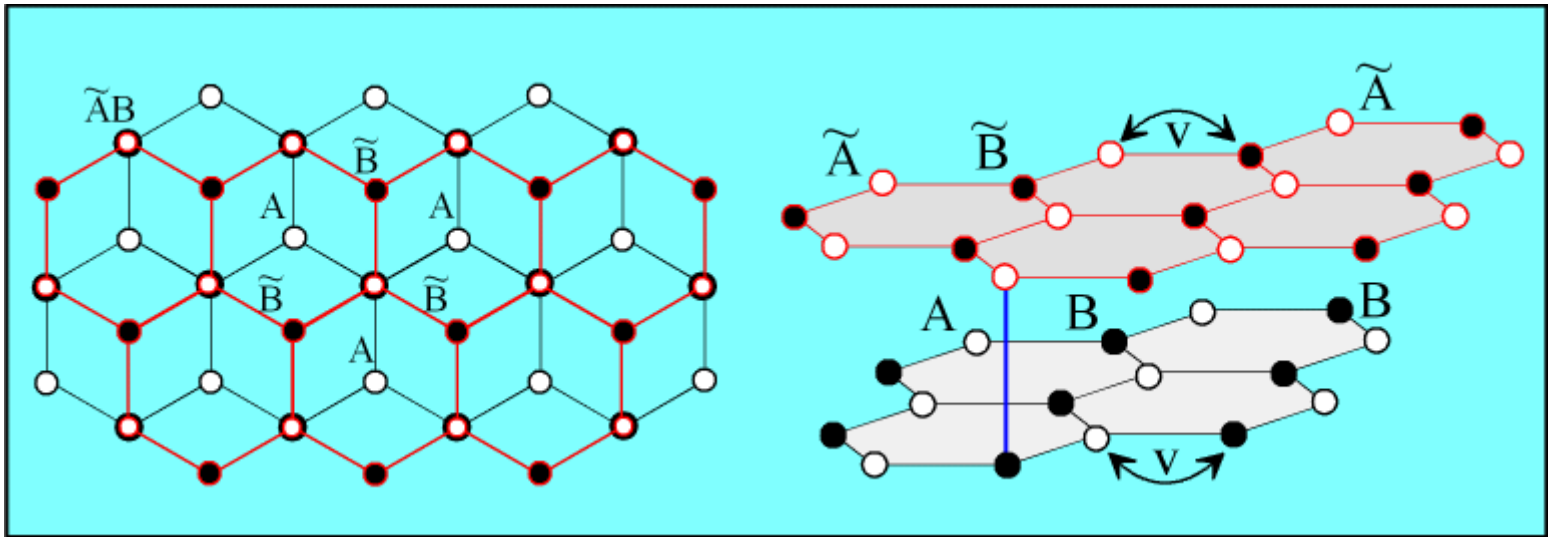
Bilayer graphene

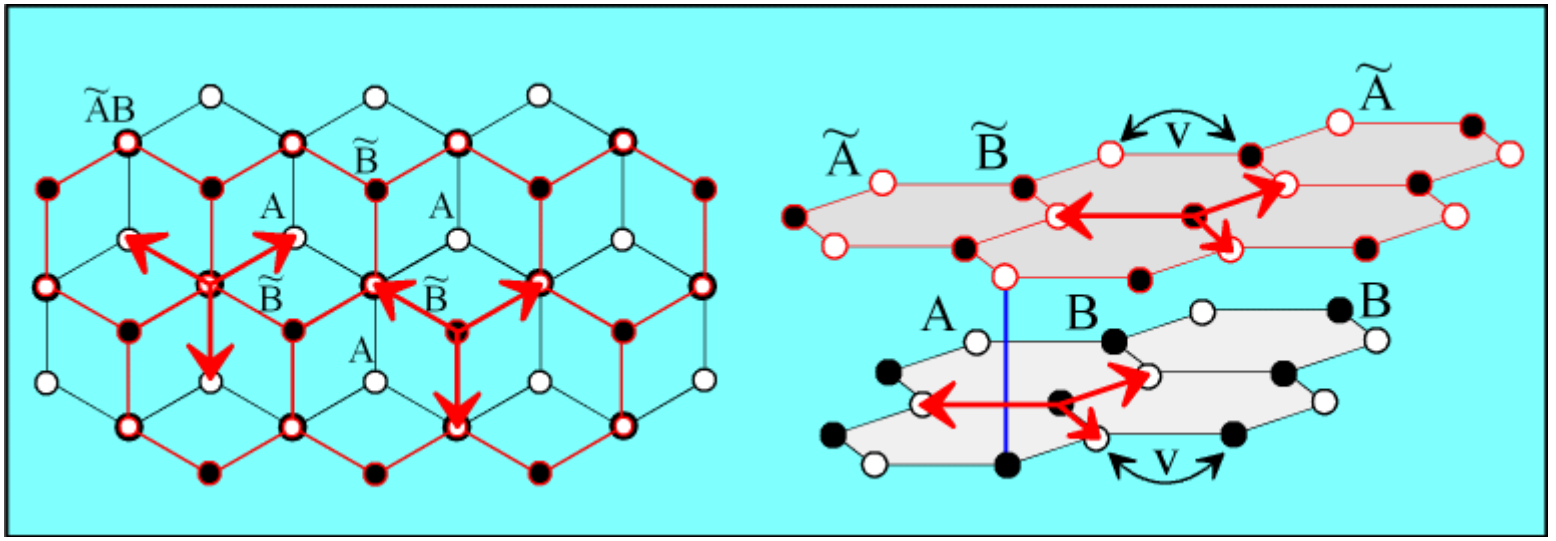


4 atoms per unit cell

$$\mathcal{H} = \begin{array}{c} \begin{matrix} A & \tilde{B} & \tilde{A} & B \end{matrix} \\ \begin{pmatrix} & & & \\ & & & \\ & & & \\ & & & \end{pmatrix} \end{array} \begin{matrix} A \\ \tilde{B} \\ \tilde{A} \\ B \end{matrix}$$

Images: V. Falko, Lecture notes

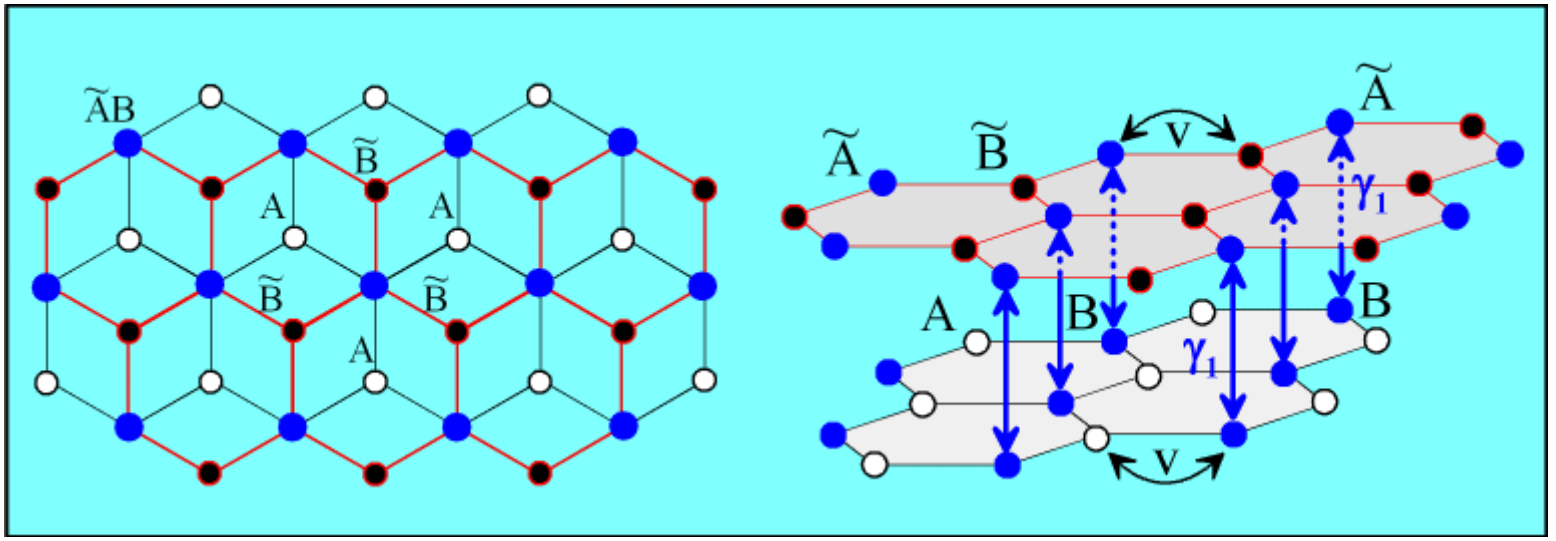
Bilayer graphene



(B to A) and ($\tilde{B}$ to $\tilde{A}$) hopping given by $\pi^+ = p_x - ip_y$

$$H = \begin{array}{c} \begin{matrix} A & \tilde{B} & \tilde{A} & B \end{matrix} \\ \begin{pmatrix} & & v\pi^+ & \\ & v\pi & & \\ v\pi^+ & & & \\ v\pi & & & \end{pmatrix} \begin{matrix} A \\ \tilde{B} \\ \tilde{A} \\ B \end{matrix} \end{array}$$

