Dielectric materials

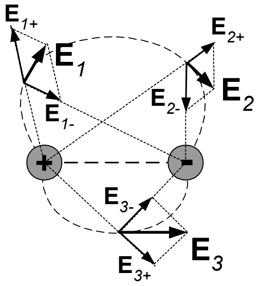
Electrical insulator materials do not contain free charge carriers. Atomic and molecular charges of an insulator can be described by bound charges that are localized at a given atom or molecule. When the + and – charges of a molecule do not overlap (i.e. are displaced from each other) the charge distribution can be modelled by an electric dipole. The electric dipole is composed of a positive and a negative charge of equal absolute value, separated by a distance *l.* The definition of the electrical dipole moment vector is the following:

$$\mathbf{d}_e = Q \cdot \underline{l}$$

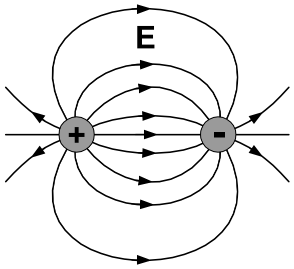
where the vector l is pointing from the negative to the positive charge.

Field of an electrical dipole:

The field of the electrical dipole can be determined by the superposition of the field of the + and – charges according to the figure.



a)

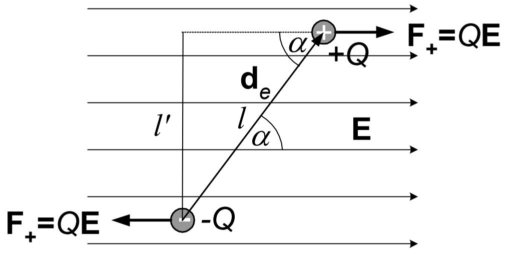


b)

Electric dipole in an external electric field

Uniform (homogenous) field

The net force acting on the dipole is zero, but a torque can be observed that rotates the dipole towards an orientation parallel to the external field:
$M = Fl' = Fl\,sin\,\alpha = QEl\,sin\,\alpha$.
expressed with the dipole moment:
$M = d_e E\,sin\,\alpha$.



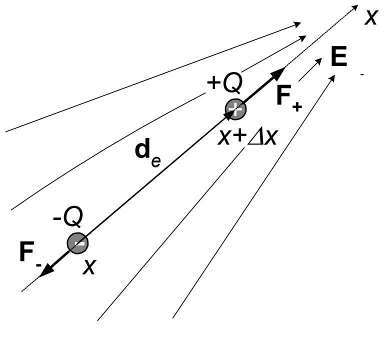
In vector form:

$$\mathbf{M} = \mathbf{d}_e \times \mathbf{E} .$$

This torque aligns the dipole moment vector parallel to the external field.

Non-uniform (inhomogenous) field
In a non-uniform field the dipole aligns parallel to the local field. As the field is different at the + and – charges a net force is acting on the dipole:
$F_x = F_+ - F_- = QE(x + \Delta x) - QE(x)$.
For small distances:

$$E(x + \Delta x) \approx E(x) + \frac{dE(x)}{dx}\Delta x .$$



$$F_x = QE(x) + Q\frac{dE(x)}{dx}\Delta x - QE(x) = Q\Delta x\frac{dE(x)}{dx}.$$

finally using the definition of the dipole moment:

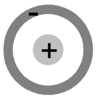
$$F_x = d_e\frac{dE(x)}{dx}.$$

So the net force is pointing towards the higher electric field region.

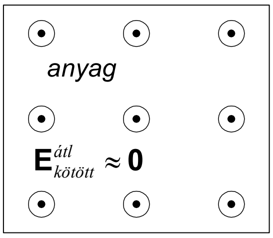
Laws of static electricity in dielectric materials

In the absence of external electric field the average dipole moment of the molecules is usually zero. For **apolar molecules** the center of + and – charges overlap, so the dipole moment of each molecule is zero (a and b on the figure).

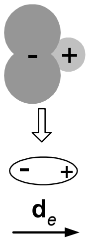
For polar molecules (like water) the molecule can be considered as a microscopic dipole (c.). Due to the interaction between the microscopic dipoles and to the thermal movement of the dipoles, the orientation of the billions of dipoles will be disordered. This also results a zero average dipole moment (d).



