



Laser Physics 13.

Coherent optical amplifier

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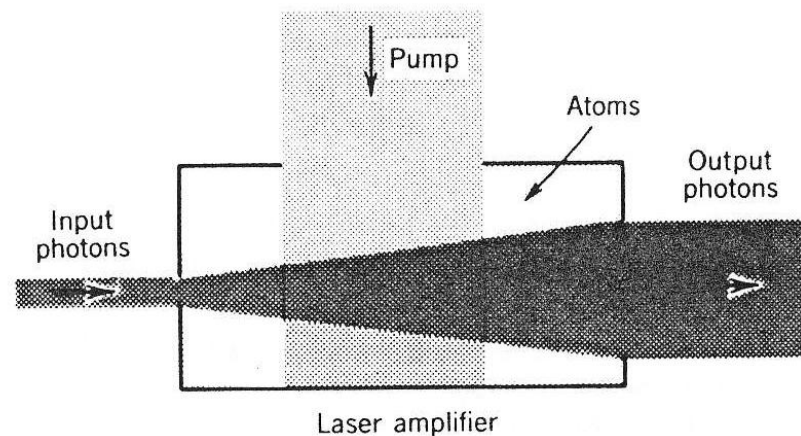


Optical amplifiers

Incoherent optical amplifier – the signal intensity increases without preserving the phase

Coherent optical amplifier – increases the amplitude of an optical field while maintaining its phase

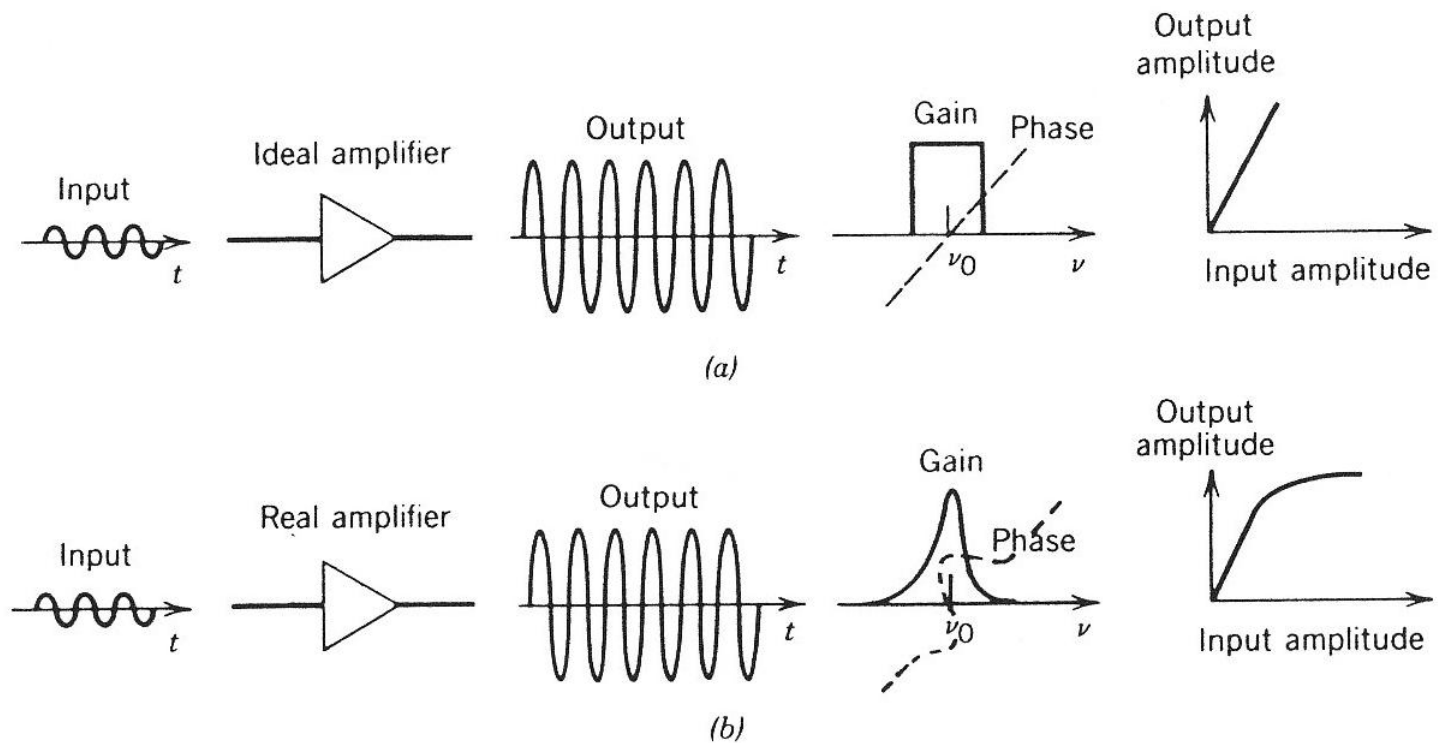
E.g. in lasers or when giant pulses are generated (ignition of nuclear fusion)



Stimulated emission: selective pumping of the upper state is necessary!



Coherent optical amplifier



Ideal (a) and real (b) amplifier - comparison.



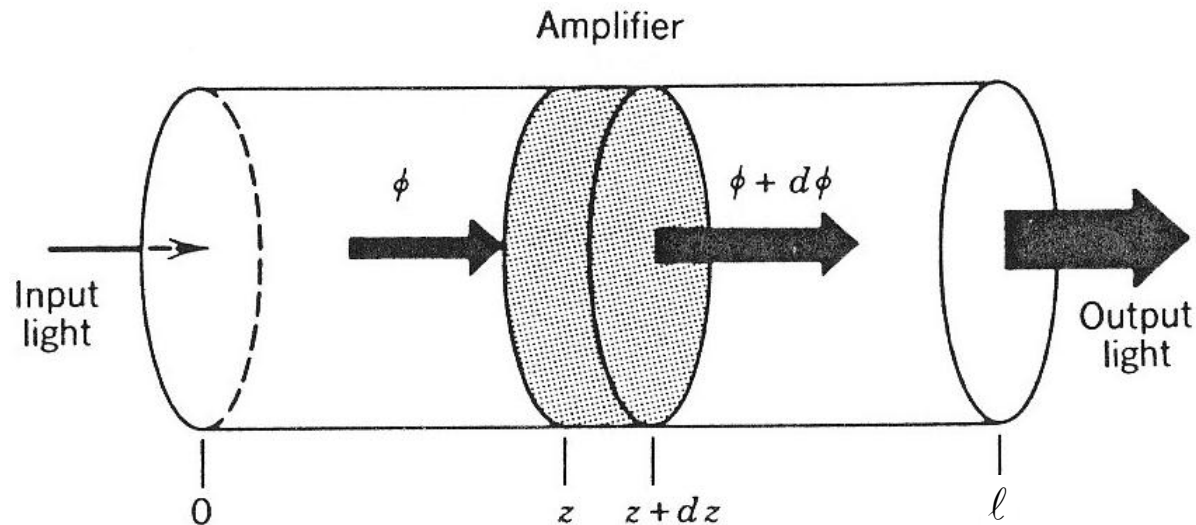
Coherent optical amplifier

Gain, bandwidth, phase shift, power source, nonlinearity and noise

The monochromatic optical plane wave traveling in z direction in the laser material with frequency ν can be characterized:

$$\text{Re}\{E(z)e^{j2\pi\nu t}\}, \quad I(z) = \frac{E^2(z)}{2\eta}, \quad \Phi(z) = \frac{l(z)}{h\nu},$$

η is the impedance of the dielectric medium.





Coherent optical amplifier

Amplifier gain

The upper level is E_2 , the light-matter interaction is between levels 2 and 1, the resonance frequency is ν_0 . N_1 and N_2 are particle densities in the levels (number of particles in unit volume in the given state).

