Laser Physics 15.

Passive optical resonators (cont.)

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Passive optical resonators (rev.)

Characteristics of passive optical resonators

open (modes with loss), dimensions >> λ_{laser}

Estimation of the resonator lifetime - τ_r

spectral bandwidth of the modes - $$\Delta\nu_r = \frac{1}{2\pi\tau_r}$$

Q-factor $$Q = \frac{\nu}{\Delta\nu_r}$$

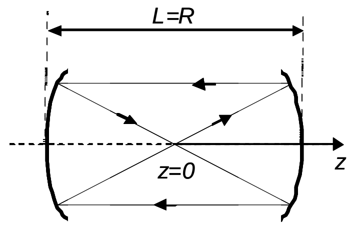
Types

Plane parallel resonator - approximate determination of the $\nu_{l,m,n}$ frequencies

- transverse amplitude distribution TEM_{lm} and loss

$$\Delta\nu_n = \nu_{l,m,n+1} - \nu_{l,m,n} = \frac{c}{2L}$$

Passive optical resonators



Confocal resonator – normalized field distribution on the mirrors

Analytic solution when $L >> a$ (a is the radius of the round mirror), $\cos\theta \sim 1$, $r \sim L$ and $N >> 1$ – lossless confocal resonator in paraxial approximation.

$$U_2(P_2) = -\frac{i}{2\lambda}\int_1 \frac{U_1(P_1)\exp(ikr)(1+\cos\theta)}{r}dS_1$$

Fresnel-Kirchoff diffraction integral

Normalized field distribution on the mirrors:

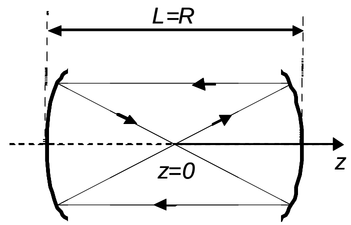
$$U_{lm}(x,y) = H_l H_m \exp\left[-\left(\frac{\pi}{L\lambda}\right)\left(x^2+y^2\right)\right]$$

H_l, H_m are the Hermite polynomials, l and m are integer and show the order of the polynomials:

$$H_0(x) = 1, \quad H_1(x) = 2x,$$

$$H_2(x) = 2(2x^2-1), \cdots$$

Passive optical resonators



Confocal resonator - normalized field distribution on the mirrors (cont.)

The lowest order mode (TEM_{00}) has a Gaussian distribution, the normalized field distribution:

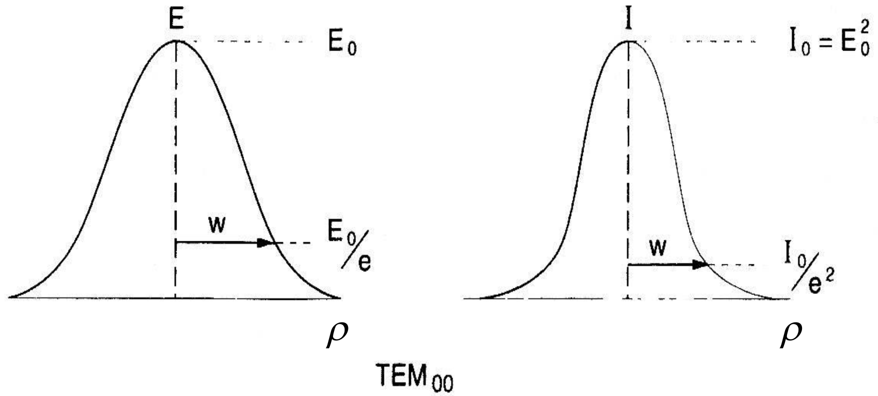
$$U_{00}(x,y) = exp\left[-\frac{\pi}{L\lambda}\left(x^2+y^2\right)\right], \quad H_0 = 1.$$

The amplitude falls to the ratio of e^{-1} in the x or y direction when $\rho^2 = x^2 + y^2$

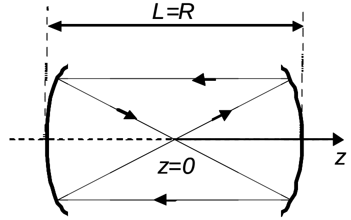
$$\rho = w_s = \left(\frac{\lambda L}{\pi}\right)^{1/2},$$ w_s is the radius of the laser spot on the mirror.

spot

Gaussian beam is characterized by the beam radius and not with the FWHM!