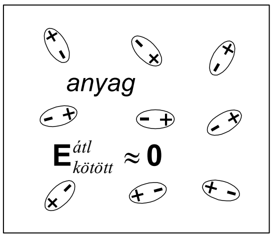
a)



b)

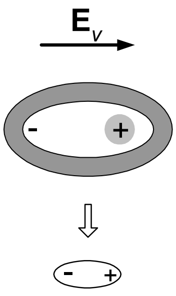


c)

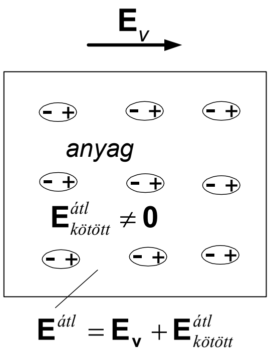


d)

However, when the material is placed in an external electric field + and – charges of apolar molecules will slightly move (displace) parallel and anti-parallel to the field creating microscopic dipoles aligned parallel to the field. Polar molecules will rotate towards the field, but will keep some degree of disorder due to the interaction and to the thermal movement of the dipoles. This phenomenon is called the *polarization of an insulator* material. For most materials and for low field strength values the average dipole moment grows linearly with the field.



a)



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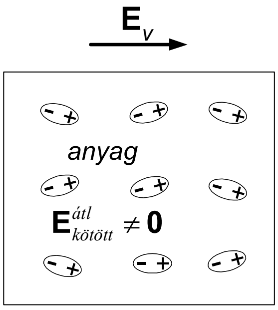
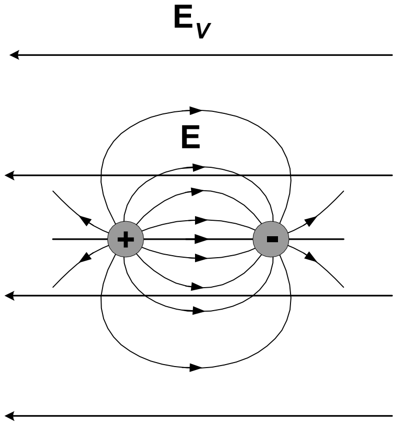


c)

As a result for both polar and apolar molecules the average dipole moment will align to the external field. The total electric field can be calculated as the superposition of the external field and that of the billions of microscopic dipoles. We can see from the figure (or prove by the addition of plenty point charge fields) that the overall electric field is reduced by the polarization of the material.



First law of static electricity in insulators

The presence of polarized charges does not change the conservative nature of static electric field, so the first law remains unchanged for the resulting average field:

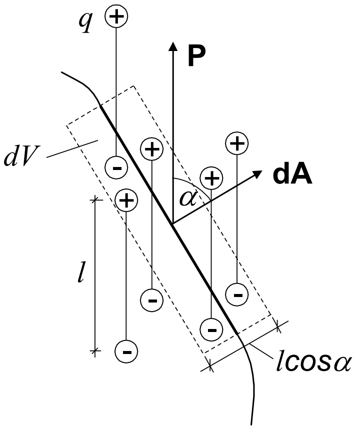
$$\oint_A \mathbf{Edr} = 0 .$$

Second law of static electricity in insulators (Gauss's law)

In Gauss's law we can separate the effect of free and bound charges:

$$\oint_A \mathbf{EdA} = \frac{Q_{free}}{\varepsilon_0} + \frac{Q_b}{\varepsilon_0} .$$

To evaluate the effect of bound charges, we have to sum up all charges enclosed by the closed surface A. Suppose that the surface is within the material. For dipoles that are outside the closed surface the result is obviously zero. The same holds for dipols having both + and – poles inside the surface, since the net contribution is zero again. So we have to sum up dipolse that have exactly one pole inside the surface (and the other pole outside). These are dipoles in the vicinity of the surface, as shown on the figure below:



The volume in which the center of these dipoles sit: $dV = ldA|\cos\alpha|$.

let us denote the dipole density (number of dipoles / unit volume) by

$$n = \frac{dN}{dV}$$

the number of dipoles intersecting the surface dA

$$dN = ndV = nldA|\cos\alpha| ,$$

and the bound charge within the closed surface:

$$dQ_p = -\frac{dN}{dV} qldA\cos\alpha ,$$

where the – sign is due to the fact that $\cos\alpha$ is positive for dipoles having the negative pole enclosed by the surface.

$$dQ_p = -\frac{dN}{dV}\mathbf{d}_e\mathbf{dA} = -\mathbf{PdA}.$$

where **P** is the dipole momentum density defined by: $\mathbf{P} = \frac{\Delta\left(\sum_i \mathbf{d}_{ei}\right)}{\Delta V}$

P is also called **polarization vector**.