Absorption: $N_1 W_i$ (in unit time and volume)

Stimulated emission: $N_2 W_i$ (in unit time and volume)

$(N_2 - N_1) W_i > 0$ amplification

$(N_2 - N_1) W_i < 0$ attenuation

$$N_2 - N_1 = N$$

population difference
or inversion density

$N > 0$ amplifier medium

$N < 0$ attenuator medium

$N = 0$ transparent medium

$$W_i = \Phi \sigma(\nu).$$

$$\sigma(\nu) = S g(\nu) = \frac{\lambda^2}{8 \pi t_{sp}} g(\nu)$$



Coherent optical amplifier

Amplifier gain (cont.)

The increase of photon-flux density in unit length:

$$d\Phi = NW_i dz,$$

$$\frac{d\Phi}{dz} = NW_i = \underbrace{N\sigma(\nu)}_{\gamma(\nu)} \Phi(z),$$

$$\boxed{\gamma(\nu) = N\sigma(\nu)} = N \frac{\lambda^2}{8\pi t_{sp}} g(\nu),$$

net gain of the photon-flux density in unit length of the medium.

In case of an ideal amplifier with gain independent on the photon-flux density:

$$\Phi(z) = \Phi(0)e^{\gamma(\nu)z},$$

$$I(z) = h\nu \Phi(z) = I(0)e^{\gamma(\nu)z}.$$

If $N < 0$,

$$\Phi(z) = \Phi(0)e^{-\alpha(\nu)z},$$

$$\alpha(\nu) = -\gamma(\nu) = -N\sigma(\nu)$$

attenuation coefficient of the medium.



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Amplifier bandwidth – determined by the lineshape function of the atomic transition:

$$\gamma(\nu) = N\sigma(\nu) = N \frac{\lambda^2}{8\pi t_{sp}} g(\nu).$$

Phase shift

If

$$I(z) = I(0)e^{\gamma(\nu)z},$$

phase shift per unit length of the amplifier medium

$$E(z) = E(0)e^{\frac{1}{2}\gamma(\nu)z} e^{-j\phi(\nu)z},$$

Taylor-series approximation

$$E(z + \Delta z) = E(z)e^{\frac{1}{2}\gamma(\nu)\Delta z} e^{-j\phi(\nu)\Delta z} \approx E(z) \left[1 + \frac{1}{2}\gamma(\nu)\Delta z - j\phi(\nu)\Delta z \right].$$

$$\frac{\Delta E(z)}{dz} = \frac{E(z + \Delta z) - E(z)}{\Delta z} = E(z) \left[\underbrace{\frac{1}{2}\gamma(\nu) - j\phi(\nu)}_{\text{transfer function}} \right].$$

transfer function

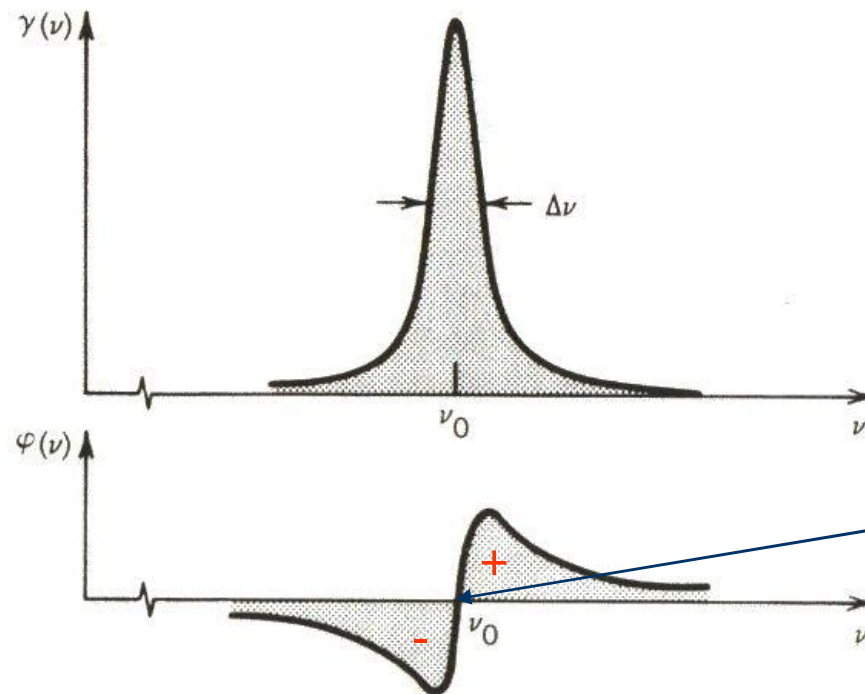


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Phase shift (cont.)

In case of homogeneous amplifier, when $\Delta\nu \ll \nu_0$

$$\varphi(\nu) = \frac{\Delta\nu}{\nu - \nu_0} \gamma(\nu).$$



$$g_L(\nu) = \frac{\Delta\nu / 2\pi}{(\nu - \nu_0)^2 + (\Delta\nu / 2)^2},$$

$$\Delta\nu = \frac{1}{2\pi\tau}, \quad g_L(\nu_0) = \frac{2}{\pi \Delta\nu}.$$

There is no phase shift at the resonance frequency!

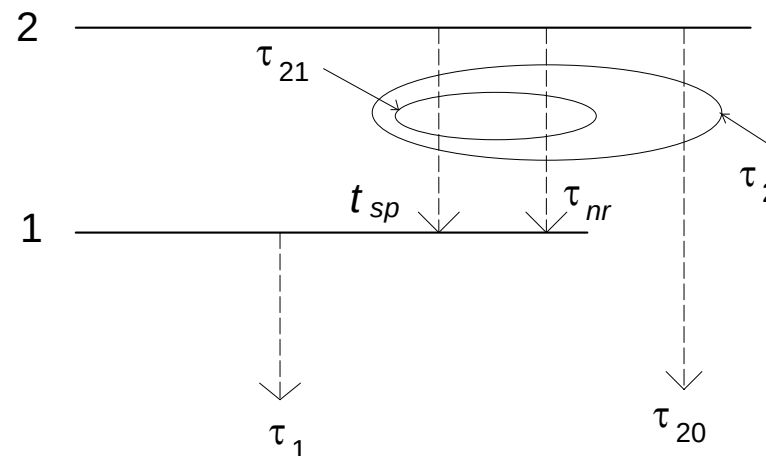


Coherent optical amplifier

Amplifier power source – pumping the laser medium

The task is to generate $N = N_2 - N_1 > 0$. Methods depend on the laser medium.

Are any two different levels suitable for laser action, or does exist an ideal selection?



