Problem 1

An elastic object has been deformed due to external forces. The displacement if the points of the object are described by the displacement field:

$$x'_i = x_i + s_i(x) \quad (1)$$

- Consider two infinitesimally close points of the object. Express the distance of these points after the deformation.
- Consider a small volume around the point x. Determine the change of this volume.

Problem 2

Consider a square prism made of homogeneous, isotropic, and elastic material. The bottom of the prism has been softly fixed to a vertical wall, while the top of the prism is pulled by a horizontal force F. (Remark: the soft fixing of the bottom means that the shearing at the wall is negligible.)

The Lame parameters of the material are μ and λ, the mass density of the material is negligible.

(a) Write down the elements of strain tensor in the prism.

(b) Using Hooke's law, determine the deformation tensor in the prism.

(c) What is the relative length change of the prism?

(d) What is the relative change in the cross section of the prism?

(e) Using the results of c.) and d.), express the Young's modulus and the Poisson number for the material.

Problem 3

Concrete is one of the most popular construction materials. It can be casted easily, but after it solidifies it sustains large compressive stresses. However, tensile and shearing stresses can easily break the concrete, therefore we use ferro-concrete. We use ferro-concrete even in the building of pillars, where – as one naively thinks – only compressive stresses are present.

Consider a not too high concrete pillar of cross section $A = 0.01m^2$. We push the top of the pillar vertically with force F. The compressive strength of the pillar is very large, but the shearing strenght of concrete is finite: the shearing stress cannot be larger than σ_{max}. (If the shearing stress is larger than the maximal value the concrete breaks.)

(a) Write down the elements of the stress tensor in the pillar. The vertical axis of the pillar is the z axis, the horizontal plane is the x-y plane.

(b) What is the shearing stress?

(c) Now rotate the coordinate system around the x axis by an angle of ϕ. What is the stress tensor in the rotated coordinate system?

(d) We see, that as a function of ϕ there is some shearing stress in the concrete. What is the maximal shearing stress? (as a function of ϕ)

(e) How large can be F, if we don't want to break the pillar?

(f) If F is larger than the maximal value, qualitatively how will the pillar break?

Problem 4

A thin and long elastic rod is bent. In this problem our goal is to describe the stress and deformation tensors in the rod.

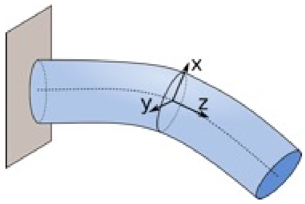
(a) Using the fact that at the surface of the rod there are no external forces present, and the rod is thin, argue that in the rod only longitudinal stress can be present.

(b) In a short (lenght dl) piece of the rod, the radius of curvature of the bent rod is R, that is quite large (weakly bent rod). Determine the elements of the deformation tensor for that piece in the coordinate system where the axis of the rod is the z-direction, and the rod is (locally) bent in the $x - z$ plane. (see figure!)

(c) Determine the so-called bending moment.

(d) Derive the expression for the rods total elastic energy.

(e) Sketch the deformation of the rods cross-section.



Problem 5

A thin and havy rod of length L is horizontally fasten in a wall (see figure). The other end of the rod is free. The axis of the (non bent) rod is the z axis while the vertical axis is the x axis. The Young's modulus of the rods material is E, the cross-section parameter is Θ, and the rod's linear mass density is ρ.

(a) Determine the bending moment in the rod as a function of z.

(b) Write down the differential equation that describes the shape of the bent rod.

(c) Determine the shape of the rod. What is the prolapse of the free end of the rod?