Bilayer graphene



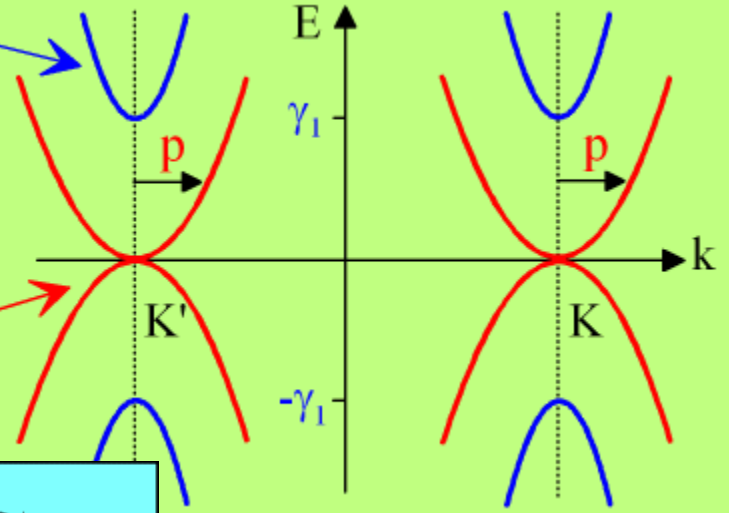
Bilayer Hamiltonian

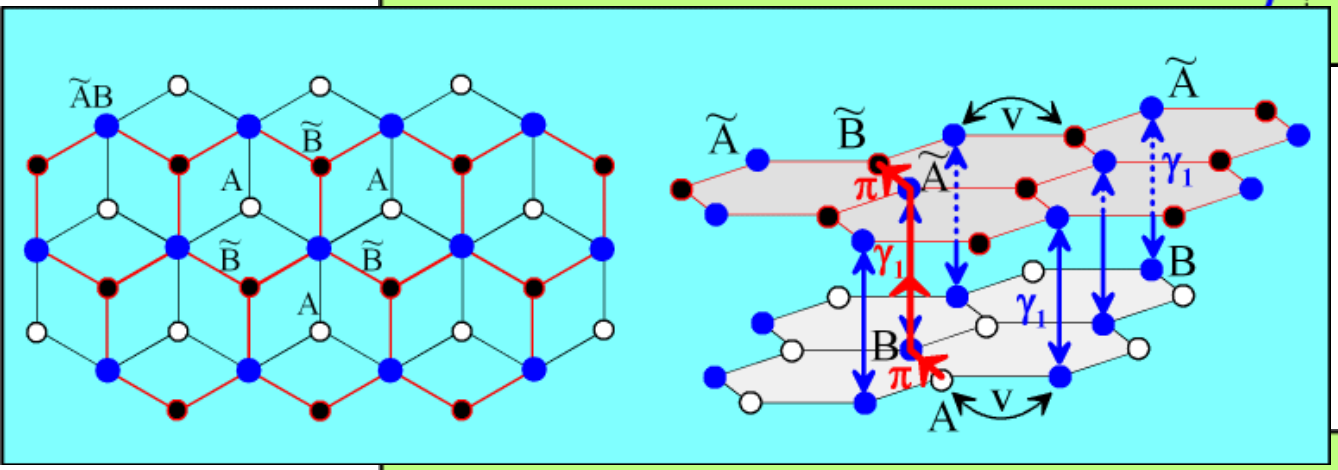
$$H = \begin{array}{c} \begin{matrix} A & \tilde{B} & \tilde{A} & B \end{matrix} \\ \begin{pmatrix} 0 & 0 & 0 & v\pi^+ \\ 0 & 0 & v\pi & 0 \\ 0 & v\pi^+ & 0 & \gamma_1 \\ v\pi & 0 & \gamma_1 & 0 \end{pmatrix} \end{array} \begin{matrix} A \\ \tilde{B} \\ \tilde{A} \\ B \end{matrix}$$

$\tilde{A}B$ orbitals form dimers with energy $|E| \gtrsim \gamma_1$

Quadratic dispersion at low energy:

$$E = \pm \frac{p^2}{2m}$$





Bilayer Hamiltonian written in a 2 component basis of A and $\tilde{B}$ sites

$$H = \frac{-1}{2m}\begin{pmatrix} 0 & (\pi^+)^2 \\ \pi^2 & 0 \end{pmatrix}$$

mass

$m = \gamma_1 / v^2$

A to $\tilde{B}$ hopping

- bottom layer A → B (factor π)
- switch layers via dimer $B\tilde{A}$ (γ_1^{-1})
- top layer $\tilde{A}$ → $\tilde{B}$ (factor π)

$$\pi = p_x + ip_y$$

Backscattering?

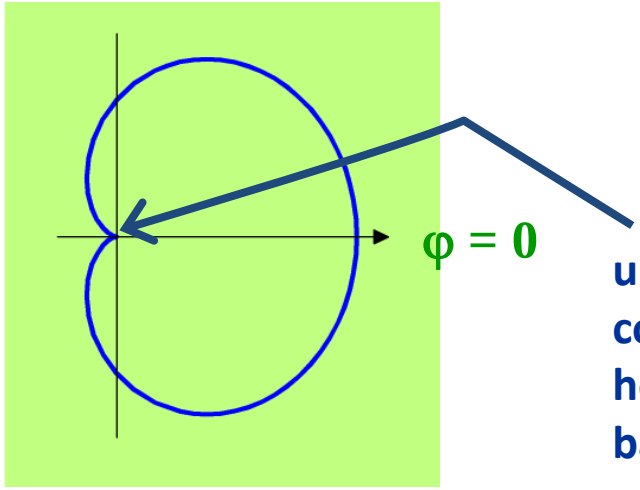
Single layer:

$$H = v\begin{pmatrix} 0 & \pi^+ \\ \pi & 0 \end{pmatrix} = vp\begin{pmatrix} 0 & e^{-i\varphi} \\ e^{i\varphi} & 0 \end{pmatrix}; \qquad E = vp \quad \Leftrightarrow \quad \psi(\varphi) = \frac{1}{\sqrt{2}}\begin{pmatrix} e^{-i\varphi/2} \\ e^{i\varphi/2} \end{pmatrix}$$

Bilayer:

$$H = -\frac{1}{2m}\begin{pmatrix} 0 & (\pi^+)^2 \\ \pi^2 & 0 \end{pmatrix} = -\frac{p^2}{2m}\begin{pmatrix} 0 & e^{-2i\varphi} \\ e^{2i\varphi} & 0 \end{pmatrix}; \qquad E = \frac{p^2}{2m} \quad \Leftrightarrow \quad \psi(\varphi) = \frac{1}{\sqrt{2}}\begin{pmatrix} e^{-i\varphi} \\ e^{i\varphi} \end{pmatrix}$$

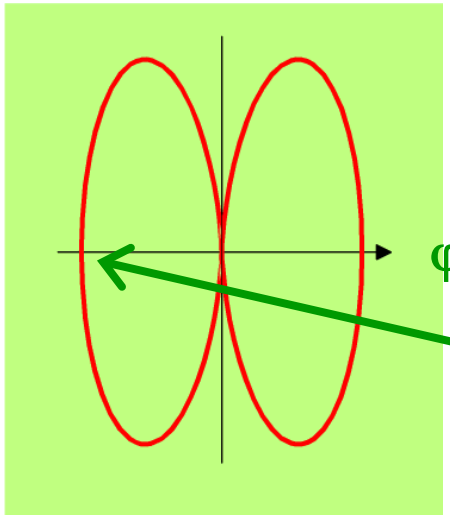
Single layer:



under pseudospin conservation, helicity suppresses backscattering in a monolayer

$$\left|\langle\psi(\varphi)|\psi(\varphi=0)\rangle\right|^2 = \cos^2(\varphi/2)$$