$$\frac{1}{\tau_{21}} = \frac{1}{t_{sp}} + \frac{1}{\tau_{nr}},$$

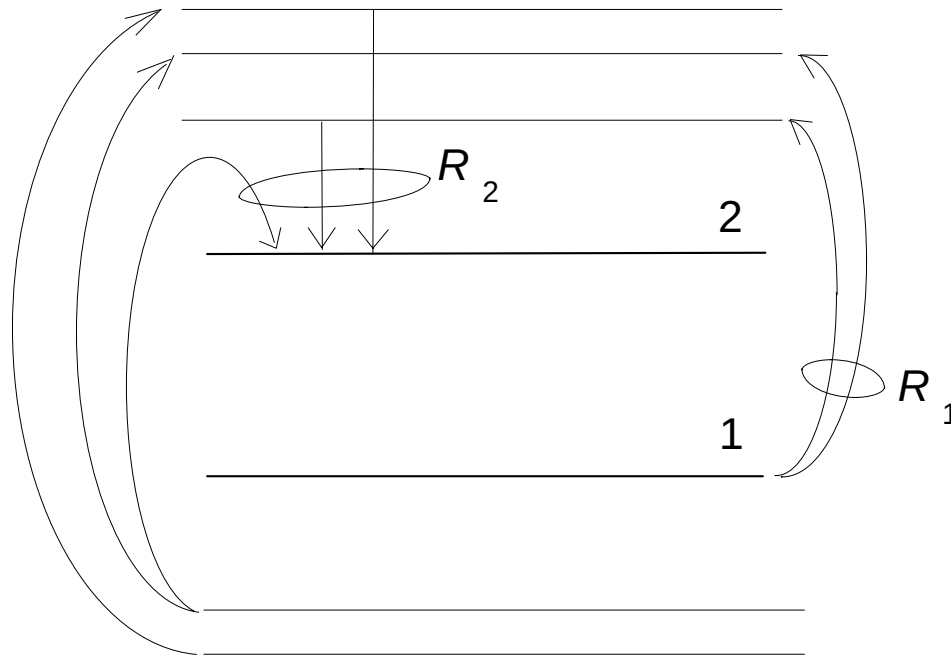
$$\frac{1}{\tau_2} = \frac{1}{\tau_{21}} + \frac{1}{\tau_{20}}.$$

Decay processes of the two selected energy levels



Coherent optical amplifier

Amplifier power source – pumping the laser medium (cont.)



R_2 is the pumping rate of level 2 (velocity of pumping in unit volume and in unit time),

R_1 is the depletion rate of level 1 (in unit volume and in unit time).

Pumping processes of the two-level amplifier embedded in the multi-level energy system of the medium.

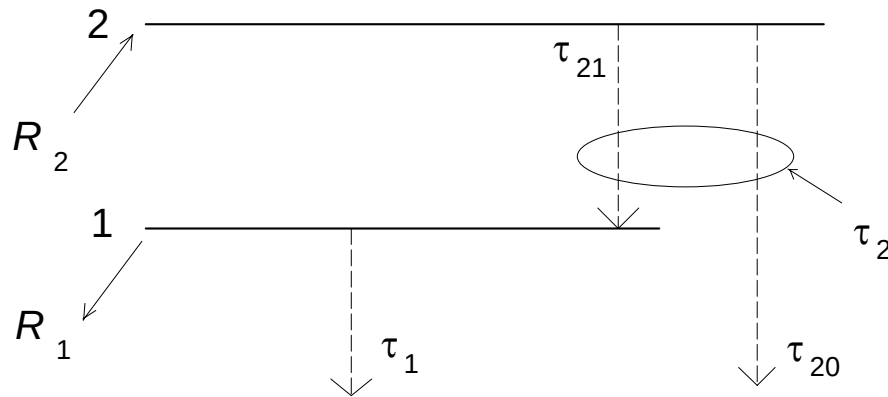


Coherent optical amplifier

Amplifier power source – pumping the laser medium (cont.)

Rate equations in the absence of the amplifier radiation in unit volume

The rates of changes in the population densities of levels 2 and 1 caused by pumping and decay:



$$\frac{dN_2}{dt} = R_2 - \frac{N_2}{\tau_2},$$

$$\frac{dN_1}{dt} = -R_1 - \frac{N_1}{\tau_1} + \frac{N_2}{\tau_{21}}.$$

Under steady-state conditions $\frac{dN_1}{dt} = \frac{dN_2}{dt} = 0$ the solution for N

$$N_2 = R_2 \tau_2, \quad N_1 = -R_1 \tau_1 + N_2 \frac{\tau_1}{\tau_{21}} = -R_1 \tau_1 + R_2 \frac{\tau_2 \tau_1}{\tau_{21}},$$

$$N = N_0 = N_2 - N_1 = R_2 \tau_2 \left(1 - \frac{\tau_1}{\tau_{21}} \right) + R_1 \tau_1.$$

N_0 is the steady-state population difference in the absence of resonant radiation – that is to be amplified.

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Coherent optical amplifier

Amplifier power source – pumping the laser medium (cont.)

The steady-state population difference in the absence of the radiation to be amplified

$$N_0 = N_2 - N_1 = R_2 \tau_2 \left(1 - \frac{\tau_1}{\tau_{21}} \right) + R_1 \tau_1.$$

N_0 is high, when:

R_1, R_2 have high values,

τ_2 is high (the upper level has long lifetime),

τ_1 is short - if $R_1 < \frac{\tau_2}{\tau_{21}} R_2$.

Intensive pumping of the upper level, its slow decay (long lifetime) and quick decay of the lower level are necessary. In ideal case:

$$\tau_{21} \approx t_{sp} \ll \tau_{20} \rightarrow \tau_2 = t_{sp} \quad \text{és} \quad \tau_1 \ll t_{sp},$$

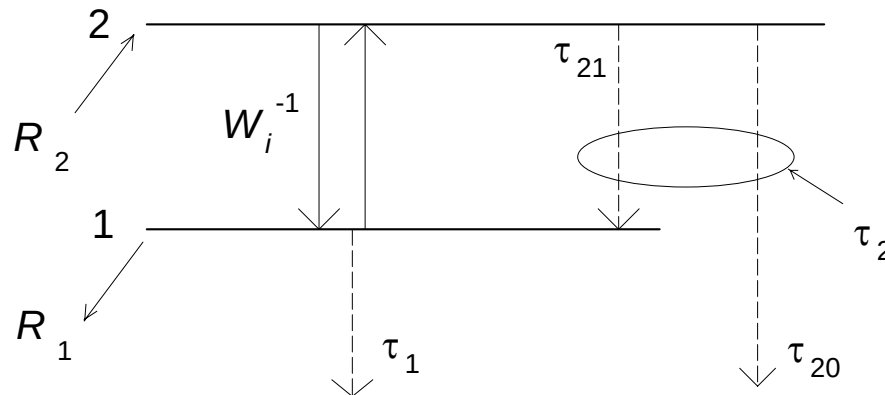
$$N_0 = R_2 t_{sp} + R_1 \tau_1.$$



Coherent optical amplifier

Amplifier power source – pumping the laser medium (cont.)

Rate equations in the presence of the radiation to be amplified - in unit volume



$$\frac{dN_2}{dt} = R_2 - \frac{N_2}{\tau_2} - N_2 W_i + N_1 W_i,$$

$$\frac{dN_1}{dt} = -R_1 - \frac{N_1}{\tau_1} + \frac{N_2}{\tau_{21}} + N_2 W_i - N_1 W_i.$$

Under steady-state conditions:

$$N = \frac{N_0}{1 + W_i \tau_s},$$

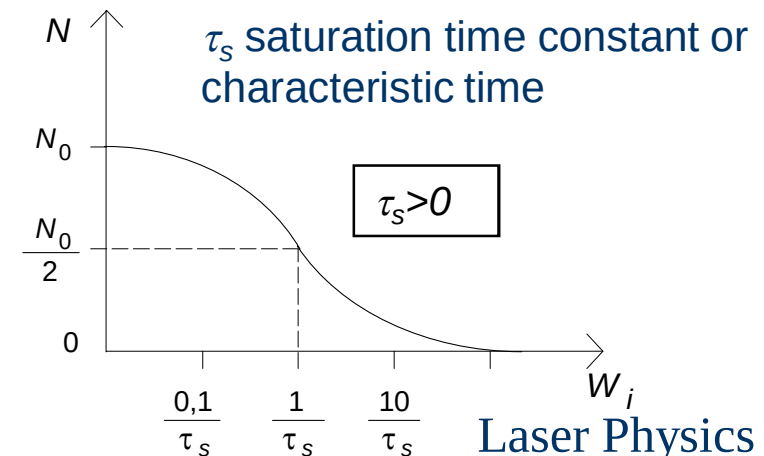
$$\tau_s = \tau_2 + \tau_1 \left(1 - \frac{\tau_2}{\tau_{21}} \right).$$