Passive optical resonators



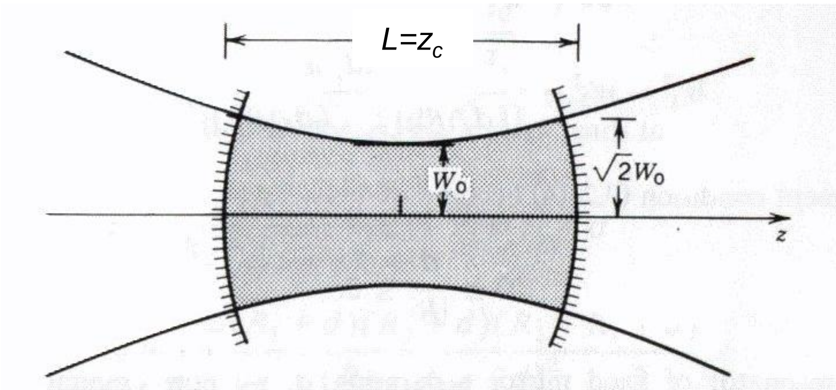
Confocal resonator – normalized field distribution at any place of the resonator

$$U(x,y,z)=\frac{w_0}{w(z)}H_l\left[\frac{\sqrt{2}x}{w(z)}\right]H_m\left[\frac{\sqrt{2}y}{w(z)}\right]e^{-\frac{x^2+y^2}{w^2(z)}}\cdot e^{-i[kz-(1+l+m)\zeta(z)]}\cdot e^{-\left[ik\left(x^2+y^2\right)/2R(z)\right]}$$

amplitude factor — longitudinal, transversal phase factors

$$w(z)=w_0\left[1+(2z/L)^2\right]^{1/2},\quad w(z=0)=w_0=(\lambda L/2\pi)^{1/2},\quad L=2\pi w_0^2/\lambda,$$

$$R(z)=z\left[1+(L/2z)^2\right],\quad \zeta(z)=tg^{-1}(2z/L).$$



w_0 is the minimal beam radius or the beam waist at z=0!

$$w_s=\sqrt{2}w_0$$

z_c is the confocal parameter

$$z_c=2\pi w_0^2/\lambda.$$

Passive optical resonators

Confocal resonator – determination of $\nu_{l,m,n}$ frequencies by the longitudinal phase factor

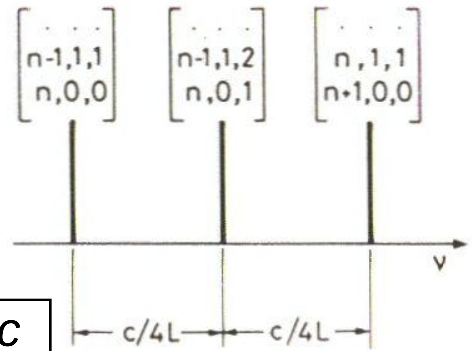
$$e^{-i[kz-(1+l+m)\zeta(z)]} = e^{-i[kz-(1+l+m)tg^{-1}(2z/L)]}, \quad \zeta(z) = tg^{-1}(2z/L)$$

$$\left[k\frac{L}{2} - (1+l+m)\overbrace{tg^{-1}1}^{\pi/4}\right] - \left[-k\frac{L}{2} - (1+l+m)\overbrace{tg^{-1}(-1)}^{-\pi/4}\right] = n\pi$$

$$kL - (1+l+m)\frac{\pi}{2} = n\pi, \quad k = \frac{2\pi\nu}{c},$$

$$\nu_{l,m,n} = \frac{c[2n+(1+l+m)]}{4L}.$$

Degenerated modes!



Frequency difference of consecutive longitudinal modes:

$$\Delta\nu_n = \nu_{l,m,n+1} - \nu_{l,m,n} = \frac{c}{2L}$$

Frequency difference of consecutive transversal modes:

$$\Delta\nu_m = \nu_{l,m+1,n} - \nu_{l,m,n} = \frac{c}{4L}$$

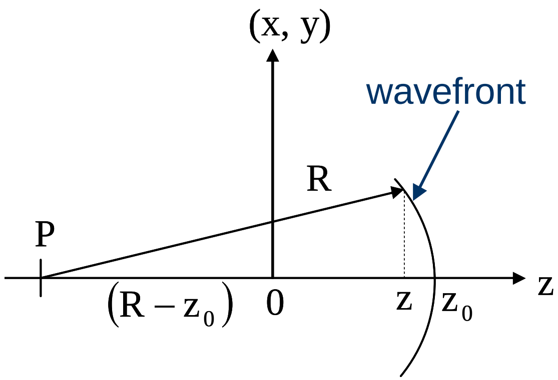
Passive optical resonators

Confocal resonator – wavefronts

Because of the transversal phase factor the surfaces of z = constant are not wavefronts $\rightarrow$ therefore the wavefronts are not plane!

$$e^{-i[kz-(1+l+m)\zeta(z)]} \cdot e^{-[ik(x^2+y^2)/2R(z)]}$$

We can determine the surfaces of constant phase by neglecting $(1+l+m)\zeta(z)$:



$$\frac{k(x^2+y^2)}{2R} + k\ z = k\ z_0$$

The left side is the equation of a paraboloid of revolution around the z axis with a radius of R at $z=z_0$!

