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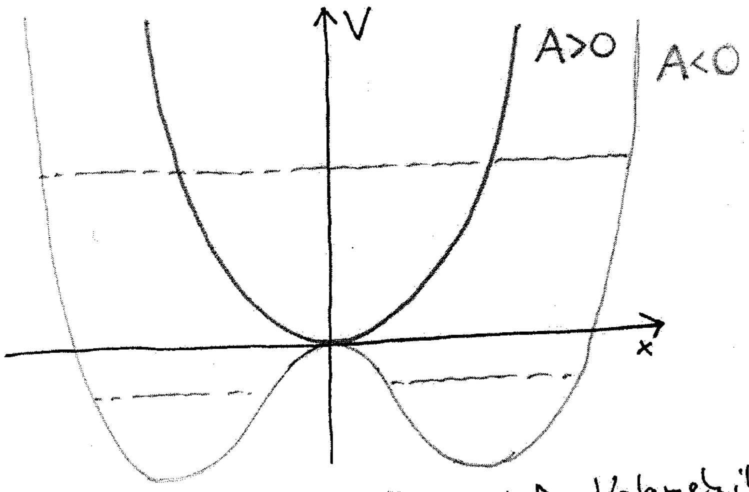
$$V(x) = Ax^2 + Bx^4 \qquad (B>0)$$



$E > 0 \Rightarrow$ "hasonló" eset A előjelétől függetlenül.

$E < 0 \Rightarrow$ Valamelyik völgyben rezeg.

i.) Fordulópontok

$\{E > 0\}$

$$E = Ax^2 + Bx^4$$

$$\underbrace{x^2}_{} = \frac{-A \pm \sqrt{A^2 + 4EB}}{-2B}$$

→ csak a pozitív gyököt tartjuk meg

$$x^2 = \frac{-A}{2B} + \frac{\sqrt{A^2+4EB}}{2B}$$

$$x_{1,2} = \mp\sqrt{\left(\frac{-A}{2B} + \sqrt{\frac{A^2}{4B^2} + \frac{E}{B}}\right)}$$

$\{E < 0\}$ (csak $A<0$ esetén értelmes)

$$x^2 = \frac{-A}{2B} \pm \sqrt{\frac{A^2}{4B^2} + \frac{E}{B}} \quad \Rightarrow 4 \text{ valós gyök}$$

$$x_1^L = -\sqrt{-\frac{A}{2B} + \sqrt{\frac{A^2}{4B^2} + \frac{E}{B}}} \qquad x_1^R = \sqrt{-\frac{A}{2B} - \sqrt{\frac{A^2}{4B^2} + \frac{E}{B}}}$$

$$x_2^L = -\sqrt{-\frac{A}{2B} - \sqrt{\frac{A^2}{4B^2} + \frac{E}{B}}} \qquad x_2^R = \sqrt{-\frac{A}{2B} + \sqrt{\frac{A^2}{4B^2} + \frac{E}{B}}}$$

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$\{E>0\}$

$$T=\sqrt{2m}\int_{-\sqrt{\frac{A}{2B}+\sqrt{\frac{A^2}{4B^2}+E/B}}}^{\sqrt{\frac{A}{2B}+\sqrt{\frac{A^2}{4B^2}+E/B}}}\frac{dx}{\sqrt{E-Ax^2-Bx^4}}$$

Legyen $\gamma=\frac{A^2}{EB}$

$$x=\sqrt{\frac{A}{2B}}\,y$$

$$T=\sqrt{2m}\int_{-\sqrt{1+\sqrt{1+\frac{4EB}{A^2}}}}^{\sqrt{1+\sqrt{1+\frac{4EB}{A^2}}}}\frac{dy\sqrt{\frac{A}{2B}}}{\sqrt{E-\frac{A^2y^2}{2B}-\frac{A^2}{4B}y^4}}=$$

$$T=\sqrt{2m}\int_{-\sqrt{1+\sqrt{1+4/\gamma}}}^{\sqrt{1+\sqrt{1+4/\gamma}}}\frac{dy\sqrt{\frac{A}{2BE}}}{\sqrt{1-\frac{1}{2}\gamma y^2-\frac{1}{4}\gamma y^4}}$$

$$T=\sqrt{2m}\cdot\sqrt{A}\cdot\underbrace{\int_{-\sqrt{1+\sqrt{1+4/\gamma}}}^{\sqrt{1+\sqrt{1+4/\gamma}}}\frac{dy\sqrt{\gamma}}{\sqrt{1-\frac{1}{2}\gamma y^2-\frac{1}{4}\gamma y^4}}}_{I(\gamma)}$$

Mit jelent γ?

