A1

A particle of resting mass m and electric charge q moves in a homogeneous electric field whose strength is E. The field points in the y direction while the particle's velocity is initially $v_0 = 0.8c$ and points in the x direction.

(a) Determine the initial (usual) momentum vector of the particle.

(b) Write down the relativistic equations of motion for the particle's momentum vector. Solve the equation, i.e. determine the particle's momentum as a function of time.

(c) Consider the moment when the x and y components of the particle's momentum are equal. Determine the particle's 4-momentum in that moment. Use the known Minkowski-length of the 4-momentum for a particle of resting mass m.

(d) What is the particle's velocity vector in that moment? What are the x and y coordinates of the velocity?

A2

The model of a relativistic rocket was considered in class. Now you have to generalize the results for the model of a "photonic-rocket". The initial resting mass of the rocket is M_0. The power unit emits a strong photon ray, that accelerates the rocket. The rocket starts from rest, and moves along the x axis.

(a) Consider the moment, when the resting mass of the rocket is M, and choose the (instantaneously) comoving frame of the rocket. The power unit emits photons of energy $d\varepsilon$, i.e. the 4-momentum of the emitted photons is $(d\varepsilon, d\varepsilon, 0, 0)$, where we used $c = 1$. Write down the 4-momentum conservation for the process.

(b) Determine the change dM of the rocket's resting mass.

(c) Determine the dv change in the rocket's velocity (seen from the instantaneously comoving frame). Convert it to the change of rapidity $d\theta$.

(d) What is the velocity of the rocket when it has lost half of its resting mass?

A3

Starting from the expression

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix} \tag{1}$$

work out the transformation rules of the fields $\vec{E}, \vec{B}$ under Lorentz-boost in the x direction. (or you could choose the z direction, as we discussed in class) Which components are invariant? Show that certain pairs of components transform as Lorentzian 2-vectors!

B1

A particle of resting mass m_0 and charge q moves in a homogeneous magnetic field B that points in the z direction. The initial velocity of the particle is $\vec{v} = (0, v_0/\sqrt{2}, v_0/\sqrt{2})$, i.e. it is not perpendicular to the magnetic field.

(a) Write down the relativistic equations of motion.

(b) Show (similarly to class) that the size of the particles (usual) velocity vector is constant.

(c) Using that, write down the equations of motion for $d\vec{v}/dt$.

(d) Show that the following expression solves the equations, and it is also compatible with the initial conditions.

$$\vec{v}(t) = \left(-\frac{v_0}{\sqrt{2}}\sin(\Omega t), \frac{v_0}{\sqrt{2}}\cos(\Omega t), \frac{v_0}{\sqrt{2}}\right) \tag{2}$$

What is Ω?

(e) Sketch the trajectory of the particle qualitatively.

(f) For practice, solve the whole problem using the covariant form of the equations of motion:

$$m_0 \frac{du^\mu}{d\tau} = qF^{\mu}_{\ .\nu} u^\nu \tag{3}$$

(in class we did not treat this, the idea is to compute all quantities as a function of τ, and to express everything afterwards using t, the lab time coordinate)

B2

A particle of resting mass m, and electric charge q is in a static homogeneous electric and magnetic fields E and B that are perpendicular to each other. The initial velocity of the particle is zero. Determine the motion of the particle. The magnetic inducion points in the z direction while the electric field points in the y direction.

(a) Write down the relativistic equations of motion for the particle in the covariant form.

(b) Let us apply the trick that was mentioned in class: We want to apply a Lorentz transformation such that either the electric or the magnetic field is zero in the new frame. Which one of them can be transformed to zero, depending on the initial, lab frame values?

(c) Let us assume that we can transform away the electric field. The Lorentz boost can be found either using the results of problem A3, or using a simple trick. We want to find a uniform linear motion, where the magnetic Lorentz-force and the electric force cancel each other and thus the moving particle does not "feel" any force. If we boost to a frame that moves with the velocity of that motion, the electric field strength must be zero, because our particle is in rest in that frame. We can then solve the original problem in this moving frame, and afterwards transform back to the lab frame.

(d) What is the velocity of the uniform linear motion? When is it physically meaningful?

(e) Transform the field-strength tensor to that frame!

(f) Solve the problem in the moving frame!

(g) Transform back to the original frame, and express $x(t)$ (not solvable in an explicit way). Sketch the trajectory of the particle.

(h) What happens if the velocity in b.) is not physically meaningful? How does the trajectory of the particle look like in that case?