Bilayer:



no suppression of back-scattering in a bilayer

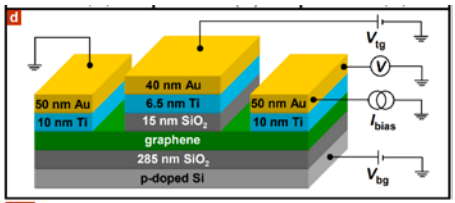
$$\left|\langle\psi(\varphi)|\psi(\varphi=0)\rangle\right|^2 = \cos^2(\varphi)$$

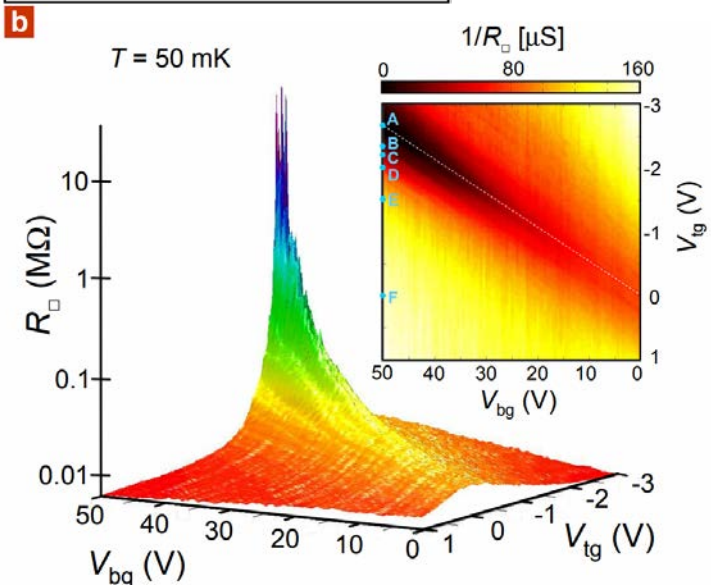
Bilayer graphene in electric fields

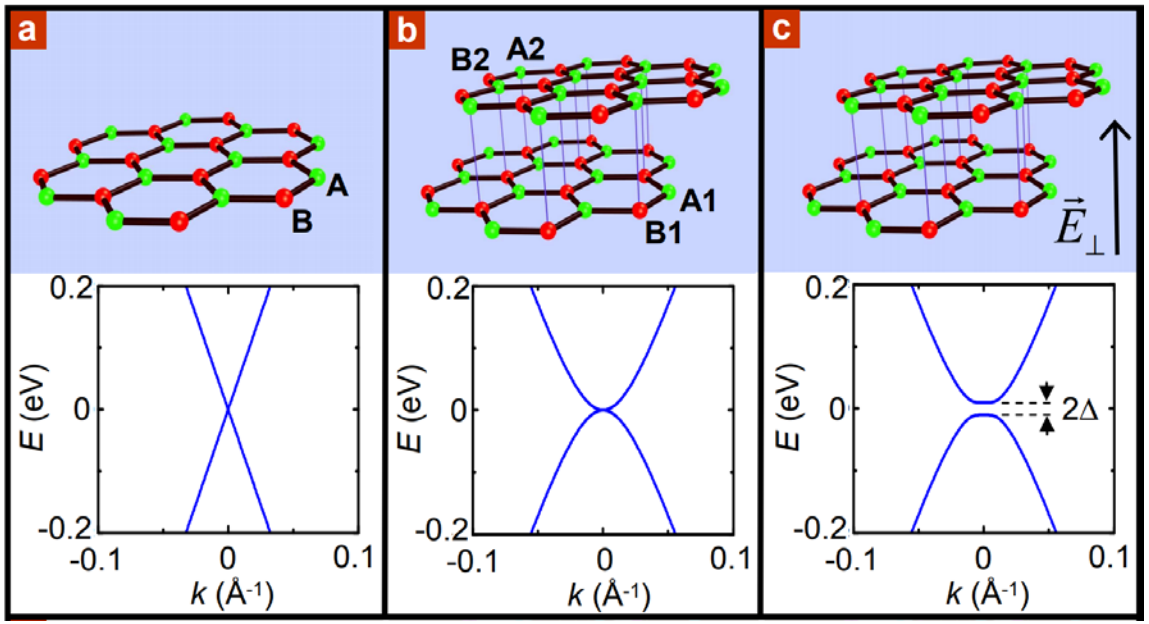
Effective 2x2 bilayer Hamiltonian acts on A1 and B2 subblattices, where 1 and 2 are on different layers.
-> Applying an electric field perpendicular to carbon plane generates a finite Δ term (breaks inversion).
→ Band gap opens in K and K' with size of 2Δ

$$H = \begin{pmatrix} \Delta & -\frac{\hbar^2}{2m}\left(k_x - ik_y\right)^2 \\ -\frac{\hbar^2}{2m}\left(k_x + ik_y\right)^2 & -\Delta \end{pmatrix}$$

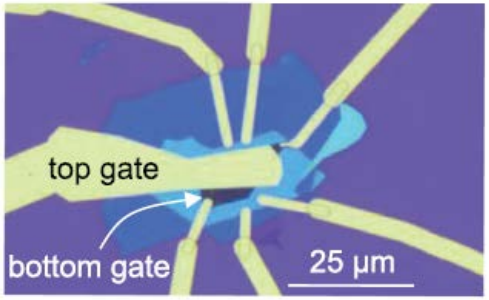
$$\psi = \begin{pmatrix} \phi_{A1} \\ \phi_{B2} \end{pmatrix}$$

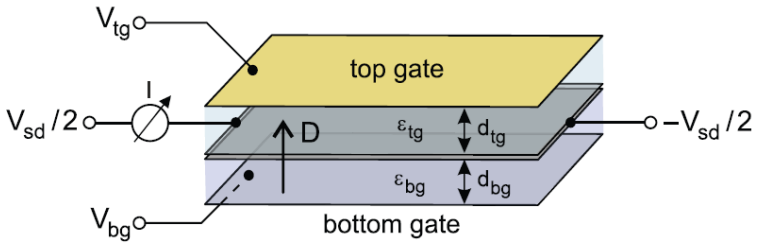






J. B. Oostinga, Nature Mat., 7, 151 (2008)

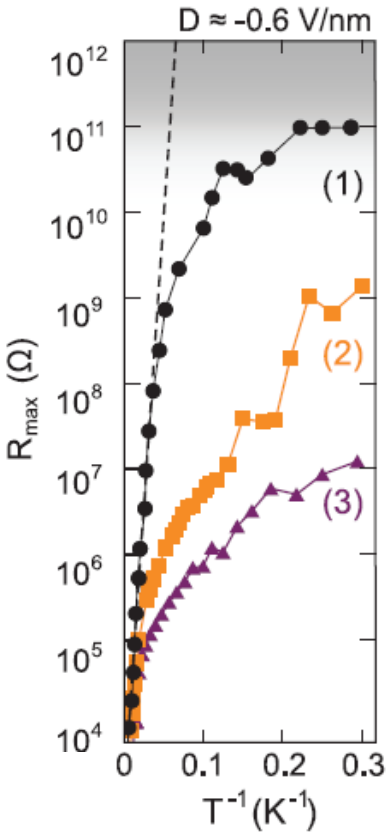


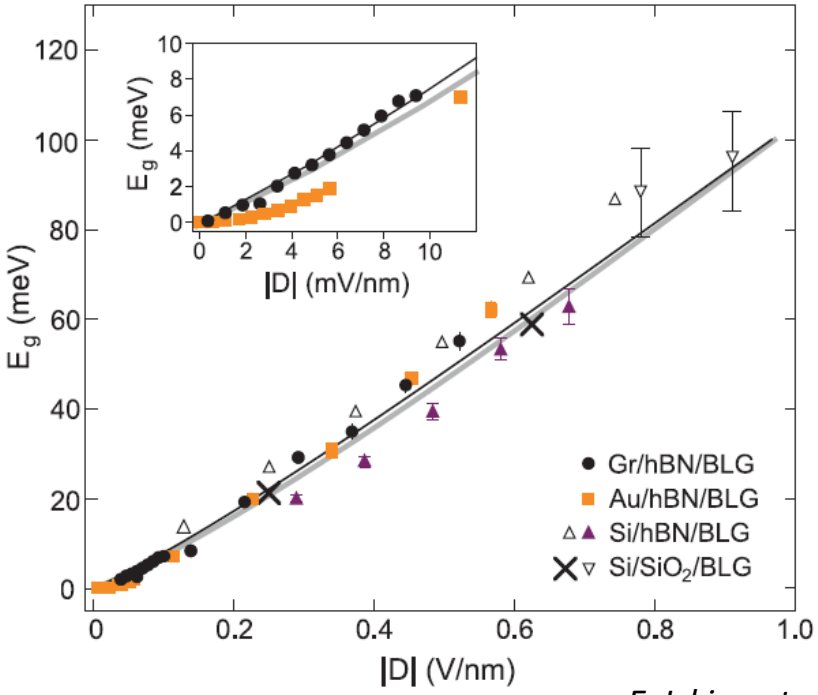


hBN encapsulated samples with high quality and graphite gates have large, "clean" gaps with resistance up to 10^{10} Ohm