Passive optical resonators

Confocal resonator – wavefronts

The equation of a spherical surface from point P is:

$$x^2+y^2+(z+R-z_0)^2=R^2$$

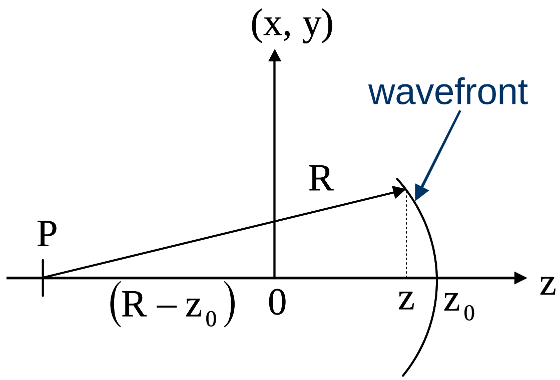
in case $z-z_0<<R$

$$(z+R-z_0)^2=R^2+2R(z-z_0)+\cancel{(z-z_0)^2},$$

$$x^2+y^2+R^2+2R(z-z_0)=R^2$$

$$x^2+y^2+2R(z-z_0)=0$$

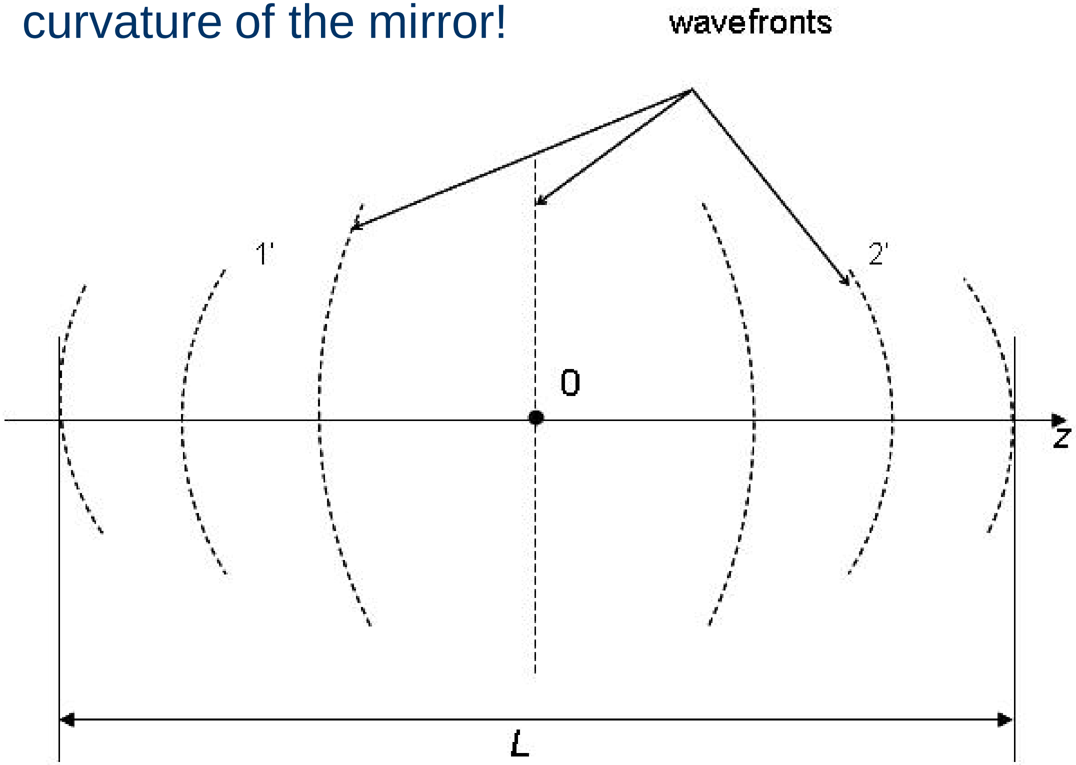
$$\frac{k(x^2+y^2)}{2R}+k\,z=k\,z_0$$



Passive optical resonators

Confocal resonator – wavefronts

The surfaces of constant phase are ~ spheres, in the middle and in the infinity the wavefronts are plane, on the mirrors the radius of curvature of the wavefront is exactly equal with the geometrical radius of curvature of the mirror!



