Problem 1

A spacraft, accelerated by a rocket, departs from an intergalactic space station. (The spacecraft is far from any source of gravitational force.) In the beginning, the total resting mass of the spacecraft and its fuel is M_0. The burned fuel goes out from the rocket with velocity u (relative to the rocket), that can be relativistically large.

(a) First focus on the moment, when the total resting mass of the spacecraft is M. Investigate the rocket from the inertial system, where its velocity is zero in that moment. In a very short time a small amount of the burned fuel goes out of the rocket. The resting mass of the exhausted fuel is dm. Write down the conservation of 4-momentum. Express the change dM of the spacecraft's resting mass and the velocity dv of the spacecraft after the process.

(b) On the last class we saw that in 1-dimensional motions (like the spacecrafts motion in our case) it's worth to use rapidities instead of velocities. Transform the dv velocity of the rocket into rapidity $d\theta$.

(c) Using the previous results determine the resting mass of the rocket, its rapidity, when the total resting mass of the exhausted fuel is m.

(d) What is then the velocity of the spacecraft?

(e) We see, that the decrease of the resting mass of the rocket is not m. Why?

Problem 2

A particle of resting mass m_0, and electric charge q is in a static homogeneous electric field E. The particle starts from rest. Solve the equations of motion for the particle. The particle is initially in the origin, and the electric field points in the x direction.

(a) First solve the equations in the nonrelativistic approximation.

(b) Write down the relativistic equations of motion.

(c) Solve the equation for the momentum of the particle.

(d) From the known momentum-time function $p(t)$, express the particles velocity $v(t)$.

(e) Draw the $v(t)$ function in a graph. Compare it with the nonrelativistic solution!

(f) Express the position $x(t)$ of the particle by integrating $v(t)$. Draw this function.

Problem 3

A particle with resting mass m, and electric charge q is in a static homogeneous magnetic field B. The particle moves in the plane $x - y$ that is perpendicular to the field, that points in the z direction. The (inital) velocity of the particle is v and points initially in the x direction.

(a) First solve the problem in the nonrelativistic approximation.

(b) Write down the relativistic equations of motion.

(c) Exploiting the fact that the Minkowski-lenght of the particle's 4-momentum is constant, show that the length of the (usual) velocity vector remains also invariant.

(d) By using the result c.), express the equations of motion for $d\vec{v}/dt$.

(e) Remark: the equations are no more complicated than the ones in a.). Let's solve them.

(f) The particles motion is a uniform circular motion. Express the radius of the orbital and the time period of the motion. Compare the results with the nonrelativistic ones.

Problem 4

Solve the problems 2.) and 3.) using the covariant form of the equations of motion.

(a) Express the equations in the following form:

$$\frac{dp^\mu}{d\tau} = F^\mu_{.\nu} u^\nu \tag{1}$$

where $F^\mu_{.\nu}$ is the electromagnetic 4-tensor that contains the electric and magnetic fields together. What are the components of that tensor? Write down the equations of motion for the case of homogeneus electric and magnetic fields.

(b) We should get a simple set of linear differential equations. Solve them! What are the initial conditions?

(c) Express the solution $u^\mu(\tau)$ that is compatible with the initial conditions.

(d) Integrate $u^\mu(\tau)$ to determine $x^\mu(\tau)$.

(e) The x^0 coordinate of the particle is simply the time of the coordinate system (multiplied by c). From $x^0(\tau)$ derive the $\tau(t)$ function.

(f) Using the result of e.), express the usual $x(t)$ position-time functions of the particle.

Problem 5

A particle of resting mass m, and electric charge q is in a static homogeneous electric and magnetic fields E and B that are perpendicular to each other. The initial velocity of the particle is zero. Determine the motion of the particle. The magnetic inducion points in the z direction while the electric field points in the y direction.

(a) Write down the relativistic equations of motion for the particle in the covariant form (like in Problem 4).

(b) We could solve simply the equations of a.) (as a practice you can do it.), but now it's worth to follow a different way. Our argument is the following: in a crossed electric and magnetic field one can figure out a uniform linear motion, where the magnetic Lorentz-force and the electric force cancel each other. If we boost to a frame that moves with the velocity of that motion, the electric field strength must be zero, because our particle is in rest in that frame. Here we can solve the much easier problem, where only a magnetic field is present, and finally we transform back to the original frame of reference, and get the solution of our problem.

(c) What is the velocity of the uniform linear motion? When is it physically meaningful?

(d) Transform the field-strength tensor to that frame!

(e) Solve the problem in the moving frame!

(f) Transform back to the original frame, and express $x(t)$. Sketch the trajectory of the particle.

(g) What happens if the velocity in b.) is not physically meaningful? How looks like the trajectory of the particle in that case?