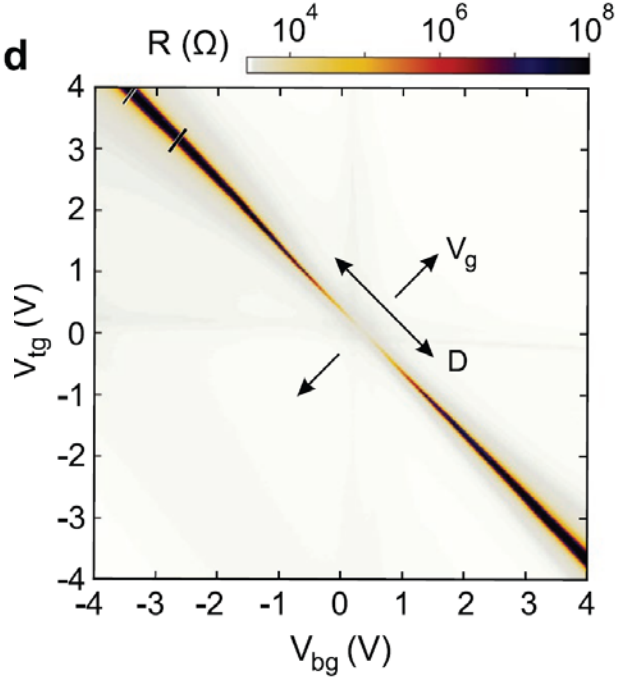
$$R \sim e^{\frac{E_g}{k_B T}}$$

Thermal activation measurements on different devices
Graphite gates give best devices

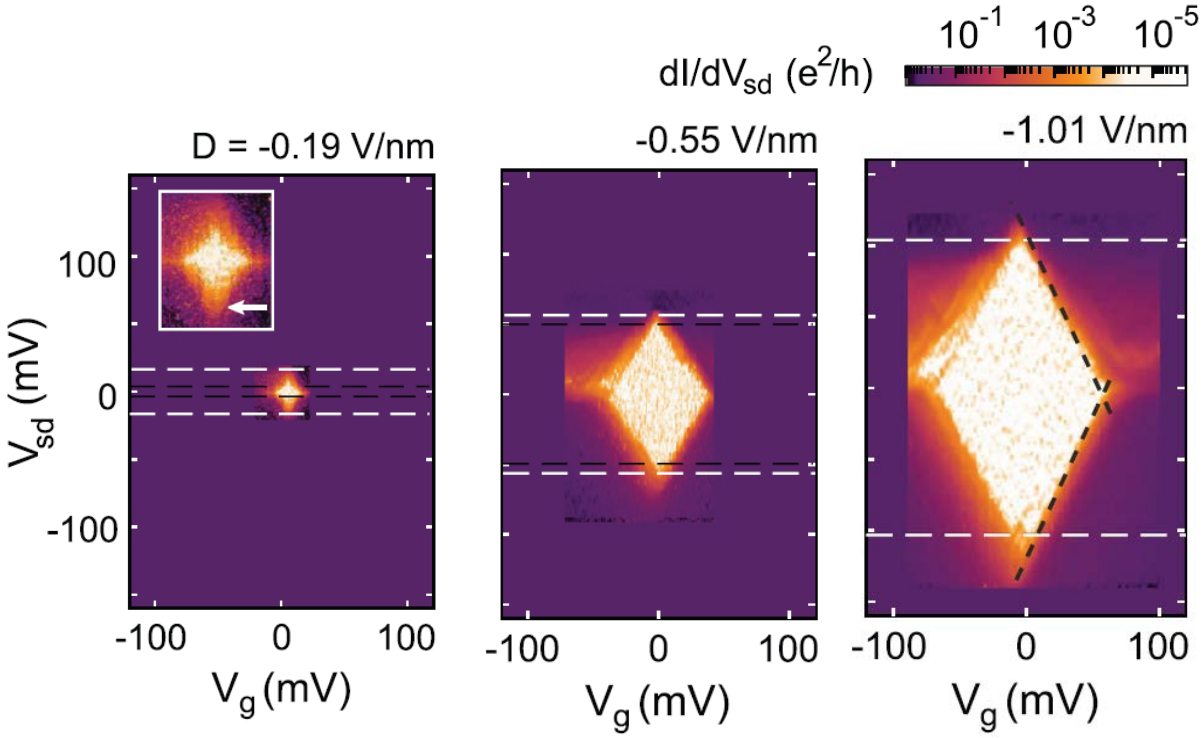




Gate-gate map



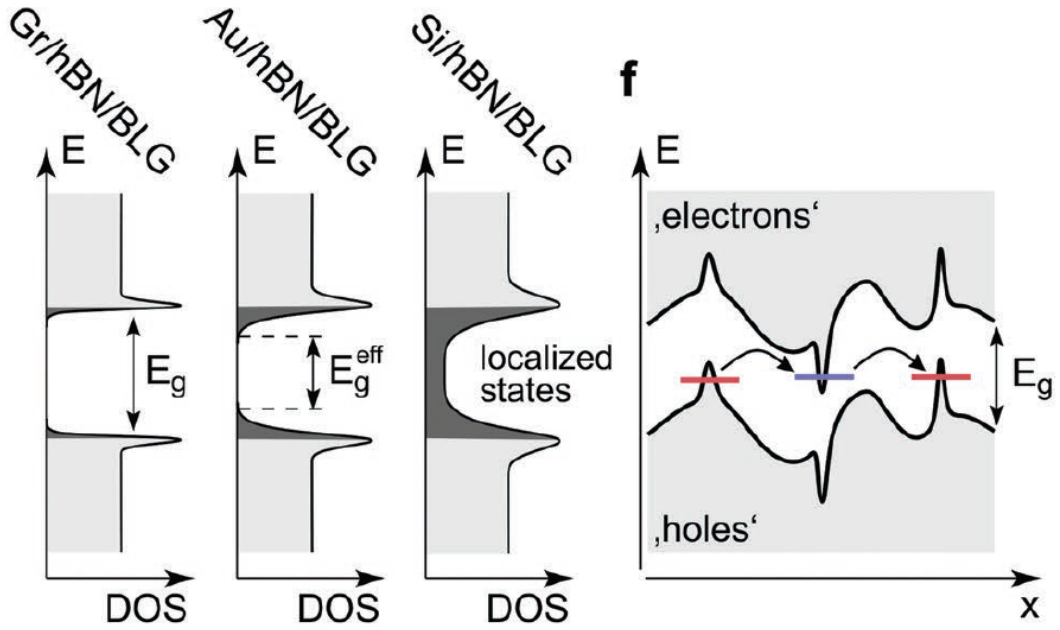
E. Icking et al., Adv. Electron. Mater. 8, 2200510 (2022)



Bias spectroscopy to address the gap
Diamond-like structures (like for QD)
Grows with displacement field

Disorder model

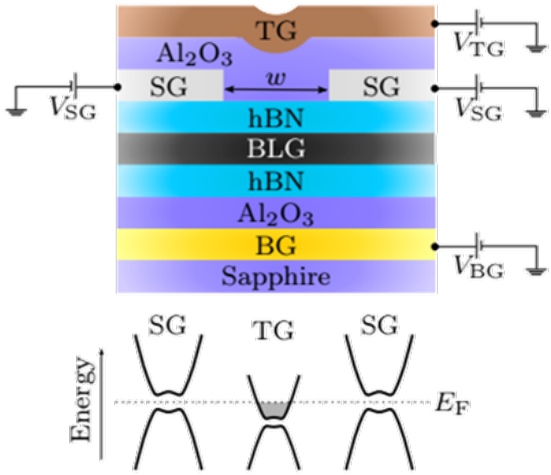
Imperfect substrates induce potential fluctuation which lead to local modulation of the gap and disorder-based hopping