$$R(z) = z\left[1 + (L/2z)^2\right],$$

$$z = 0, \quad R(0) = \infty,$$

$$z = \infty, \quad R(\infty) = \infty,$$

$$z = \frac{L}{2}, \quad R\left(\frac{L}{2}\right) = L.$$

Passive optical resonators

Confocal resonator – complex q parameter

The transversal part of the amplitude is:

$$U(x,y,z)=\frac{w_0}{w(z)}H_m\left[\frac{\sqrt{2}x}{w(z)}\right]H_l\left[\frac{\sqrt{2}y}{w(z)}\right]e^{-\frac{x^2+y^2}{w^2(z)}}\cdot e^{-i[kz-(1+m+l)\zeta(z)]}\cdot e^{-[ik(x^2+y^2)/2R(z)]}$$

$$U_t\sim exp\left[-\left(i\frac{k(x^2+y^2)}{2R}+\frac{x^2+y^2}{w^2}\right)\right]$$

With the notation

$$\frac{1}{q}=\frac{1}{R}-i\frac{\lambda}{\pi w^2}$$

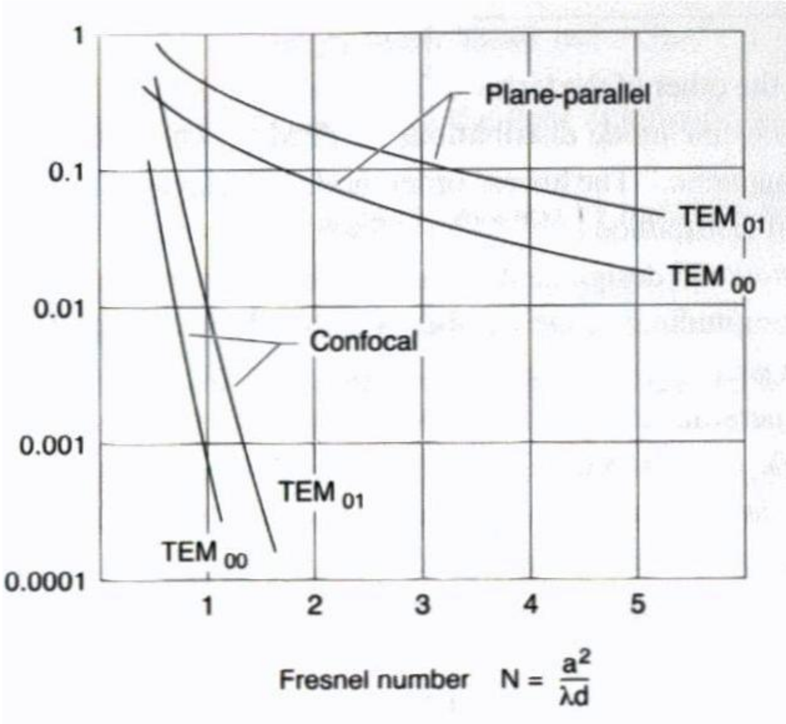
$$\mathrm{U_t}\sim exp\left[-\left(i\frac{k(x^2+y^2)}{2q}\right)\right].$$

q is the complex radius of the beam.

Important: q is full complex when the wavefront is plane!

Passive optical resonators

Confocal resonator – calculation of the loss (numerically)



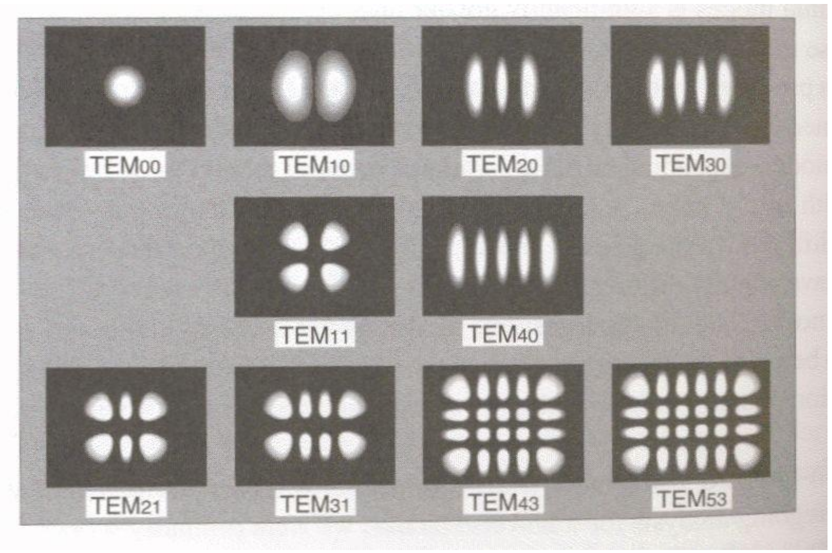
The loss in the resonator with spherical mirrors is significantly smaller $\rightarrow$ in practice the resonators consist of spherical mirrors and the radius of curvature of the mirrors is usually large compared to the length of the resonator.

