Lecture 1, Problem 4

February 12, 2020

Problem:

Einstein's famous train was hit by two lightnings at the two ends. According to the workers who worked on the fields nearby, the two events happened exactly at the same time. The velocity of the train is V.

a.) Draw the world lines of the trains endpoints in the Minkowski plane. Mark the two lightning hits.

b.) Draw the world lines of the lightnings' light in the figure.

c.) At the middle of the train an observer is traveling. Draw her/his world line in the figure!

d.) Which lightning happend earlier from the observers point of view?

e.) How large is this time difference, if the (resting) length of the train is 100meters, and its velocity is $V = 180\text{km/h}$?

Solution:

The lighning at the front of the train happened earlier according to the observer on the train.

Let L be the physical length of the train. This is the length that the observer *on the train* sees.

The contracted length for the observer on the ground *lab frame* is

$$L' = L\sqrt{1 - v^2/c^2} \tag{0.1}$$

Let t_1 and t_2 be the time coordinates in *lab frame* when the light after the lightning arrives at the observer in the middle of the train.

We have

$$t_1 = \frac{L'/2}{c - v} \qquad t_2 = \frac{L'/2}{c + v} \tag{0.2}$$

The time difference in the lab frame is

$$t_1 - t_2 = \frac{L'v}{c^2 - v^2} = \frac{Lv}{c^2} \frac{1}{\sqrt{1 - v^2/c^2}} \tag{0.3}$$

The proper time passed for the observer on the train is

$$\tau = (t_1 - t_2)\sqrt{1 - v^2/c^2} = \frac{Lv}{c^2} \tag{0.4}$$

Numerically $L = 100m$, $v = 50m/s$, $c = 3 \cdot 10^8 m/s$, so

$$\tau = 5,5 \cdot 10^{-14} s \tag{0.5}$$

This result means that according to the observer the lightning strikes happened with a time delay of τ, because τ is the time difference he measures, and he knows that he is sitting on the middle of the train. So for him, the Minkowski-vector $(\delta t, \delta x)$ describing the difference between the two events (two lightning strikes) is

$$(\tau, L), \tag{0.6}$$

becaue the two events happen at distance L.

For the observer in the lab frame the vector describing the difference between the two events is

$$(0, L'), \tag{0.7}$$

because this observer sees the two events at the same time, but at a distance of L'.

Note that the Minkowski-length of the two vectors is the same:

$$c^2\tau^2 - L^2 = \frac{L^2v^2}{c^2} - L^2 = -\left(L\sqrt{1 - v^2/c^2}\right)^2 = 0^2 - (L')^2 \tag{0.8}$$

This is a check of our computations.