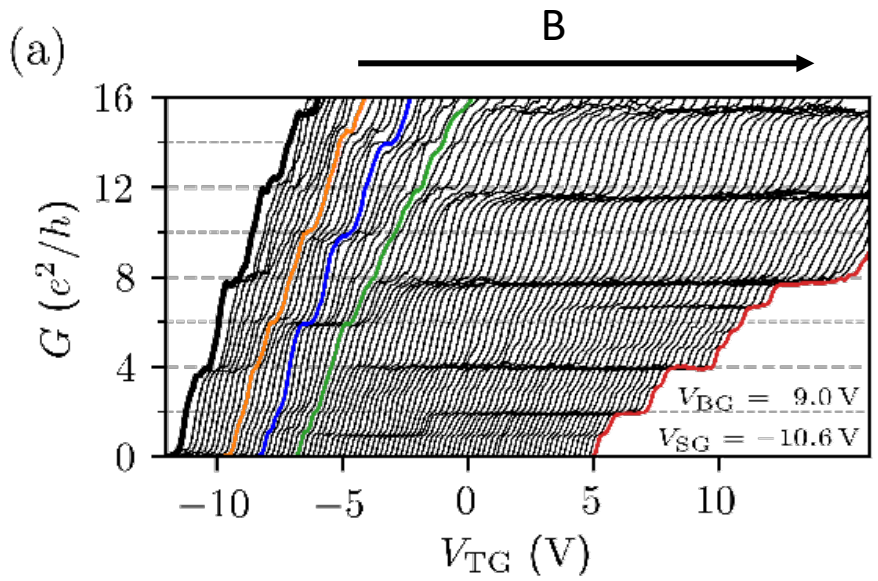


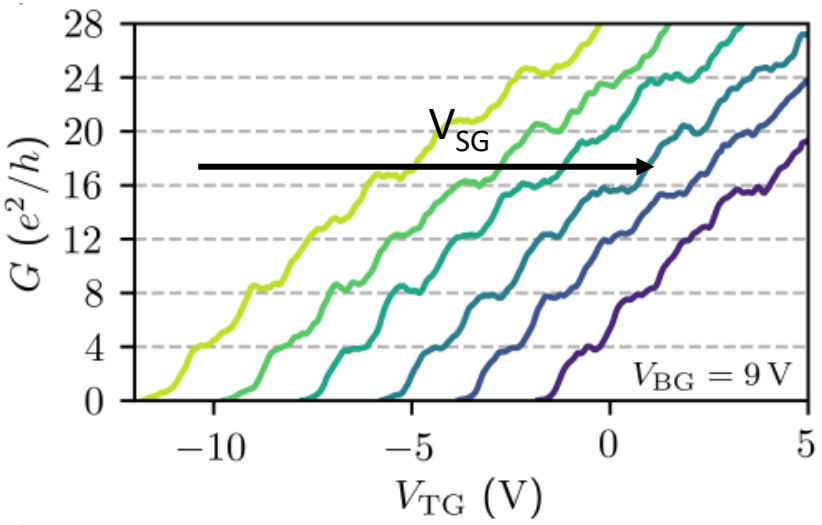
QPCs in bilayer graphene

- High quality BLG due to hBN stack
- Electric field is defined by BG, TG and SG.
- SG and BG generate perpendicular E field and open gap and define confinement
- TG is used to set Fermi level

- Conductance quantization 4e2/h
2spin x 2valley degree of freedom
- In B field 4 fold degeneracy changes.







R. Kraft Phys. Rev. Lett. 121, 257703 (2018)

QPCs in bilayer graphene

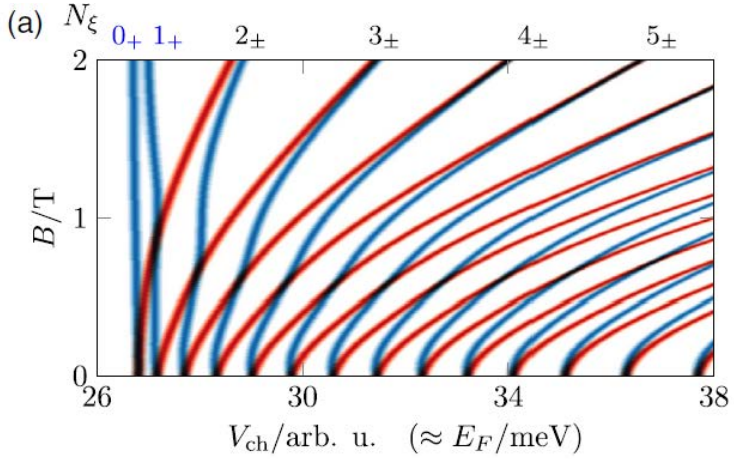
Simple theory (e.g. GaAs):
Transition from QPC to QHE

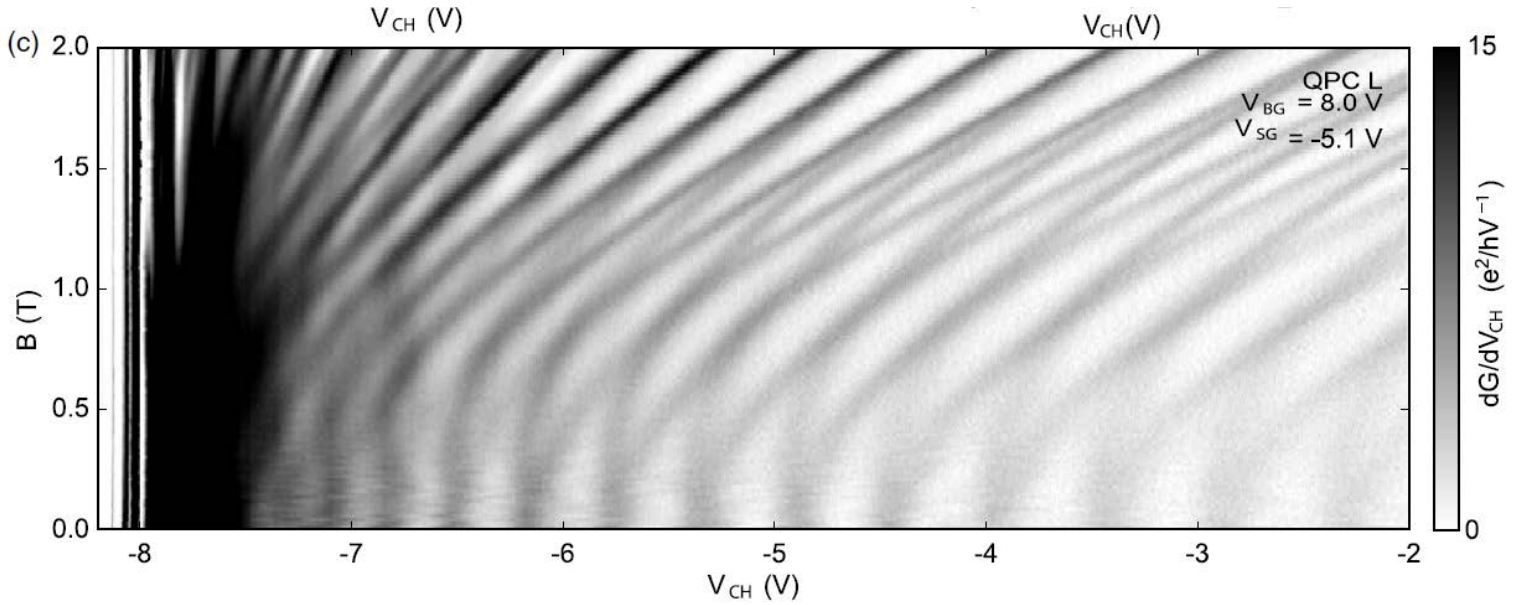
$$V(y) = \frac{1}{2} m\omega_0 y^2$$

$$\omega(B) = \sqrt{(\omega_0^2 + \omega_c^2)}$$

$$E = \hbar\omega(B)(n + \frac{1}{2})$$

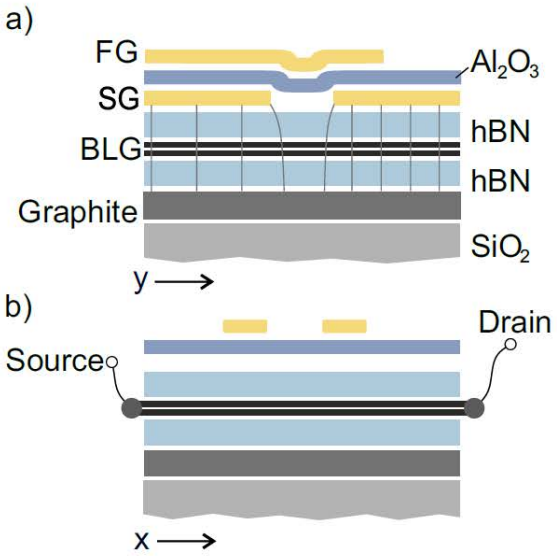
- Graphene: more complicated: splitting and crossing and merging again and high magnetic field.
- Can be nicely reproduced theoretically