For the entire surface A:

$$Q_p = -\oint_A \mathbf{PdA}.$$

Substituting to Gauss's law:

$$\oint_A \mathbf{EdA} = \frac{Q_{free}}{\varepsilon_0} - \frac{1}{\varepsilon_0}\oint_A \mathbf{PdA}.$$

$$\oint_A \left(\mathbf{E} + \frac{1}{\varepsilon_0}\mathbf{P}\right)\mathbf{dA} = \frac{Q_{free}}{\varepsilon_0}$$

$$\oint_A (\varepsilon_0\mathbf{E} + \mathbf{P})\mathbf{dA} = Q_{free}$$

We can introduce

$$\mathbf{D} = \varepsilon_0\mathbf{E} + \mathbf{P}.$$

called dielectric displacement vector. With this definition Gauss's law simplifies to the form:

$$\oint_A \mathbf{DdA} = Q_{free}.$$

Polarization in homogenous, isotropic, linear dielectrics

In in homogenous, isotropic, linear dielectrics the average dipole is parallel and proportional to the field, thus $\mathbf{P} \sim \mathbf{E}$. It is customary to define

$$\mathbf{P} = \varepsilon_0\chi\mathbf{E}$$

where χ is a material constant called dielectric susceptibility. In vacuum $\chi = 0$, and in all practical materials at static external fields or low frequency. For air $\chi^{air} = 0.00059$).

in that case:

$$\mathbf{D} = \varepsilon_0\mathbf{E} + \varepsilon_0\chi\mathbf{E} = \varepsilon_0(1+\chi)\mathbf{E} = \varepsilon_0\varepsilon_r\mathbf{E} = \varepsilon\mathbf{E}.$$

where $\varepsilon_r = 1 + \chi$ is the relative permittivity of the material and $\varepsilon = \varepsilon_0\varepsilon_r$ is the absolut permittivity.

Anizotropic linear materials

In anizotropic linear materials the direction of the polarization is not necessarily parallel to the electric field***:***

$$P_x = \varepsilon_0 \chi_{xx} E_x + \varepsilon_0 \chi_{xy} E_y + \varepsilon_0 \chi_{zx} E_z$$

$$P_y = \varepsilon_0 \chi_{yx} E_x + \varepsilon_0 \chi_{yy} E_y + \varepsilon_0 \chi_{yz} E_z$$

$$P_z = \varepsilon_0 \chi_{zx} E_x + \varepsilon_0 \chi_{zy} E_y + \varepsilon_0 \chi_{zz} E_z .$$

The relation is similar to the connection between the angular velocity and the angular momentum of a rigid body. The polarization can be calculated by multiplying the electric field and the 3 by 3 element susceptibility tensor.

Nonlinear materials

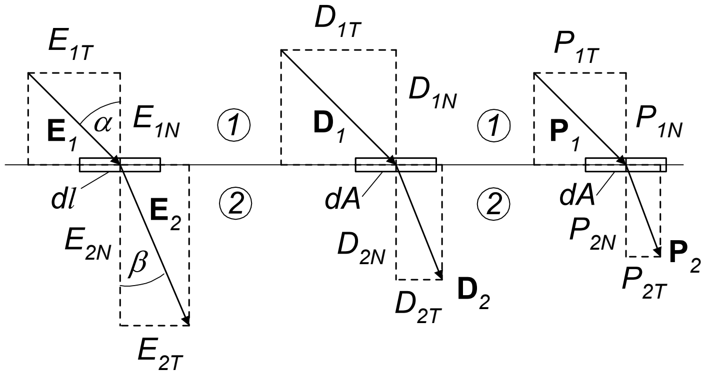
Almost all materials show some nonlinearity at very high electric fields, but some special materials exhibit nonlinear polarization at low or even zero external field. So called remanent polarization can be observed in pyroelectric and ferroelectric materials.

Boundary conditions on a planar interface of two materials.

By applying the laws of static electricity on a surface (or volume) across the interface, we can see that:

$$\oint_L \mathbf{Edr} = E_{1T} dl - E_{2T} dl = 0 ,$$

$$E_{1T} = E_{2T} ,$$



and

$$\oint_A \mathbf{DdA} = -D_{1N} dA + D_{2N} dA = Q .$$

$$D_{2N} - D_{1N} = \frac{Q}{dA} = \sigma$$

D lines can only start or end on free surface charge. If there is no free surface charge:

$$D_{2N} = D_{1N} ,$$