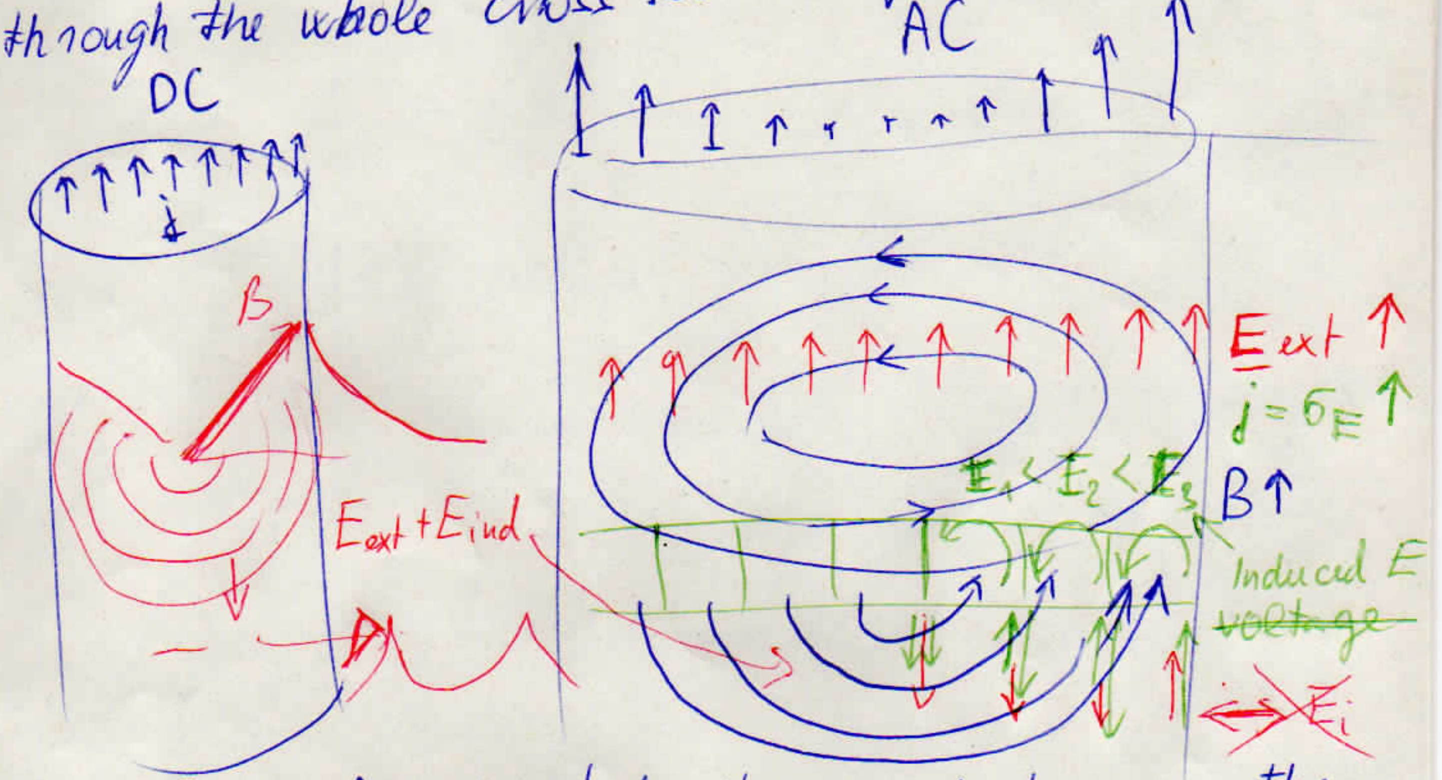
Skin effect

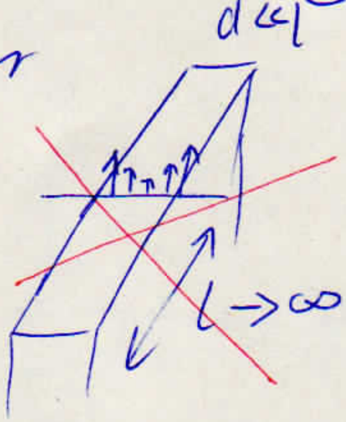
In a DC current the current density is constant through the whole cross section of the conductor



while in AC current density concentrates on the outside of the conductor Why?