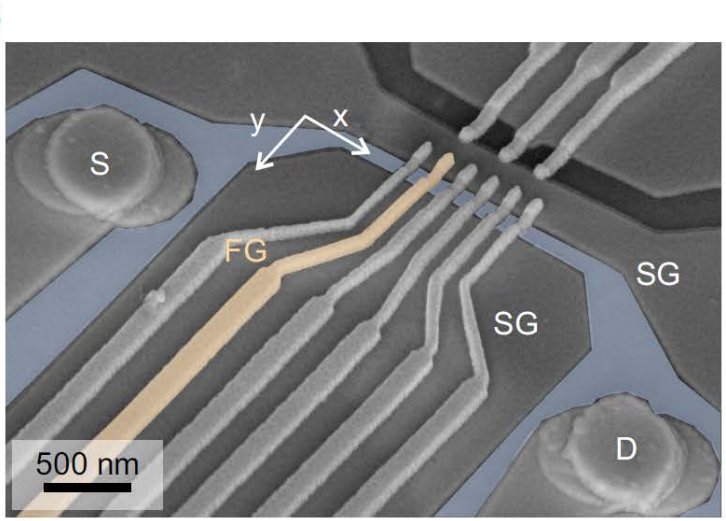


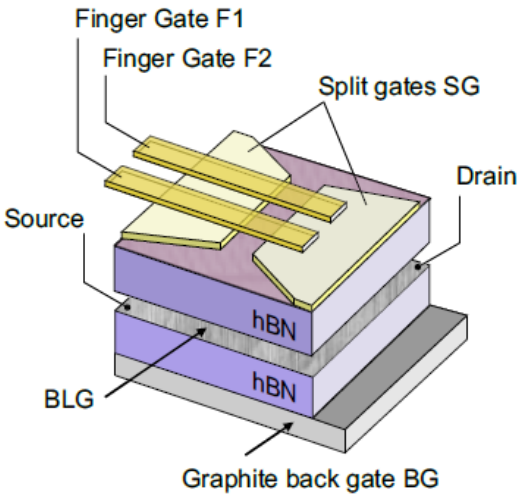


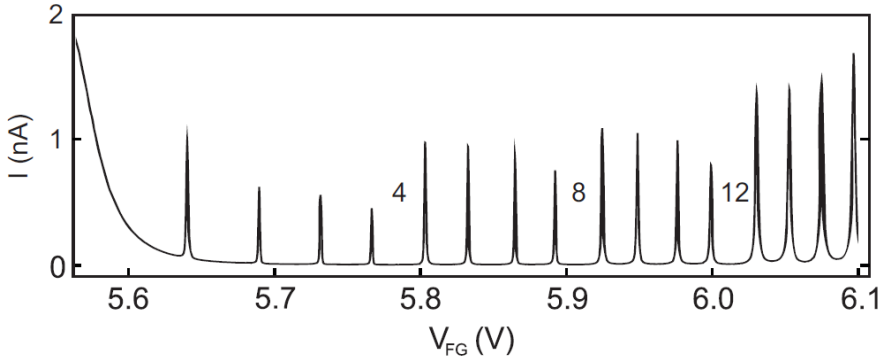
H. Overweg et al., PRL 121, 257702 (2018)

