A1

In a simplified relativistic model of nucleus-nucleus collisions, a nucleus of (resting) mass m_1 having (usual) velocity v_1 collides with an other nucleus of (resting) mass m_2 that rests initially. After the collision a highly excited composite particle is produced that has velocity v_0 and "resting" mass m_0. (Note: the word "resting" is marked, because for such a highly excited particle the word "rest" is not very accurate.)

(a) Write down the 4-momenta of the nuclei before the collision.

(b) Write down the 4-momentum conservation for the collision, and express the 4-momentum of the composite after the collision.

(c) The Minkowski-length of the 4-momentum vector is Lorentz invariant. Use this fact, and express the "resting" mass m_0 of the composite particle.

(d) Determine the velocity v_0 of the composite! Calculate its numerical value, if $m_1 = 40u_a$, $m_2 = 238u_a$ and $v_1 = 0.8c$, with u_a denoting the atomic mass unit.

A2

Let's consider the propagation of two photons. One has 4-momentum $q_1^\mu = (q, q, 0, 0)$, while the other has $q_2^\mu = (2q, -q, \sqrt{3}q, 0)$.

(a) Determine the Minkowski length squares of the 4-momenta to show, that they can be the 4-momenta of (massless) photons.

(b) Express the total momentum of the two photons. Which kind of 4-vector is that (space-like/time-like/light-like)?

(c) Find a frame of reference (moving in direction y) where the total (3-) momentum vanishes. (We can call this the "center of mass" frame of the two photons.)

(d) Express the 4-momenta $q_1'^\mu$ and $q_2'^\mu$ of the photons in this "center of mass" frame. What is the energy of the photons? What can you say about the direction of their propagation?

B1

Consider the 4-index Levi-Civita tensor $\varepsilon^{\alpha\beta\gamma\delta}$!

- Show that it is invariant for those Lorentz transformations which have $\det \Lambda = 1$. Show that the lower-index tensor $\varepsilon_{\alpha\beta\gamma\delta}$ is also invariant! What is its relation to the upper-index representation?
- Show that for any $A^{\mu\nu}$ tensor the combination

$$A^{\mu\nu} A^{\delta\kappa} \varepsilon_{\mu\nu\delta\kappa}$$

is invariant!

B2

In the lecture the 4-velocity vector $u^\mu = \frac{dx^\mu}{d\tau}$ has been introduced, and it has been shown that this is a proper 4-vector.

(a) Write down the connection between the 4-velocity and the usual (3-)velocity vector.

(b) Let's suppose, that watching the sky, we see two spacecrafts that are flying towards each other, and both have velocity 0.6c. We use a coordinate system, where the trajectories of the spacecrafts lie on the x-axis.

Determine the 4-velocities of the spacecrafts.

(c) Write down a Lorentz-transformation that transforms into the frame of one of the spacecrafts.

(d) Express the 4-velocities of the spacecrafts in that frame of reference.

(e) What is the usual 3-velocity of the spacecrafts in that frame? Let suppose now, that – as we see from the Earth – the two spacecrafts travel in perpendicular directions, x and y.

(f) Determine the modified 4-velocities of the spacecrafts

(g) Transform to the frame of the spacecraft travelling in the x direction. What is the 4-velocity of the other spacecraft in this frame?

(h) What is the usual 3-velocity of the other spacecraft in this frame?

B3

A spacecraft has departed from Earth, and now travels with constant 3-velocity $v = (c/2, c/2, 0)$.

(a) Determine the 4-velocity vector of the spacecraft.

(b) A physics student must find a Lorentz-transformation, that transforms to the frame of reference of the spacecraft. Because of some internet-connection issues he cannot search the result on Wikipedia, and he only remembers the specific Lorentz matrices that are boosts in the directions of some of the three axes. He doesn't give up, and figures out the following solution:

- First he performs a boost in the x-direction. He chooses the velocity of the boost in such a way, that the spacecraft will move precisely in the y'-direction. He denotes the velocity of the boost by V_1.
- As a second step, he performs a boost in the y' direction such, that in the final frame the spacecraft is in rest.

Determine the velocity V_1 of the first boost.

(c) Determine the 4-velocity of the spacecraft in this "auxiliary" frame of reference. What is the (usual) velocity V_2 of the spacecraft in this frame?

(d) Using the results of the previous questions express the total Lorentz transformation that transforms from the frame of Earth to the frame of the spacecraft, by using matrix multiplication.

(e) The student finds the previous result very strange: this is not a symmetric matrix. As a cross-check he calculates the transformation matrix in a different way: first he performs a boost in the y-direction with velocity V_1 , and then in the x'-direction with velocity V_2. What is the resulting matrix now?

(f) Finally the student remembers that the usual 3-rotations of the coordinate system are also Lorentz-transformations. Therefore first he performs a rotation in the original frame, to rotate the velocity vector of the spacecraft in the x-direction. Then he performs an appropriate boost, and finally he rotates back the coordinate system (using the same angle as in the fist rotation). What is the resulting matrix in this case?