A1

During your introductory physics classes you learned about Pascal's law, which states that in a resting fluid the pressure is isotropic, i.e. the pressing force on a test surface does not depend on the direction of the surface.

This law can be reformulated after the introduction of the stress tensor. We know that in a resting fluid there are no shearing stresses, because the shearing stress is proportional to the derivates of the velocity field: in rest the velocity is zero leading to zero shearing stress.

Pascal's reformulated law thus states: if in a medium there are no shearing stresses then the stress tensor is proport ional to the unit matrix. In this problem you have to prove this theorem, using an indirect proof.

First assume that the stress tensor is diagonal (there are no shearing stresses), but the diagonal elements are not equal:

$$\sigma = \begin{pmatrix} \sigma_{xx} & 0 & 0 \\ 0 & \sigma_{yy} & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix} \tag{1}$$

For example let's suppose that $\sigma_{xx} \neq \sigma_{yy}$.

(a) Write down the matrix that rotates the coordinate system around the z axis by an angle of α.

(b) Determine the elements of the stress tensor in this rotated coordinate system.

(c) Express the shearing stress σ_{xy} in this rotated coordinate system.

(d) Show that the shearing stress is zero for all angles only if $\sigma_{xx} = \sigma_{yy}$, implying that our initial assumption was wrong.

A2

A light, elastic medium is put in a rigid box, see picture below. The bottom of the box is a square with side a, while the height of the box is b. The medium does not stick to the walls of the box, but it fills it like a jelly. The Lamé parameters of the medium are λ and μ.

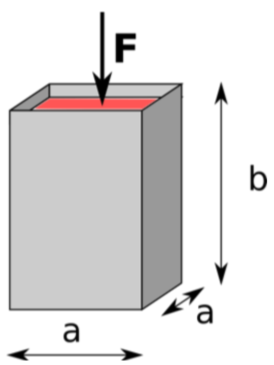
We start to push the surface of the medium with an unknown force F, that leads to a compression Δb of the surface.

(a) Express the deformation tensor of the medium.

(b) Write down Hooke's law for the medium.

(c) Express the nonzero elements (σ_{xx}, σ_{yy} and σ_{zz}) of the stress tensor.

(d) Knowing the compression Δb determine the unknown force F.



B1

In the lecture you saw that in the problem of bending rods the cross-section parameter

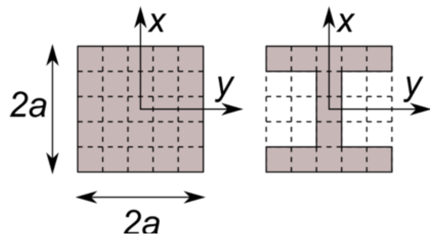
$$\Theta = \int dA\, x^2 \tag{2}$$

is very important. Here the integration is over the cross section of the rod, and x denotes the distance from the neutral surface in the rod.

We made two rods of steel. The cross section of the first one is a square with sides $2a$, while the other has an "I" shaped cross section (see figure). The width of the parts of the I-shape is $2a/5$. (see figure)

(a) Determine the cross-section parameter Θ_{square} for the square. What is the ratio Θ_{square}/A for the square, where A denotes the cross section?

(b) What is the cross-section parameter for the "I"-shaped rod? What is the ratio Θ_I/A for the "I"-shaped rod?



B2

A rod of length L, cross-section A and mass density ρ is softly fixed to the ceiling. Due to its own weight the rod elongates. In the first part of the problem you must consider this "self-elongation". The Lamé-parameters of the rod are λ and μ.

Let the origin be on the ceiling and let the z axis point vertically down.

(a) Write down the only nonzero element σ_{zz} of the stress tensor as a function of z.

(b) Using Hooke's law express the nonzero elements ε_{xx}, ε_{yy} and ε_{zz} of the deformation tensor, as a function of z.

(c) Knowing ε_{zz}, determine the displacement s_z of the rods points as a function of z.

(d) What is the rods ΔL elongation?

(e) We start to pull down the end of the rod by a force of F. Repeat the previous computations in this case too!

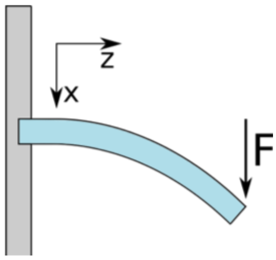
B3

A thin, light rod of length L is fastened to a wall horizontally. The free end of the rod is pushed down with a force F. The mass of the rod is negligible. The horizontal direction is z, the F points in the x direction. The Young's modulus of the rod is E, the cross-section parameter is Θ.

(a) Determine the bending moment in the rod as a function of z.

(b) Write down the differential equation that describes the shape of the rod.

(c) Determine the shape of the rod. What is the displacement of its free end?



B4

An elastic plank is laid on two wedges (see figure). The two ends of the plank are at $z = \pm L/2$. The Young's modulus of the plank is E, the cross-section parameter is Θ, the total mass of the plank is m.

(a) Determine the forces that act at the two wedges.

(b) Determine the bending moment in the plank as a function of z.

(c) Write down the differential equation for the plank's shape.

(d) Solve the differential equation. Use the boundary conditions that the endpoints of the rod are fixed.

(e) What is the displacement of the midpoint of the plank?