Problem A1

Shortly after Einstein published his theory of relativity plenty of paradoxes were created. Using these paradoxes, the critics wanted to point out inconsistencies in the theory. Some of them are really helpful in understanding the theory. Probably the most famous paradox is the "twin-paradox" that will be discussed in deatils next week. Here we discuss an other one, that is called the ladder paradox.

There is a garage that has two doors (front and back) on the two sides. The distance between the doors is L_0. There is also a ladder whose (resting) length is again L_0. Now let's run with a relativistic velocity v across the garage with the ladder in our hands. Because of the Lorentz-contraction, the ladder will be shorter: $L = L_0\sqrt{1 - v^2/c^2}$. That means it is possible (for a short time) to close both the front and back doors. However, we can describe the situation also from the running observers frame of reference. Here the ladder has length L_0, while the distance between the front and back doors is only $L = L_0\sqrt{1 - v^2/c^2}$, therefore it is impossible to close the two doors at the same time.

To see that this is not only a theoretical question, let us suppose that there is a bomb connected to the two doors. If they are closed at the same time, the bomb explodes. It is important to know whether we survive the experiment or not.

(a) Draw the world-lines in the Minkowski plane of the two doors and the two ends of the ladder, where we have chosen the reference frame of the garage. Mark the events when the front of the ladder exits the garage (crosses the back door), and when the back of the ladder enters the garage (crosses the front door). Which event happened earlier in the frame of the garage? Can one close the two doors at the same time?

(b) Draw the ct' and x' axes (follow the figure of Problem 5. from class) of the moving frame of reference in the figure.

(c) Which event happend earlier in the frame of the garage? Can one close the two doors at the same time now?

(d) Can we tell, what happens with the bomb? (Can one develop such a precise electronic control, that is described in the paradox?)

Problem A2

A train of length L is traveling along a long linear track with velocity V. At the middle of the train a passenger blinks his flash light at time $t = 0$. In the back and front of the train two mirrors are placed.

(a) Draw the world lines of the two endpoints of the train in the Minkowski plane.

(b) Draw the world lines of the forward and backward going light pulse in the figure.

(c) Mark the events when the light hits the mirrors.

(d) Which reflection happend earlier according to the passenger? And according to the observer resting at the nearby fields?

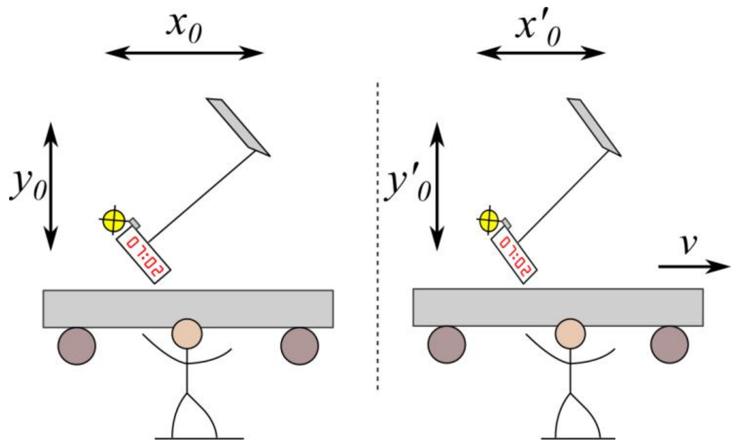
(e) Draw the word lines of the reflected light signals. Which signal is observed earlier by the passenger?

Problem B1

Consider the light-clock from Problem 1.) of the class. The arm of the clock is tilted by an angle α therefore its horizontal and vertical projections are $x_0 = d_0\cos(\alpha)$ and $y_0 = d_0\sin(\alpha)$, respectively (see picture below).

Now the car carrying the clock is moving with velocity v. Following the solution from class, we expect that the vertical projection of the clocks arm remains unchanged ($y_0' = y_0$), while the horizontal projection is contracted to $x_0' = x_0\sqrt{1 - v^2/c^2}$.

In this problem our goal is to show, that this hypothesis – that will be prove later – is consistent in the sense that the clocks time unit $t_0' = \frac{2d_0}{c}\frac{1}{\sqrt{1-v^2/c^2}}$ is independent of the angle α and coincides with the time unit we computed in class.



(a) Draw the light ray that is reflected from the mirror to the detector in the case of a moving car.

(b) Using the Pythagorean theorem, and taking the cars motion into account calculate the length of the ray until it hits the mirror and also from the mirror to the detector.

(c) Express the time unit of the moving clock, and show it remains unchanged.