$$\frac{dN_1}{dt} = \frac{dN_2}{dt} = 0$$

$$N = N_0, \text{ if}$$

$$W_i = 0$$

Inversion density

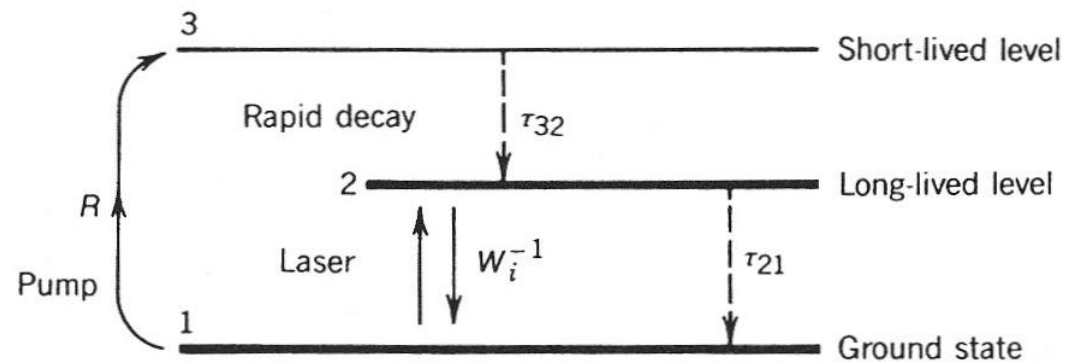




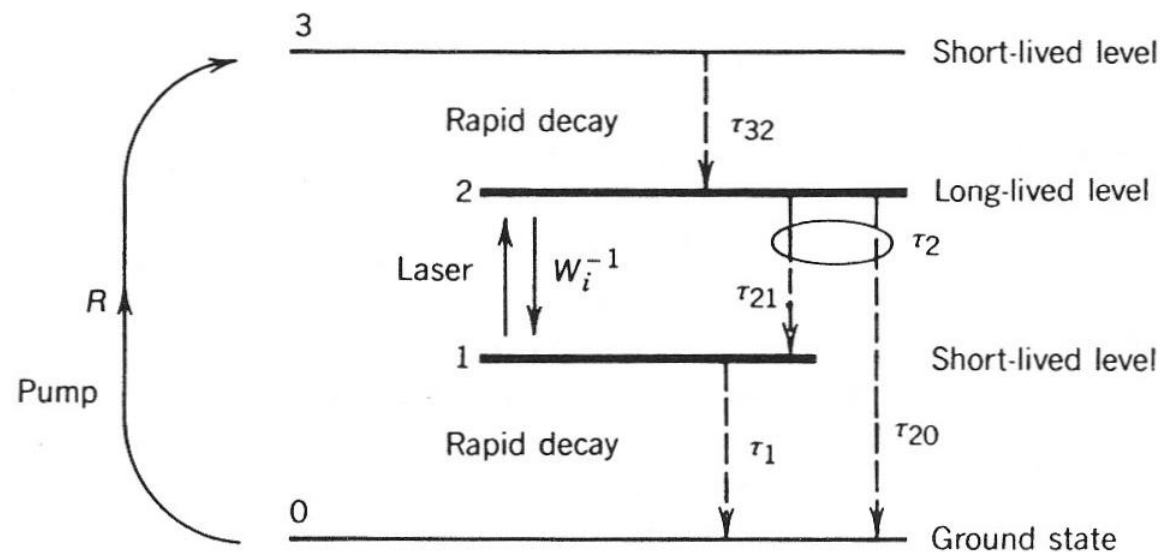
Coherent optical amplifier

Three- and four-level pumping schemes

Three-level



Four-level





Coherent optical amplifier

Pumping in the practice

Pumping methods of commercial lasers: *optical or electrical*.

Optical pumping by

- a conventional light source (e.g. lamps pumping of solid-state or dye lasers)
- an other laser (e.g. a solid-state laser pumped by a laser diode or a dye laser pumped by Ar^+ , N_2 , excimer laser)

Electrical pumping

- gas discharge $\rightarrow$ pumping method of gas lasers, e.g. He-Ne, Ar^+ , CO_2
- direct current $\rightarrow$ current flowing through the p-n junction of the laser diode



Coherent optical amplifier

Pumping in the practice – *optical*, by conventional light sources

e.g. a solid-state laser, the form of the laser active material is rod with dimensions: diameter of mm-cm, length of 10-20 cm.

The pump efficiency:

$$\eta = \eta_r \eta_t \eta_a \eta_{pq} \approx 0.01 - 0.03$$

$$\eta_r = \frac{\text{radiated pump energy}}{\text{input electrical energy}}$$

Emission efficiency $\sim 0.25 - 0.5$

$$\eta_t = \frac{\text{optical energy entering the medium}}{\text{emitted (radiated) pump energy}}$$

Transfer efficiency (characterizes the geometry) $\sim 0.3 - 0.8$

$$\eta_a = \frac{\text{absorbed pump energy}}{\text{entrant pump energy}}$$

Absorption efficiency $\sim 0.1 - 0.6$

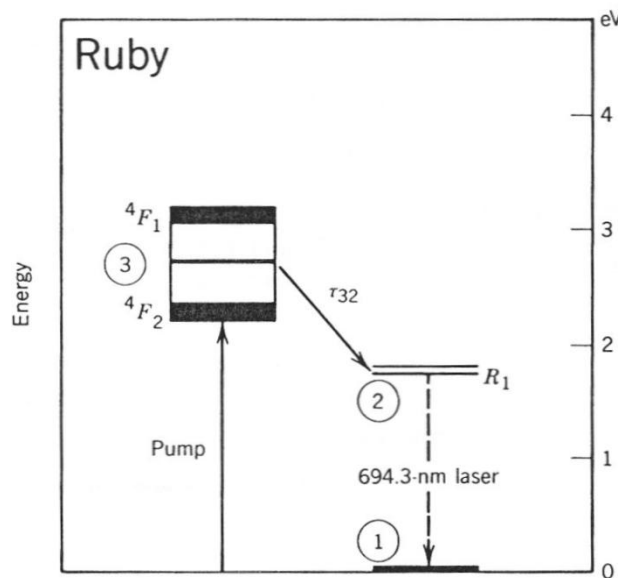
η_{pq} quantum efficiency $\sim 0.4 - 0.5$, the fraction of the absorbed energy used for pumping the upper laser level.



Coherent optical amplifier

Pumping in the practice – *optical*, by conventional light sources (examples)

3-level ruby laser

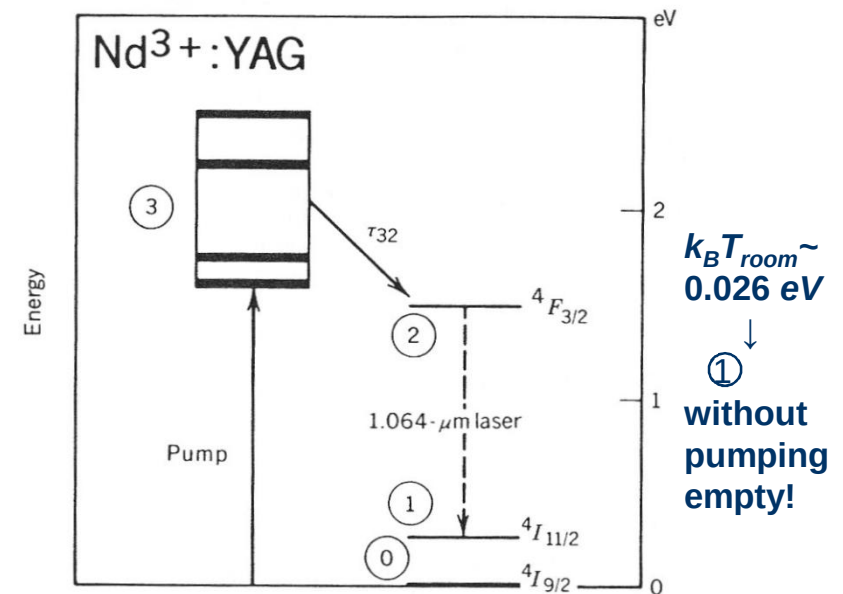


- 1 – laser lower level (top of the valence band)
- 2 – laser upper levels (2 pcs)
- 3 – conduction bands ($0.42 \mu\text{m}$ and $0.55 \mu\text{m}$ in λ from level 1)