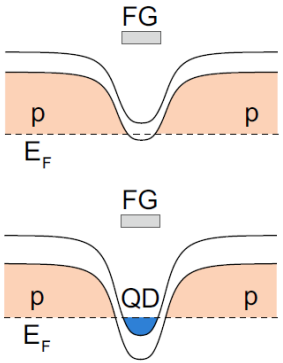
Quantum dots in BLG







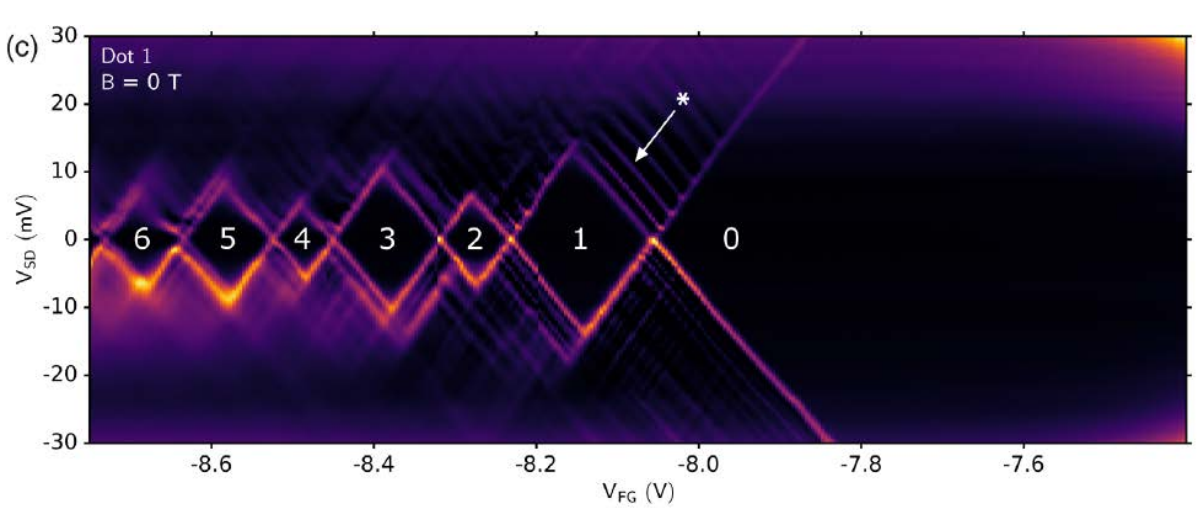


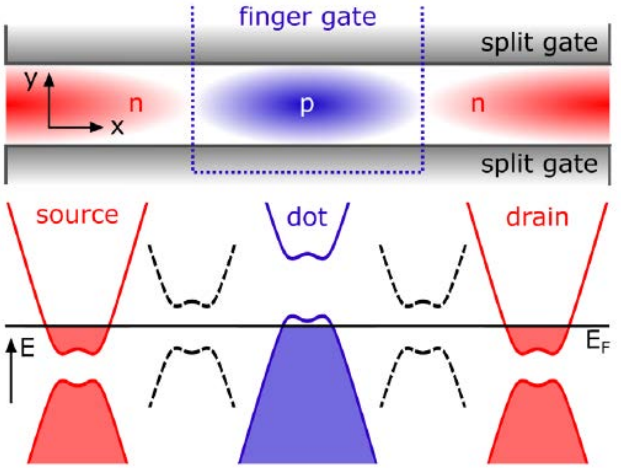


Dots can be formed by adding finger gates on split gates

Finger gate pushes down conduction band and forms electron dot with p-type leads

L. Banszerus, PhD Thesis

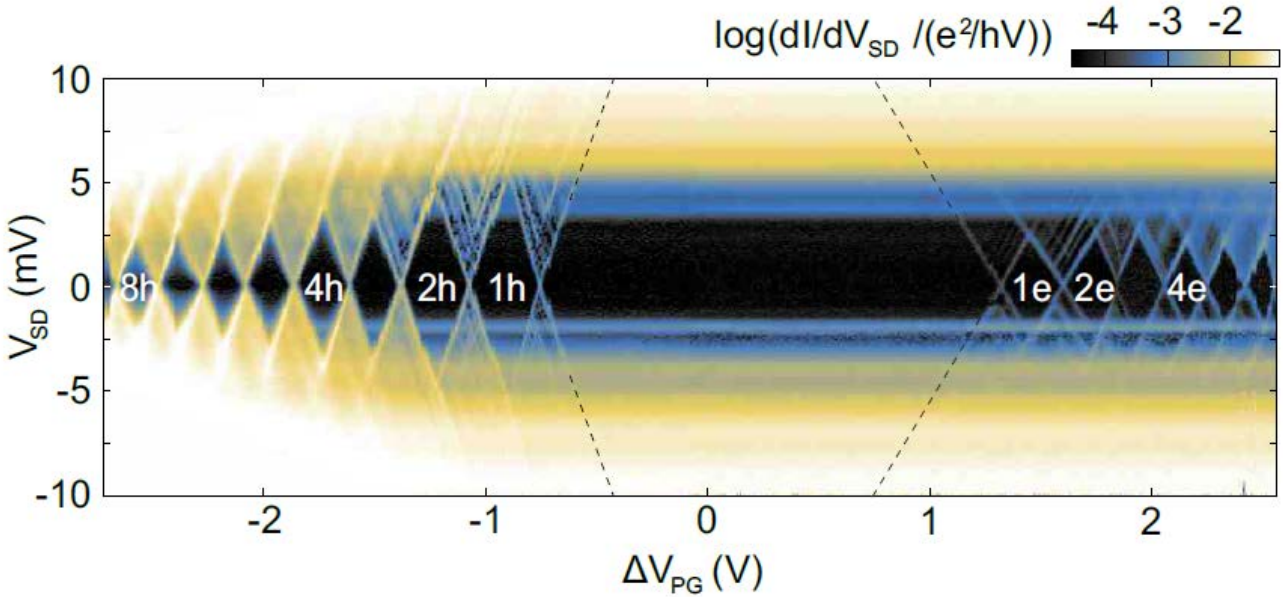




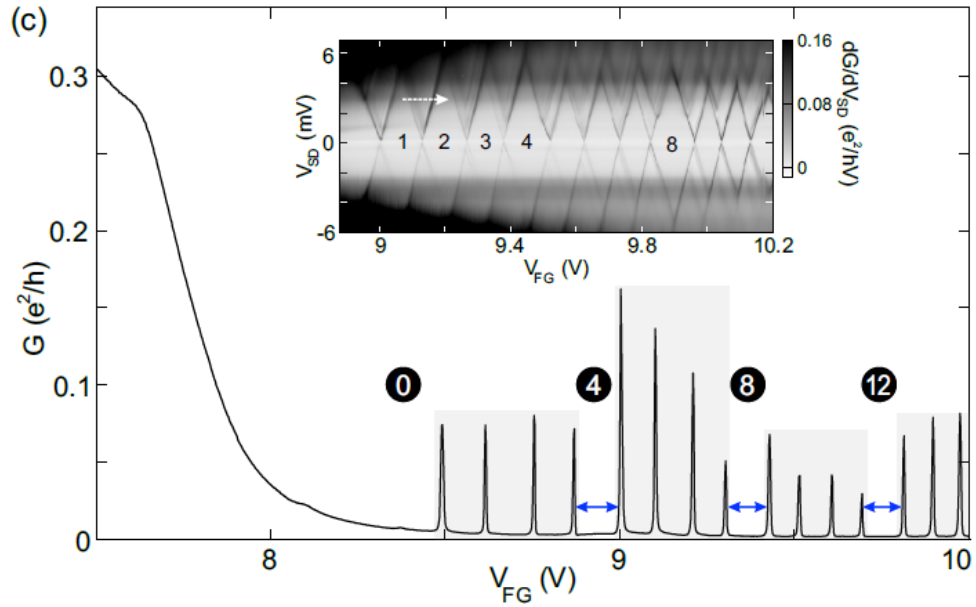
Opposite configuration can be formed as well - hole dots, with electron leads

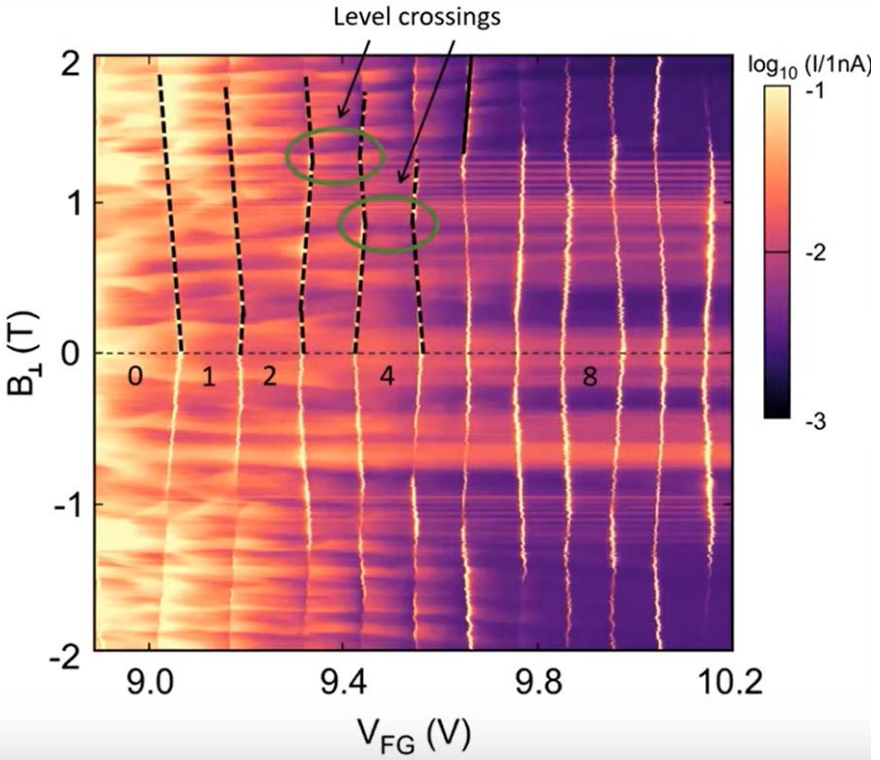
M. Eich et al., PRX , 031023 (2018)

By proper setting of the gates – electron- hole crossover can be achieved

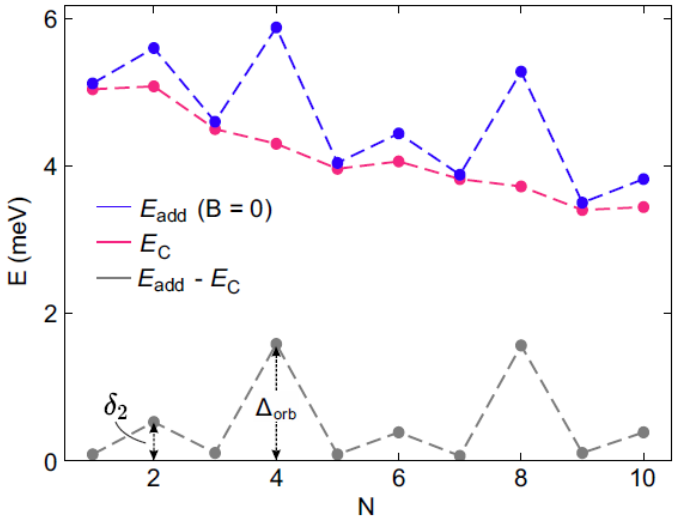


L. Banszerus, PhD Thesis

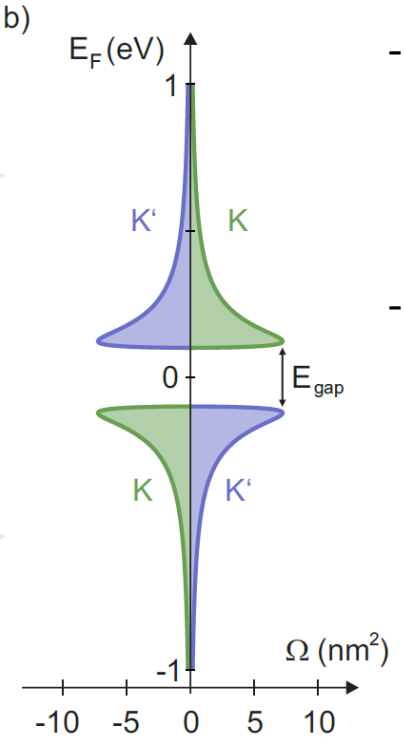




- Every fourth spacing is larger (addition energy)
- Investigate in perpendicular field
- See lines going towards each other and almost showing avoided crossing – their distance is the charging energy
- Using these charging energies level spacing can be obtained
- For full shell (valley and spin – large level spacing is visible)



