuming that muons are produced in an altitude of 10km, what fraction of them would reach the Earth's surface?

We know that Newtonian mechanics fails to describe the above questions. We want to measure the velocity of μ, therefore we perform the following experiment. We have created two identical μ detectors. One is attached to a wheather balloon and is lifted up to $h = 3$km altitude. The other one remains on the surface of Earth. We measure $n_b = 700$ counts at the balloon while only $n_s = 500$ counts on the surface in an hour.

c.) Assuming we know the V_μ velocity of the muons, what is the connection between n_b and n_s?

d.) According to the measured velues, determine the velocity of the μ particles.

Problem 2

At time $t = 0$ two spacecrafts depart from Earth in perpendicular directions with velocities $3/5c$.

a.) Determine the position vectors $r_1(t)$ and $r_2(t)$ of the two spacecrafts as a function of time. (Use a convenient coordinate-system in the reference frame of Earth.)

b.) Let's sit in the reference frame of the spacraft "1". Determine the position vector $r_2'(t')$ of the spacraft "2" in this reference frame.

c.) What is the velocity vector of the 2nd spacecraft in that reference frame? Determine also the direction of this velocity vector.

Problem 3

There are given two 4-vectors with their contravariant coordinates in some inertial system, $a^\mu = (a^0\, a^1\, a^2\, a^3)$ and $b^\mu = (b^0\, b^1\, b^2\, b^3)$. The metric tensor of the Minkowskian spacetime is simply $g_{\mu\nu}$.

a.) Using Einstein's convention, and the metric tensor, express the Minkowski length square of a^μ and the Minkowskian scalar product of a^μ and b^μ.

b.) How we define the covariant coordinates of these 4-vectors? Determine the $a_\mu = (a_0\, a_1\, a_2\, a_3)$ and $b_\mu = (b_0\, b_1\, b_2\, b_3)$ "lower index" coordinates. With the help of these covariant coordinates, express again the Minkowski length square of a_μ and the Minkowskian scalar product of a_μ and b_μ.

c.) As we see, the indices can be lowered by multiplication with the $g_{\mu\nu}$ tensor. The inverse of this manipulation is the "raising" of indices. What tensor $g^{\mu\nu}$ can be used to raise the indices?

Problem 4

Consider the following transformation,

$$\Lambda^{\mu}_{\cdot\,\nu} = \begin{pmatrix} 5/3 & 0 & -4/3 & 0 \\ -4/3 & 0 & 5/3 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

a.) Consider the 4-vector $a^\mu = (1, 1, 0, 0)$. What is its Minkowski length square? Apply the above transformation on this vector. Show that its Minkowski length square is invariant.

b.) Consider the 4-vector $b^\mu = (6, 1, 3, 1)$, and show that its Minkowski lenght square is also invariant.

c.) Show in general, that the transformation $\Lambda^{\mu}_{\cdot\nu}$ is a Lorentz transformation.

d.) Express the components b_μ. Express also the transformed b'_μ components.

e.) Determine the appropriate form of Λ that transforms the lower-index vectors, $b'_\mu = \Lambda_{\mu}^{\cdot\nu} b_\nu$.

f.) Show that $a^\mu b_\mu$ remains invariant.

g.) Show directly that $\Lambda_{\rho}^{\cdot\mu} \Lambda^{\rho}_{\cdot\nu} = \delta^{\mu}_{\cdot\nu}$, where $\delta^{\mu}_{\cdot\nu}$ stands for the Kronecker-delta.