$$\tau_{32} \sim \text{ps}, \tau_{21} \sim t_{sp} \sim 3 \text{ ms}$$

$$\Delta\nu \sim 330 \text{ GHz (300 K) – hom.}$$

4-level Nd³⁺:YAG laser



- 0 – top of the valence band,
- 1, 2 – laser levels
- 3 – 4 conduction bands (width of 30 nm : 0.52 ; 0.58 ; **0.75 and $0.81 \mu\text{m}$** in λ - distance from level 0)

$$\tau_{32} \sim 100 \text{ ns}, \tau_{21} \sim t_{sp} \sim 1.2 \text{ ms}, \tau_1 \sim 30 \text{ ns}$$

$$\Delta\nu \sim 120 \text{ GHz (300 K) – hom.}$$

$k_B T_{\text{room}} \sim 0.026 \text{ eV}$

① without pumping empty!

Pumping with diode laser!!! Laser Physics 13



Coherent optical amplifier

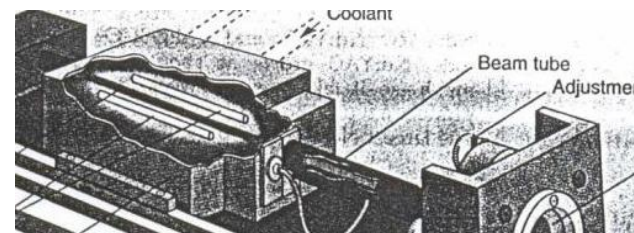
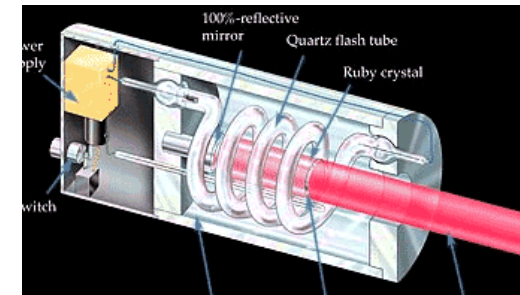
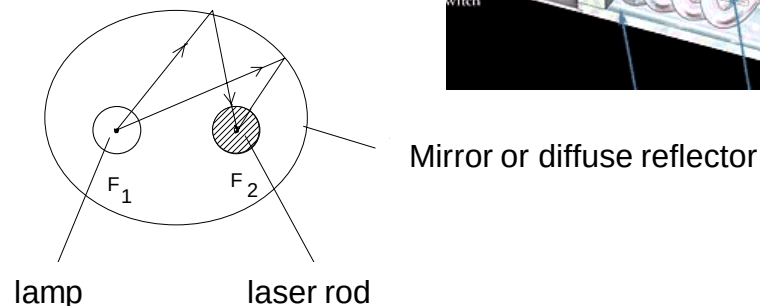
Pumping in the practice – *optical*, by conventional light sources (cont.)

Usual configurations

a) Lamp around the laser medium (1. laser)

b) Elliptical cylinder

c) Close coupling with multi lamps



High pressure lamps are used for pumping:

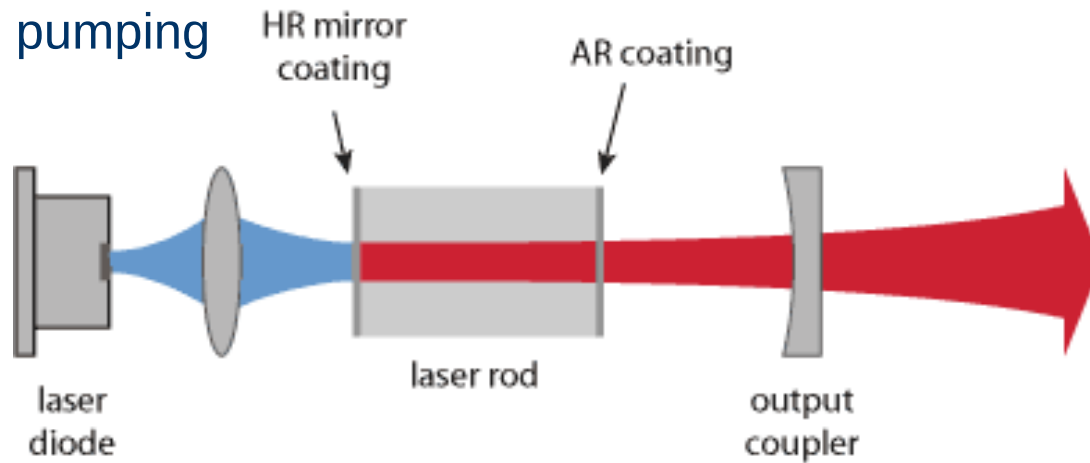
for continuous wave – cw: 4000 - 8000 torr e.g. Kr