Quantitatively:

Imagine our conductor to be rectangular



① Ohm's law $\underline{j} = \sigma \underline{E}$

② Ampère's law: $\operatorname{rot} \underline{H} = \underline{j}$

③ Faraday's law $\operatorname{rot} \underline{E}_{\cancel{ind}} = -\frac{dB}{dt}$ / rot

$\operatorname{rot} \underline{B} = \mu \underline{j} \; / \frac{\partial}{\partial t}$

$\operatorname{rot} \frac{\partial B}{\partial t} = \mu_0 \frac{\partial j}{\partial t}$

$\operatorname{rot} \operatorname{rot} E_{\cancel{ind}} = -\operatorname{rot} \frac{\partial B}{\partial t}$

$\operatorname{rot}(\operatorname{rot} E_{\cancel{i}}) = -\mu_0 \frac{\partial j}{\partial t}$

$\underbrace{\operatorname{grad}(\operatorname{div} E_{\cancel{i}})}_{\emptyset \text{ since } \frac{\rho}{\varepsilon_0} = \emptyset} - \Delta \underline{E}_{\cancel{i}} = -\mu \frac{\partial j}{\partial t}$

$\underline{E} = \frac{\underline{j}}{\sigma}$

$\Delta \underline{j} = \mu \sigma \frac{\partial \underline{j}}{\partial t}$

Test function:

$\underline{j}(\underline{r}, t) = \underline{j}_0(\underline{r}) \cdot e^{i\omega t}$

$\Delta \underline{j}_0 = i\omega\sigma\mu \cdot \underline{j}_0(r)$

We are interested in the y component

x dependence of the

y and z derivatives $= \emptyset$ $\left(\Delta \underline{v} = \frac{\partial^2 \underline{v}}{\partial x^2} + \frac{\partial^2 \underline{v}}{\partial y^2} + \frac{\partial^2 \underline{v}}{\partial z^2}\right)$

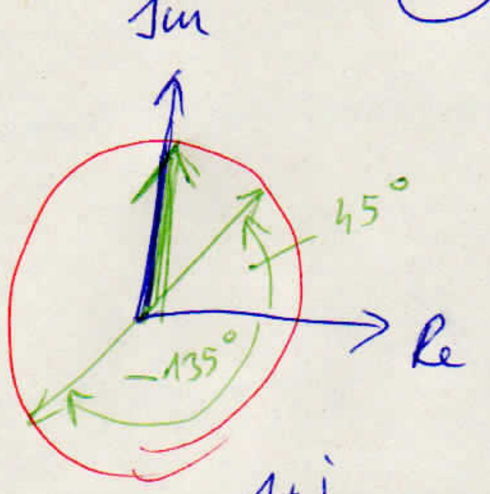
$\frac{\partial^2 j_{0y}}{\partial x^2} = i\omega\sigma\mu \, j_{0y} \; / \; j_{0y} := A e^{\lambda x}$

$$A\cdot\lambda^2\cdot e^{\lambda x} = i\omega\sigma\mu A e^{\lambda x}$$

$$\lambda^2 = i\omega\sigma\mu =$$

$$\lambda_{1,2} = \cancel{\pm}\sqrt{i}\cdot\sqrt{\omega\sigma\mu} = \cancel{\pm\sqrt{i}} =$$

$$= \pm\frac{1+i}{\sqrt{2}}\sqrt{\omega\sigma\mu}$$

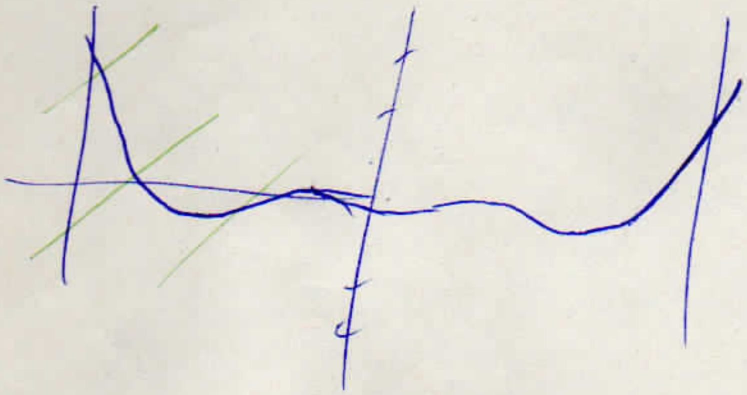


$$\pm\frac{1+i}{\sqrt{2}}$$

$$\delta := \sqrt{\frac{2}{\omega\sigma\mu}}$$

$$\left[\left(\frac{1+i}{\sqrt{2}}\right)^2 = \frac{1+2i-1}{2} = i\right]$$

$$j_{0y} = \mathrm{Re}\, A\Big[\underbrace{e^{-\frac{x}{\delta}}\cdot e^{-i\frac{x}{\delta}}}_{\text{in } \Omega'} + \underbrace{e^{\frac{x}{\delta}}\cdot e^{+i\frac{x}{\delta}}}_{\text{physical solution in } \Omega}\Big]$$



δ: skin depth

$\frac{\omega}{2\pi} = 50\,\text{Hz}$ $\quad\delta = 8\,\text{mm}$

$10\,\text{MHz}$ $\quad\delta = 21\,\mu\text{m}$

$10\,\text{GHz}$ $\quad\delta = 0{,}65\,\mu\text{m}$

Ch 20: Calculate skin eff in cylindrical Coordinates System!