γ kicsi $\longleftrightarrow$ E nagy $\quad$ ($\gamma\ll 1 \iff E\cdot B\gg A^2$, $E\gg\frac{A^2}{B}$)

$$\Rightarrow T\simeq\sqrt{2m}\sqrt{A}\cdot\int_{-\sqrt[4]{\frac{4}{\gamma}}}^{\sqrt[4]{\frac{4}{\gamma}}}\frac{dy\sqrt{\gamma}}{\sqrt{1-\frac{1}{4}\gamma y^4}}\quad\Longleftrightarrow\quad V(x)=cx^4 \text{ eredmény}$$

($\gamma y^2\ll\gamma y^4$ szinte végig)

γ nagy $\longleftrightarrow$ E kicsi $\qquad E \ll \frac{A^2}{B}$

$$T \cong \sqrt{2m} \cdot \sqrt{A} \cdot \int_{-\sqrt{2}}^{\sqrt{2}} \frac{dy \sqrt{\gamma}}{\sqrt{1 - \frac{1}{2}\gamma y^2}} \rightsquigarrow V(x) \approx b^2 x^2$$

eset.

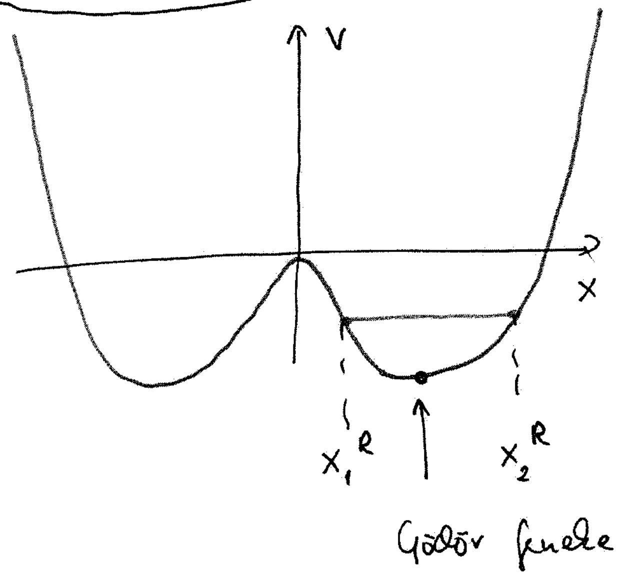
$\Updownarrow$

jogos a harmonikus közelítés

$\uparrow$

Elhagyhatjuk a γy^4 tagot! ($\gamma y^4 \ll \gamma y^2$ végig)

$\boxed{E < 0,\ A < 0}$



Harmonikus közelítés a gödör fenekénél

$$V(x) = Bx^4 + Ax^2$$

$$\frac{\partial V}{\partial x} = \underbrace{4Bx^3 + 2Ax}_{} = 0$$

$x = 0$ (hegycsúcs)

$$x^2 = \pm\sqrt{\frac{-A}{2B}}$$

$\Downarrow$

$$x_G^R = +\sqrt{\frac{-A}{2B}}$$

$$\frac{\partial^2 V}{\partial x^2} = 12Bx^2 + 2A$$

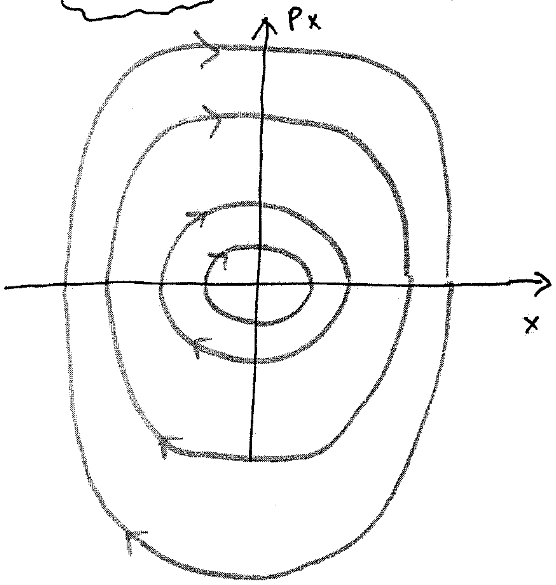
$$V(x) \approx V(x_G^R) + \frac{1}{2} V''(x_G^R)(x - x_G^R)^2 + \dots$$

$$V''(x_G^R) = 12B\,\frac{-A}{2B} + 2A = -6A + 2A = -4A$$

$$V(x) \approx \underbrace{V(x_G^R)}_{\frac{A^2}{4B} - \frac{A^2}{2B} = -\frac{A^2}{4B}} - 2A(x - x_G^R)^2 = -\frac{A^2}{4B} + \underbrace{2A}_{\frac{1}{2}D}(x - x_G^R)^2$$

$$T = 2\pi \cdot \sqrt{\frac{m}{D}} = 2\pi \cdot \sqrt{\frac{m}{4A}}$$

$A > 0$



$A < 0$