Maxwell's equations in integral and differential form

	Integral form	*Differential form*	*Differential in Descartes coordinates*
Gauss's law	$\oint_A \mathbf{D}d\mathbf{A} = \int_V \rho dV$	$div\mathbf{D} = \rho$	$\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = \rho$
Gauss's law in magnetism	$\oint_A \mathbf{B}d\mathbf{A} = 0$	$div\mathbf{B} = 0$	$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0$
Ampère–Maxwell law	$\oint_L \mathbf{H}d\mathbf{l} = \int_A \mathbf{j}d\mathbf{A} + \int_A \frac{\partial \mathbf{D}}{\partial t}d\mathbf{A}$	$curl\mathbf{H} = \mathbf{j} + \frac{\partial \mathbf{D}}{\partial t}$	$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = j_x + \frac{\partial D_x}{\partial t}$ $\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = j_y + \frac{\partial D_y}{\partial t}$ $\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j_z + \frac{\partial D_z}{\partial t}$
Faraday's law	$\oint_L \mathbf{E}d\mathbf{l} = -\frac{d}{dt}\int_A \mathbf{B}d\mathbf{A} = -\int_A \frac{\partial \mathbf{B}}{\partial t}d\mathbf{A}$	$curl\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -\frac{\partial B_x}{\partial t}$ $\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -\frac{\partial B_y}{\partial t}$ $\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\frac{\partial B_z}{\partial t}$

The integral form of Faraday's law requires that surface A does not change in time

Definition of **D** and **H**:

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}_e$$
$$\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{P}_m).$$

Derivation using Gauss's and Stokes' Theorem
($\oint_A \mathbf{V}d\mathbf{A} = \int_V (div\mathbf{V})dV$, $\oint_L \mathbf{V}d\mathbf{l} = \int_A (rot\mathbf{V})d\mathbf{A}$):

$$\oint_A \mathbf{D}d\mathbf{A} = \int_V \rho dV \quad \Rightarrow \quad \int_V (div\mathbf{D})dV = \int_V \rho dV \quad \Rightarrow \quad div\mathbf{D} = \rho$$

$$\oint_A \mathbf{B}d\mathbf{A} = 0 \quad \Rightarrow \quad \int_V (div\mathbf{B})dV = 0 \quad \Rightarrow \quad div\mathbf{B} = 0$$

$$\oint_L \mathbf{H}d\mathbf{l} = \int_A \mathbf{j}d\mathbf{A} + \int_A \frac{\partial \mathbf{D}}{\partial t}d\mathbf{A} \quad \Rightarrow \quad \oint_A (curl\mathbf{H})d\mathbf{A} = \int_A \left(\mathbf{j} + \frac{\partial \mathbf{D}}{\partial t}\right)d\mathbf{A} \quad \Rightarrow \quad curl\mathbf{H} = \mathbf{j} + \frac{\partial \mathbf{D}}{\partial t}$$

$$\oint_L \mathbf{E}d\mathbf{l} = -\int_A \frac{\partial \mathbf{B}}{\partial t}d\mathbf{A} \quad \Rightarrow \quad \oint_A (curl\mathbf{E})d\mathbf{A} = -\int_A \frac{\partial \mathbf{B}}{\partial t}d\mathbf{A} \quad \Rightarrow \quad curl\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$