A1

A cilinder having moment of inertia Θ can easily rotate around a fixed axis. The Hamiltonian of the system is

$$H(\phi, p_\phi) = \frac{p_\phi^2}{2\Theta} \tag{1}$$

(a) Write down the Hamilton-Jacobi equation for the system.

(b) Following the usual separation $S(\phi, t) = S_0(\phi) - Et$ write down the abbreviated Hamilton Jacobi equation for S_0.

(c) Solve the abbreviated equation, i.e. express the solution $S_0(\phi, E)$.

(d) At $t = 0$ the cilinder is in the position $\phi = 0$, and has angular momentum $p_\phi = L$. Using this information determine the values of the constants of motion E and $\beta_E = \frac{\partial S}{\partial E}$.

(e) From the results of d.) determine the $\phi(t)$ solution of the equations of motion.

A2

The Hamiltonian of a free particle in special relativity is

$$H(x, p) = \sqrt{m^2c^4 + p^2c^2} \tag{2}$$

(a) Write down the Hamilton-Jacobi equation for the system.

(b) Following the usual separation of time, by introducing the constant E, write down the abbreviated Hamilton-Jacobi equation.

(c) Solve the abbreviated equation. Determine also the solution of the full Hamilton-Jacobi equation.

(d) Determine β_E , which is the canonical pair of E.

B1

A particle can move along the x axis. An external force that increases linearly in time acts on the particle, therefore the Hamiltonian of the system is

$$H(x, p) = \frac{p^2}{2m} - Axt \tag{3}$$

(a) Write down the Hamilton-Jacobi equation for the system.

(b) The Hamilton function depends explicitly on time, therefore we cannot follow the usual separation technique. Try the following form instead:

$$S(x, t) = F(t)x + G(t) \tag{4}$$

Substitute this to the Hamilton-Jacobi equation.

(c) Collect the terms according to the powers of x.

(d) In order to find a solution to the equation, both the first order and zero'th order terms in x must be zero. Use this to determine the functions $F(t)$ and $G(t)$. If your calculation is correct, an integration constant appears. Denote it by α.

(e) Write down the solution $S(x, \alpha, t)$. At $t = 0$ the particle is at $x_0 = 0$ and has momentum $p_0 = 0$. Using this information determine the value of α.

(f) According to the Hamilton-Jacobi theory, $\beta = \frac{\partial S}{\partial \alpha}$ is also a constant of motion. Express its value using the initial conditions.

(g) Use the equation for β to determine the $x(t)$ solution of the equation of motion.

B2

In class we considered the particle in gravitational field. The Hamiltonian is

$$H(x,p) = \frac{p^2}{2m} + mgx \tag{5}$$

In the usual Hamilton-Jacobi approach we search a 2nd type generator function $S(x,\alpha,t)$ that makes the new Hamiltonian zero. Then we exploit the fact that the new momentum α is a constant of motion. However, this is not the only possible choice.

In this problem we consider a 3rd type generator function $S(p,\alpha)$. In this case $x = -\frac{\partial S}{\partial p}$.

(a) Write down the Hamilton-Jacobi equation.

(b) Separate the time using $S(p,E,t) = S_0(p,E) - Et$. Write down the abbreviated Hamilton-Jacobi equation for S_0.

(c) Solve the equation, and express the function $S_0(p,E)$.

(d) At the moment $t = 0$ the particle starts from the position $x = 0$ having momentum p_0. Express the value of the constant E.

(e) The quantity $\beta = \frac{\partial S}{\partial E}$ is also a constant of motion. Express its value using the initial conditions.

(f) Determine the $x(t)$ function.