for pulsed operation: 400 - 1600 torr e.g. Xe flash lamp

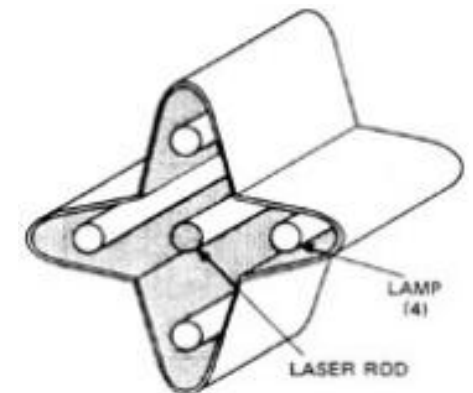


Pumping schemes

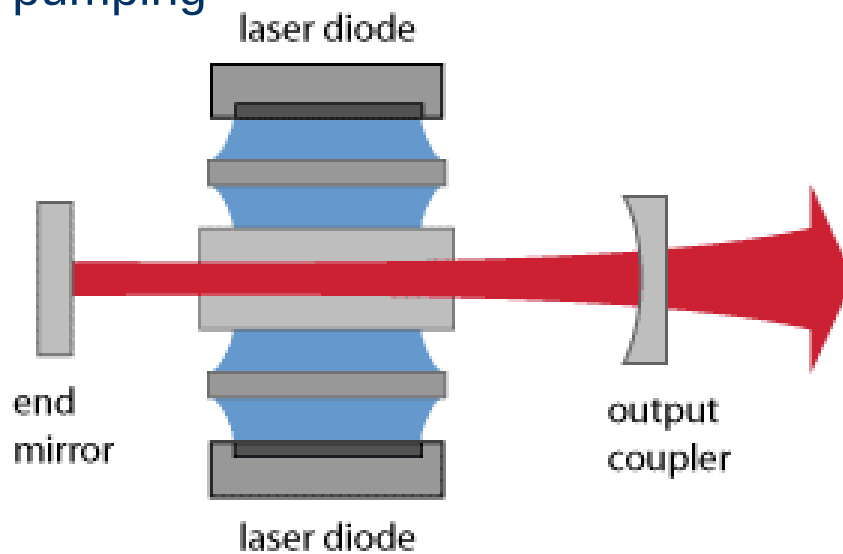
End pumping



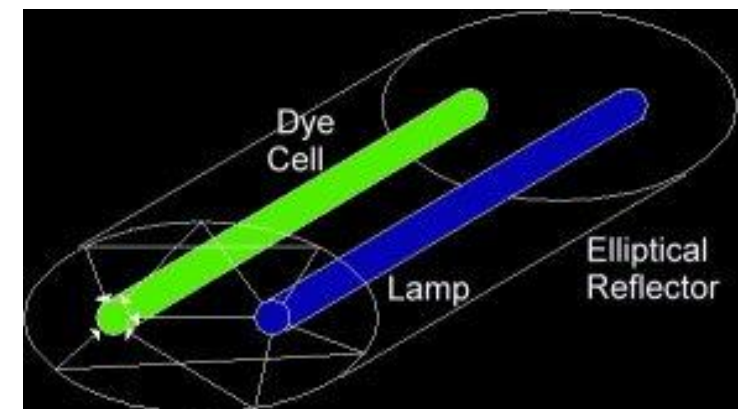
Four combined pumps



Side pumping



Flashlamp

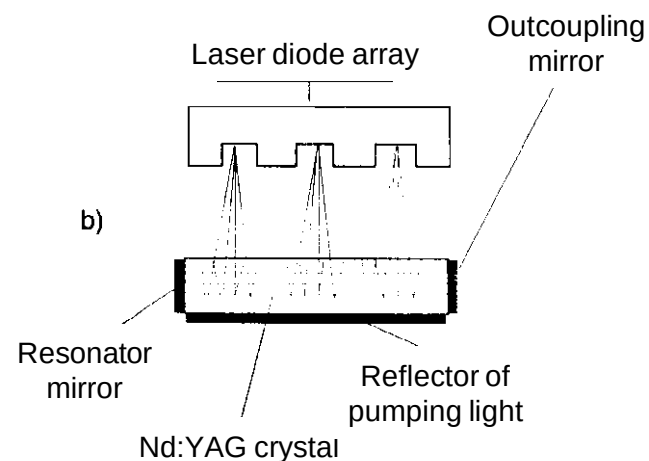
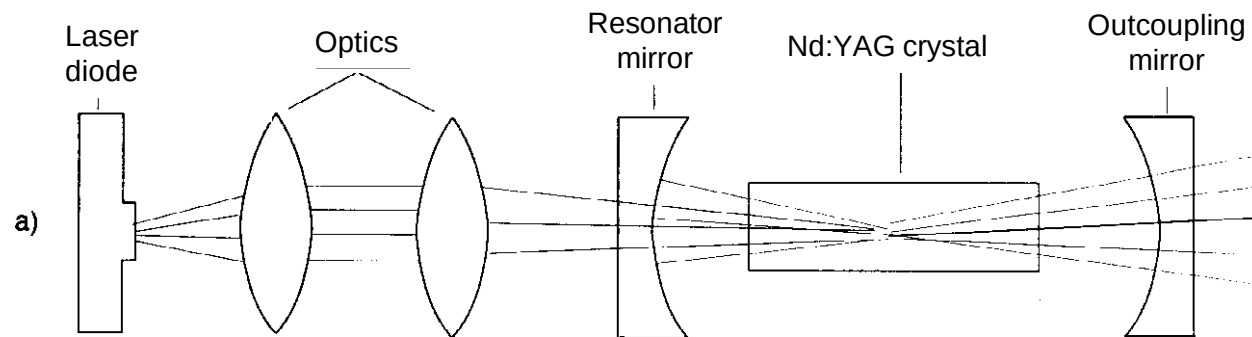




Coherent optical amplifier

Pumping in the practice – *optical* (examples)

Geometry used for pumping with a laser diode:



a) longitudinal

b) transversal

$\eta > 30\%$ at least 10x
better, than with
lamps!



Coherent optical amplifier

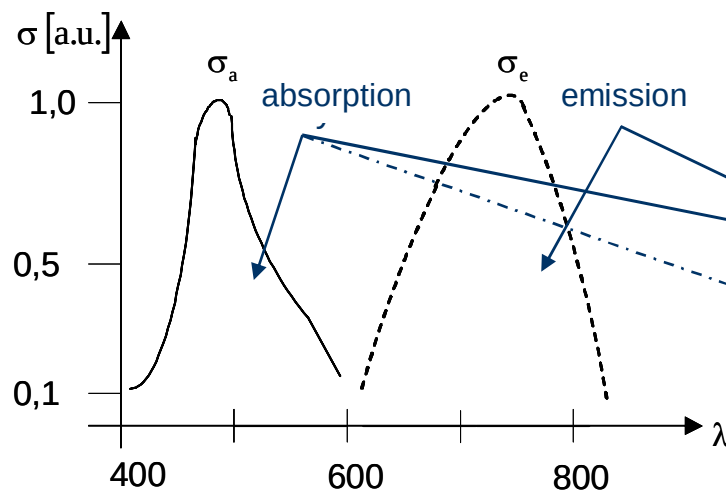
Pumping in the practice – *optical*, with lamp or with another laser (examples)

Tunable solid-state lasers

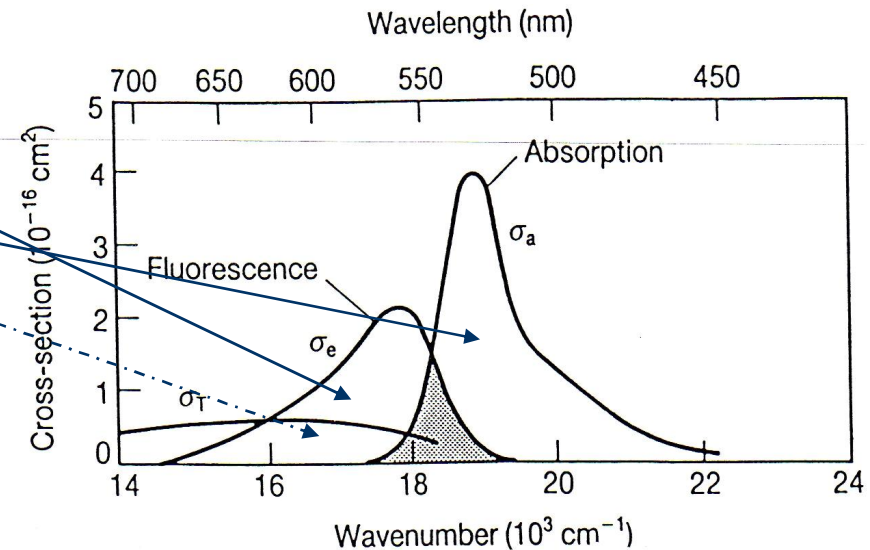
alexandrite ($\text{Cr}^{3+}:\text{BeAl}_2\text{O}_4$) and
titanium sapphire ($\text{Ti}^{3+}:\text{Al}_2\text{O}_3$)

Dye lasers

Dye	Tuning range
	$\Delta\lambda$ [μm]
Rhodamin	0,56-0,64
Coumarin	0,45-0,5
Stilbene	0,41-0,47



Pumping with a flash lamp or with an Ar^+ laser.



Pumping with *ns* laser pulse, flash lamp or cw laser (Ar^+).



Coherent optical amplifier

Pumping in the practice – *electrical* with gas discharge

Pumping of atoms, ions, and molecule lasers. The density of laser material is small → small gain in unit volume! High power gas lasers are physically large and require high voltage supply. Commercial gas lasers use discharge excitation.

First step: generation of charges

A high enough electric field is necessary to break down the gas and to create free electrons and positive ions.

The electric field can be: DC, pulsed, or RF.

The direction of the discharge can be: longitudinal (parallel with the laser axis) or transversal (perpendicular to the laser axis). Transverse dimension is usually much smaller, therefore less voltage is needed.