Passive optical resonators

<u>Gaussian modes</u> – transversal mode patterns (solutions of the diffraction integral at given symmetry)

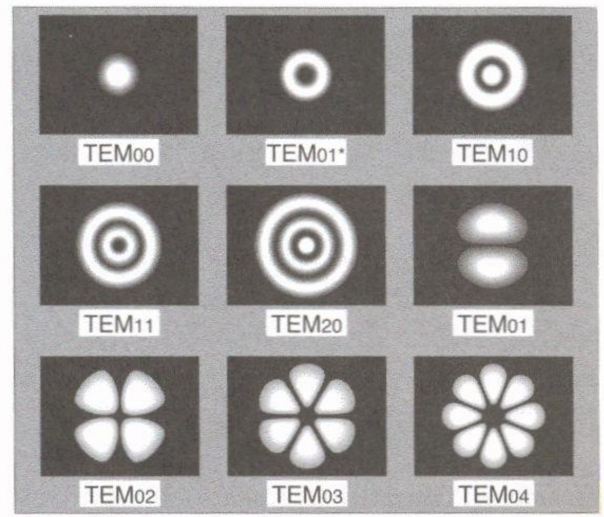
Hermite-Gaussian modes

x, *y* symmetry

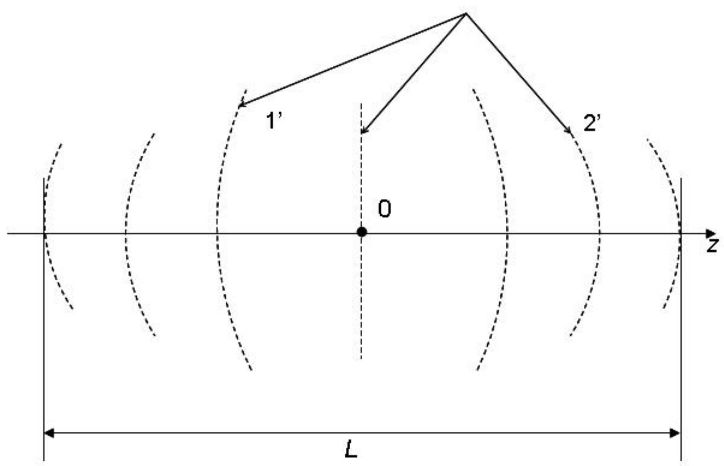


Laguerre-Gaussian modes

circular symmetry



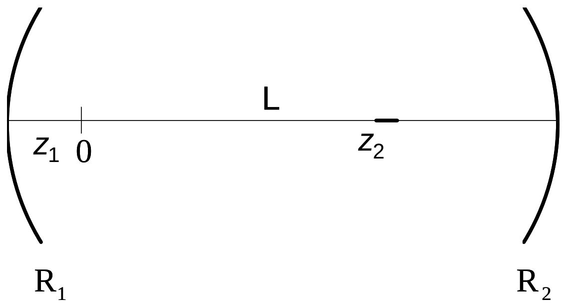
Passive optical resonators



General resonator

We can put a mirror with a suitable radius of curvature in the place where the wavefront bending is the same, e.g. in place of wavefronts 1' and 2' and we will have a general resonator.

All stable resonator with at least one spherical mirror has its equivalent confocal resonator.



$$R_1 = z_1\left[1+\left(\frac{z_c}{2z_1}\right)^2\right], \quad R_2 = z_2\left[1+\left(\frac{z_c}{2z_2}\right)^2\right],$$

$$z_1 + z_2 = L$$

In the symmetrical resonator *(R_1=R_2=R):*

$$z_c^2 = (2R - L)L, \quad \text{mert} \quad z = \frac{L}{2}$$