A7

A particle of resting mass m and electric charge q moves in a homogeneous electric field whose strength is E. The field points in the y direction while the particle's velocity is initially $v_0 = 0.8c$ and points in the x direction.

(a) Determine the initial (usual) momentum vector of the particle.

(b) Write down the relativistic equations of motion for the particle's momentum vector. Solve the equation, i.e. determine the particle's momentum as a function of time.

(c) Consider the moment when the x and y components of the particle's momentum are equal. Determine the particle's 4-momentum in that moment. Use the known Minkowski-length of the 4-momentum for a particle of resting mass m.

(d) What is the particle's velocity vector in that moment? What are the x and y coordinates of the velocity?

A8

The model of a relativistic rocket was considered in class. Now you have to generalize the results for the model of a "photonic-rocket". The initial resting mass of the rocket is M_0. The power unit emits a strong photon ray, that accelerates the rocket. The rocket starts from rest, and moves along the x axis.

(a) Consider the moment, when the resting mass of the rocket is M, and choose the (instantaneously) comoving frame of the rocket. The power unit emits photons of energy $d\varepsilon$, i.e. the 4-momentum of the emitted photons is $(d\varepsilon, d\varepsilon, 0, 0)$, where we used $c = 1$. Write down the 4-momentum conservation for the process.

(b) Determine the change dM of the rocket's resting mass.

(c) Determine the dv change in the rocket's velocity (seen from the instantaneously comoving frame). Convert it to the change of rapidity $d\theta$.

(d) What is the velocity of the rocket when it has lost half of its resting mass?

B6

A particle of resting mass m_0 and charge q moves in a homogeneous magnetic field B that points in the z direction. The initial velocity of the particle is $\vec{v} = (0, v_0/\sqrt{2}, v_0/\sqrt{2})$, i.e. it is not perpendicular to the magnetic field.

(a) Write down the relativistic equations of motion.

(b) Show (similarly to class) that the size of the particles (usual) velocity vector is constant.

(c) Using that, write down the equations of motion for $d\vec{v}/dt$.

(d) Show that the following expression solves the equations, and it is also compatible with the initial conditions.

$$\vec{v}(t) = \left(-\frac{v_0}{\sqrt{2}}\sin(\Omega t), \frac{v_0}{\sqrt{2}}\cos(\Omega t), \frac{v_0}{\sqrt{2}}\right) \tag{1}$$

What is Ω?

(e) Sketch the trajectory of the particle qualitatively.

(f) For practice, solve the whole problem using the covariant form of the equations of motion:

$$m_0 \frac{du^\mu}{d\tau} = qF^\mu_{.\nu} u^\nu \tag{2}$$

(in class we did not treat this, the idea is to compute all quantities as a function of τ, and to express everything afterwards using t, the lab time coordinate)