Physics BSc of FNS BME
4. semester

MECHANICS 2
List of entrance questions for the exam

One of the entrance questions below must be answered within a few minutes at the beginning of the exam. Only flawless answer/solution is acceptable.

1. Prove that the Minkowski scalar product of two 4-vectors is invariant with respect to the Lorentz transformation (described by the $L = \{L^{\mu}{}_{\nu}\}$ matrix)!
2. The velocity of a particle is $\mathbf{v}(t)$. What are the components of the particle's four-momentum? What is the Minkowski length of the 4-velocity?
3. Compton scattering: a photon of frequency ω scatters on a standing electron of rest mass m. Calculate the frequency ω' of the photon flying out at scattering angle ϑ!
4. Write down the Lagrangian density of a 1-dimensional medium (string), and derive the corresponding equation of motion!
5. Write down the Lagrangian density of an isotropic elastic medium with the help of the Lamé constants! Determine the energy density and the total energy!
6. A physical system is described by the real scalar field $\varphi(\mathbf{r}, t)$. The corresponding Lagrangian density $\mathcal{L} = \frac{1}{2}(\partial_t\varphi)^2 - \frac{1}{2}(\nabla\varphi)^2 - \frac{1}{2}m^2\varphi^2$. Write down the Euler-Lagrange equation!
7. The Lagrangian density of a mechanical system characterized by the displacement field $u(x, t)$ is $\mathcal{L} = \frac{\alpha}{2}u^2(\partial_t u)^2 - \frac{\beta}{2}(\partial_x u)^2$. Write down the $\pi(x, t)$ canonical momentum density, and construct the energy density as well as the $H[\pi, u]$ Hamiltonian functional!
8. The Hamiltonian of a harmonic oscillator is $H = \frac{1}{2}p^2 + \frac{1}{2}\omega^2x^2$. Calculate the I action variable, and express H with its help! Derive the equation of motion of the conjugated angle variable!
9. Write down the equation of motion of an *anharmonic oscillator*, and sketch the structure of the solution! (Give all terms and their dependence on amplitude up to the second harmonic.)
10. Calculate the Poisson bracket of $A = (x^2 - y^2)(p_x^2 - p_y^2)$ with x and p_y!
11. Write down the equations of motion of a mechanical system described by the Hamiltonian $H(\mathbf{q}, \mathbf{p}, t)$, and the total time derivative of a physical quantity $A(\mathbf{q}, \mathbf{p}, t)$ with the help of Poisson brackets!
12. The Hamiltonian of a mechanical system is $H(\mathbf{q}, \mathbf{p}, t)$. State the generalized Hamilton principle!
13. What is the definition for the transformation $(\mathbf{q}, \mathbf{p}) \to (\mathbf{Q}, \mathbf{P})$ being canonical? Give the necessary and sufficient condition!
14. Enumerate the 4 important properties of the canonical transformation, and prove one of those!
15. State Poisson's theorem! (Give formulas using Poisson brackets.)
16. What is the canonical transformation generated by the generating function $F = qP^3$?
17. Write down the Hamilton-Jacobi equation of a harmonic oscillator!