Advantageous when the gas pressure is high (e.g. high pressure CO₂ lasers with transverse discharge – transverse electric atmospheric TEA laser)!



Coherent optical amplifier

Pumping in the practice – *electrical* with gas discharge (cont.)

Collision processes:

Collision of the first kind in a gas with only one component (only atoms A):

1. $e + A \rightarrow A^* + e$ excitation by electron impact,
2. $e + A \rightarrow A^+ + 2e$ ionization,
3. $e + A^+ \rightarrow A^{+*} + e$ excited ion

Collision of the second kind or resonant energy transfer in a gas with two components (atoms A and B):

4. $A^* + B \rightarrow A + B^* \pm \Delta E$
5. $A^* + B \rightarrow A + B^{+*} + e \pm \Delta E$ Penning-ionization
6. $A^+ + B \rightarrow A + B^{+*} \pm \Delta E$ charge transfer



Coherent optical amplifier

Pumping in the practice – *electrical* with gas discharge (examples)

He-Ne laser – excited by process 4

Laser material is **neon**, the role of He is in the pumping of the upper laser levels of Ne.

He: two electrons

1^1s ground level, 2 electrons with $\uparrow\downarrow$ spins

2^3s first excited level, the spin of the electron is parallel to the spin of the ground state electron (3 different positions)

2^1s next excited level, one electron with spin opposite to the electron in the ground state (one position)

2^3s and 2^1s are metastable levels!

Ne: 10 electrons

Ground state: $1s^2 2s^2 2p^6$

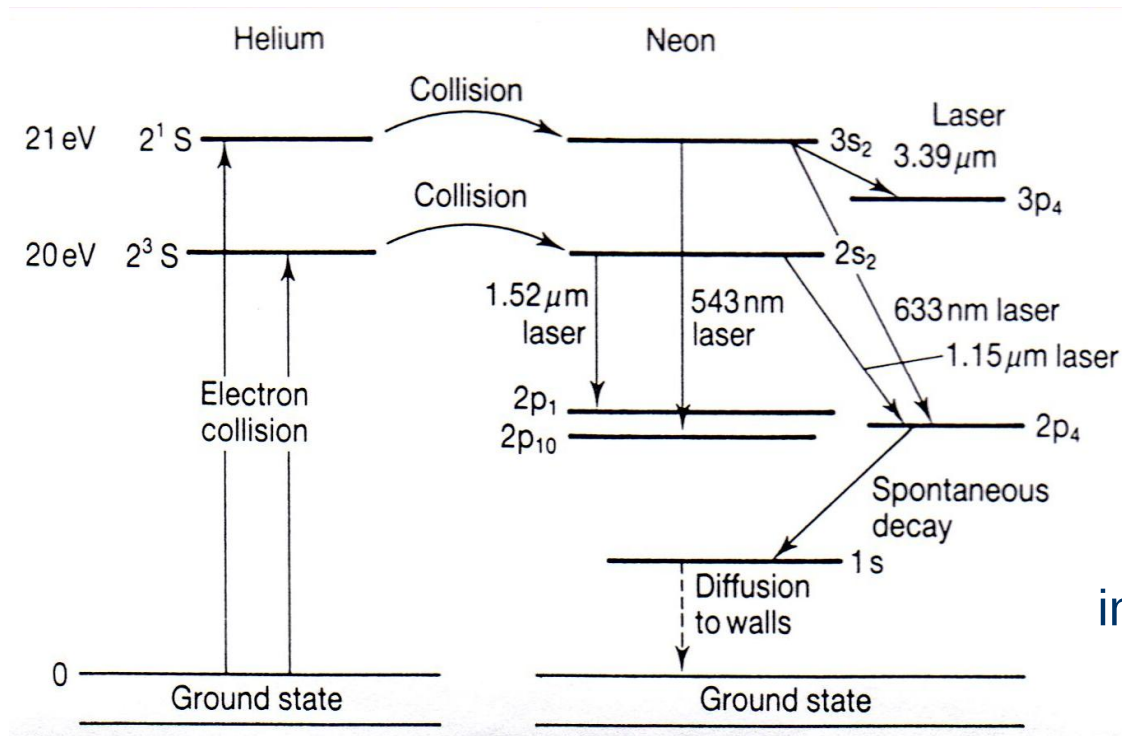
Excited states: 2p electron excited to 3s, 4s, 5s or 3p, 4p states

Lasing: between different $s \rightarrow p$ transitions



Coherent optical amplifier

Pumping in the practice – *electrical* with gas discharge (examples)
He-Ne laser (cont.)



$$\tau_s \cong 100 \text{ ns}, \quad \tau_p \cong 10 \text{ ns}$$

$$\tau^{-1} = \tau_s^{-1} + \tau_p^{-1}$$

$$\Delta\nu_n = \frac{1}{2\pi\tau} = 19 \text{ MHz}$$

$$\Delta\nu_D \sim 1.5 \text{ GHz}$$

inhomogeneous broadening!

$$\eta < 0.001$$

Typical lifetime: ~25000 hours

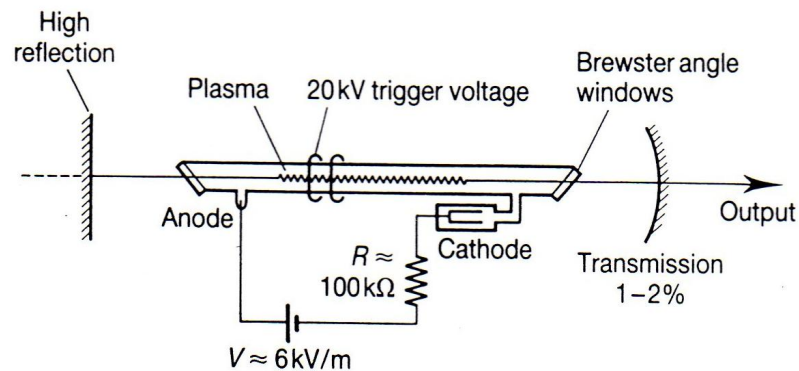


Coherent optical amplifier

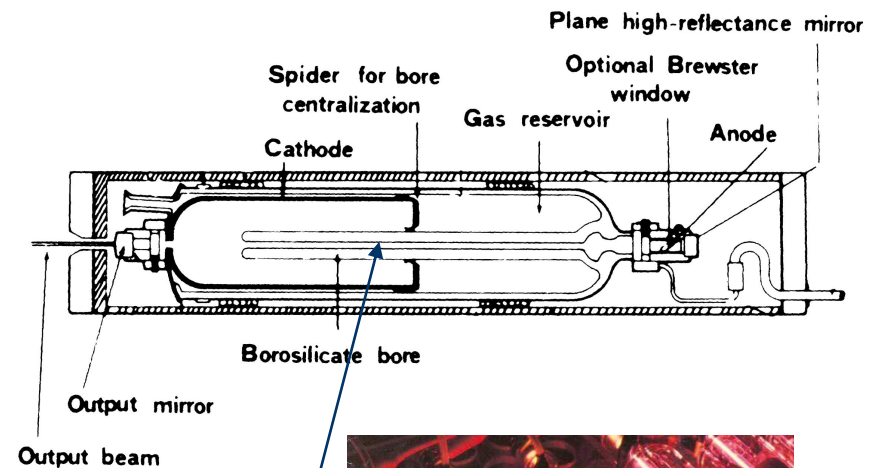
Pumping in the practice – *electrical* with gas discharge (examples)

He-Ne laser (cont.)

first solution



advanced solution



633 nm He-Ne laser typical operation parameters:

P_{out} : 5–10 mW (max. 50 mW)

p : 3–5 torr (He:Ne 5:1)

d : 0.8 – 1.5 mm

I_{op} : 3 – 5 mA

d is the bore diameter, place of the excitation



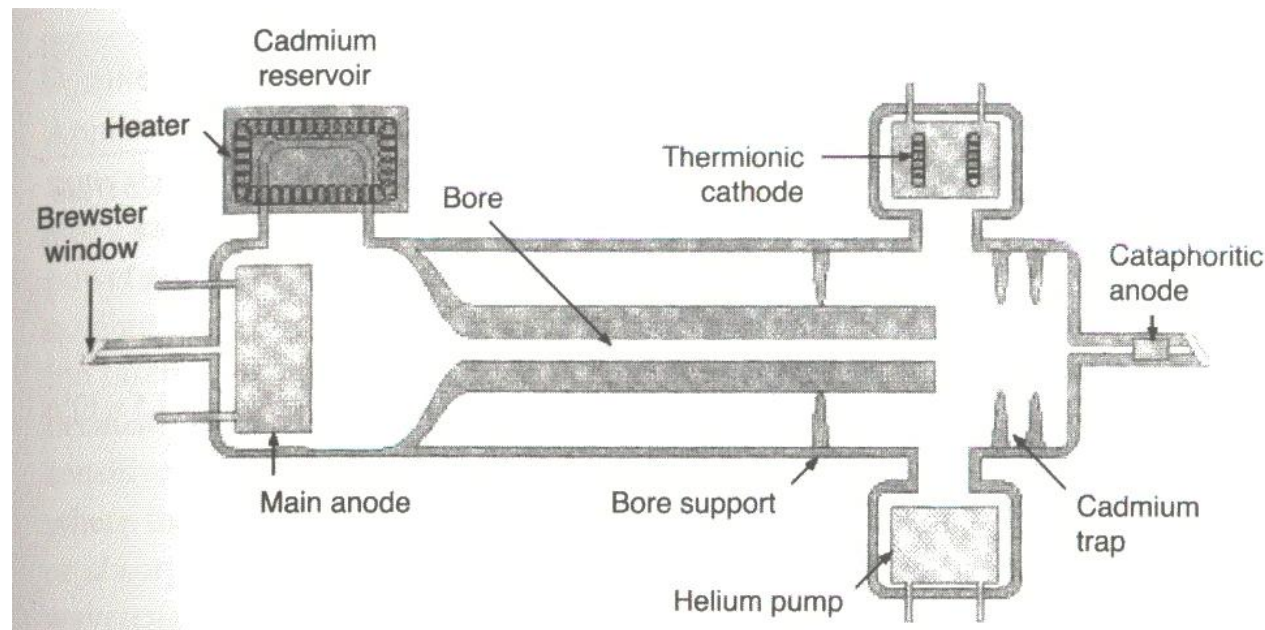


Coherent optical amplifier

Pumping in the practice – *electrical* with gas discharge (examples)

He-Cd or Cd^+ laser – excited with process 5

Metal vapor laser. Known wavelengths: 325 nm, 441,6 nm. The metal must be evaporated from a reservoir, then distributed uniformly down the laser bore. The distribution is accomplished through a process called electrophoresis. The amount of cadmium continuously decreases. Typical lifetime: ~8000 hours.





Coherent optical amplifier

Pumping in the practice – *electrical* with gas discharge (examples)

Ar⁺ laser – excited with processes 1, 2, 3 $\eta < 0.001$

Ionization energy of Ar ~16 eV, energies of excited ion levels ~20 eV.
Instead of *mA* the current is *A* → cooling is important. *mW* with air cooling, *W* with liquid cooling (water or oil).

Known wavelengths : 488 *nm*, 515 *nm*, but there are transitions in the *UV* and between 454 – 529 *nm* 10 different wavelengths

CO₂ laser

CO₂:He:N₂ gases in the discharge, N₂ takes part in the excitation, the role of He is depletion of the lower levels.

Wavelength: 10.6 μm typical, but around 9.6 and 10.6 μm tunable to several discrete lines (spectroscopic application)

Problem: **dissociation** of the CO₂ molecule in the discharge, laser material „disappears”!



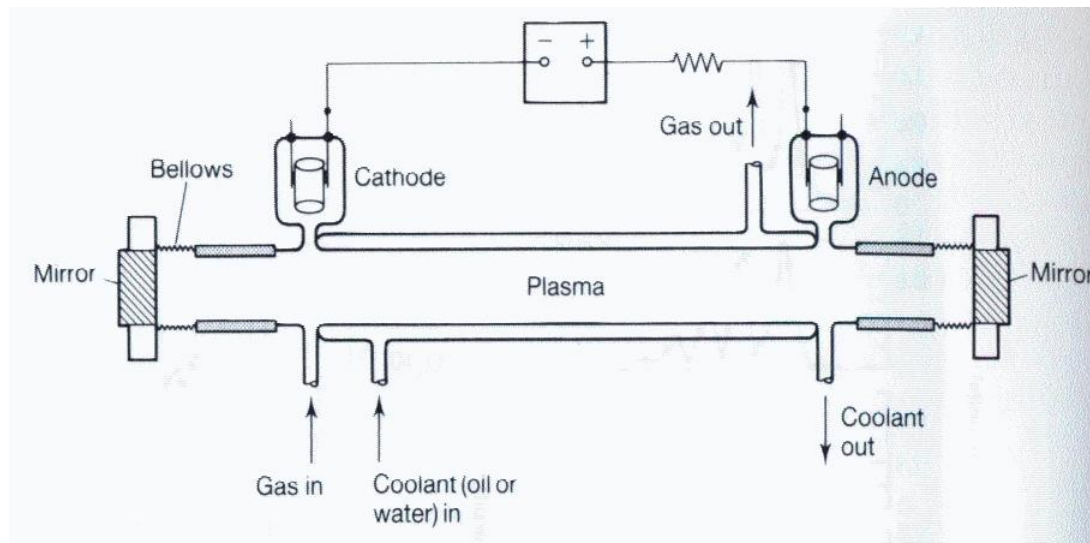
Coherent optical amplifier

Pumping in the practice – *electrical* with gas discharge (examples)

CO₂ laser (cont.)

Solution: e.g. continuous flow of the gas mixture. Laser sizes are much higher because of the wavelength. Diameter of *cm* and The material of the optical elements (mirrors, lenses) is different, glass is not transparent at $\sim 10\text{ }\mu\text{m}$! Suitable materials are Ge, ZnSe, GaAs.

Conventional design with slow flow of the gas mixture:



$$\eta > 0.1!$$



Coherent optical amplifier

Summary

Medium	$\lambda[\mu\text{m}]$	$\sigma_0[\text{cm}^2]$	t_{sp}	$\Delta\nu$	H hom.	n
					I inhom.	
He-Ne	0.6328	$1 \cdot 10^{-13}$	$0.7 \mu\text{s}$	1.5 GHz	I	~ 1
Ruby	0.6943	$2 \cdot 10^{-20}$	3 ms	330 GHz	H	1.76
Nd ³⁺ -YAG	1.064	$4 \cdot 10^{-19}$	1.2 ms	120 GHz	H	1.82
Nd ³⁺ :glass	1.06	$3 \cdot 10^{-20}$	0.3 ms	3THz	I	1.5
Er ³⁺ :fiber	1.55	$6 \cdot 10^{-21}$	10.0 ms	4 THz	H/I	1.46
Rhodamin 6G	0.56-0.64	$2 \cdot 10^{-16}$	3.3 ns	5 THz	H/I	1.33
Ti ³⁺ :Al ₂ O ₃	0.66-1.18	$3 \cdot 10^{-19}$	$3.2 \mu\text{s}$	100 THz	H	1.76
CO ₂	10.6	$3 \cdot 10^{-18}$	2.9 s	60 MHz	I	~ 1
Ar ⁺	0.515	$3 \cdot 10^{-12}$	10.0 ns	3.5 GHz	I	~ 1