S. Möller et al., Phys. Rev. B 108, 125128 (2023)



- If inversion symmetry is broken finite Berry curvature arises close to the gap edge. Opposite in the two valleys

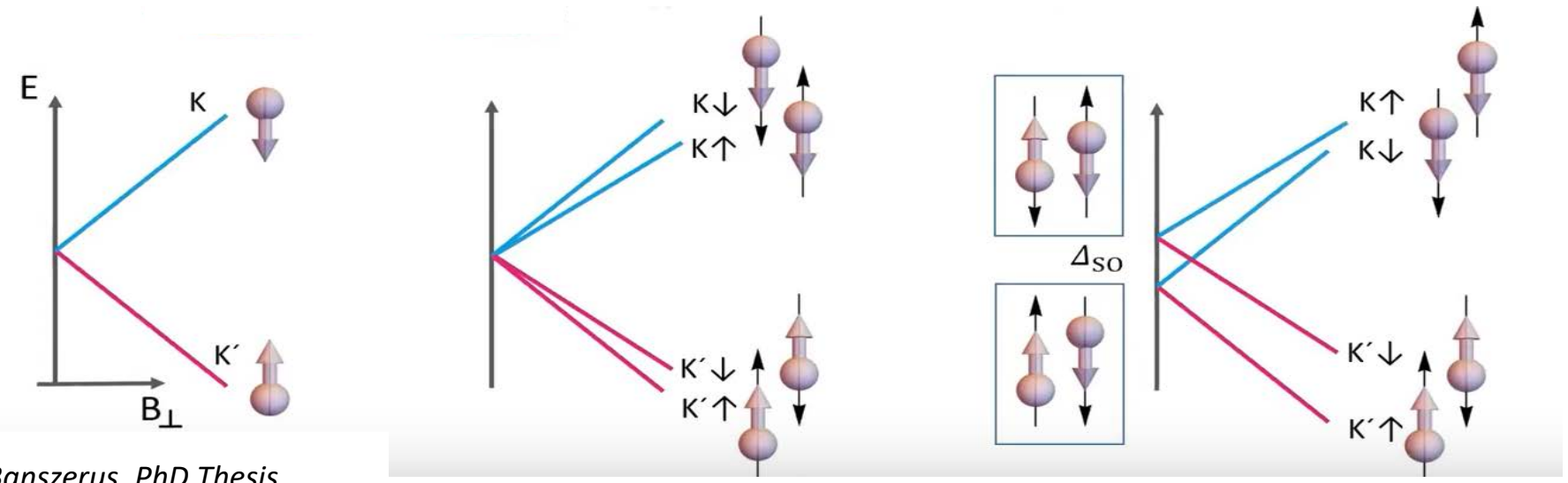
$$\Omega = i\hbar^2 \langle \nabla_{\mathrm{k}} \Psi(\mathbf{k})| \times |\nabla_{\mathrm{k}} \Psi(\mathbf{k})\rangle \, \mathbf{e}_z .$$

- This leads to a finite magnetization, which can couple to magnetic fields

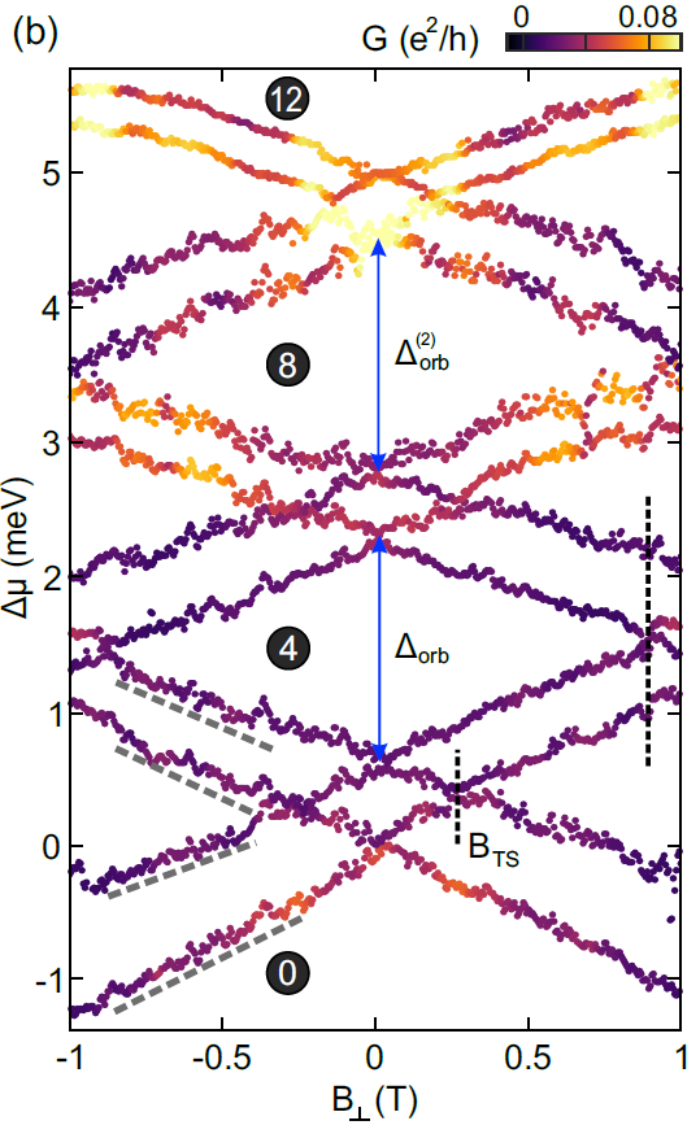
$$\mathbf{M} = -ie\hbar \langle \nabla_{\mathrm{k}} \Psi(\mathbf{k})| \times (\varepsilon(\mathbf{k}) - H(\mathbf{k})) \, |\nabla_{\mathrm{k}} \Psi(\mathbf{k})\rangle \, \mathbf{e_z}$$

$$\Delta E_{\mathrm{v}} = \pm \frac{1}{2} g_{\mathrm{v}} \mu_{\mathrm{B}} B_{\mathrm{z}}$$ Valley splitting

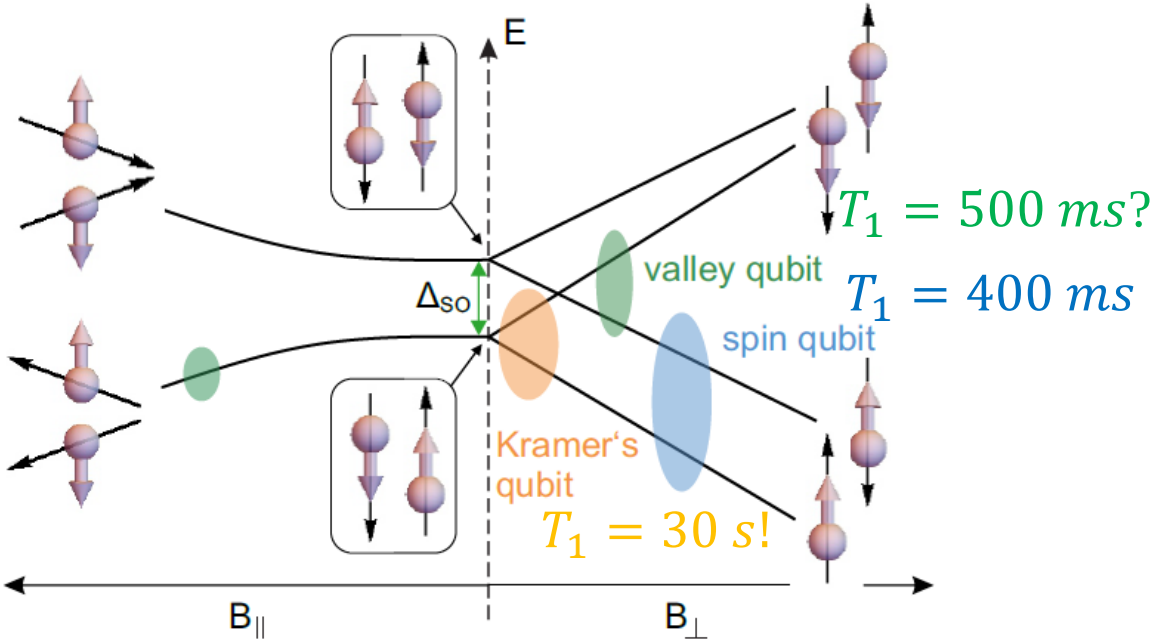
Expected level evolution including spin Zeemann, valley coupling and spin orbit coupling



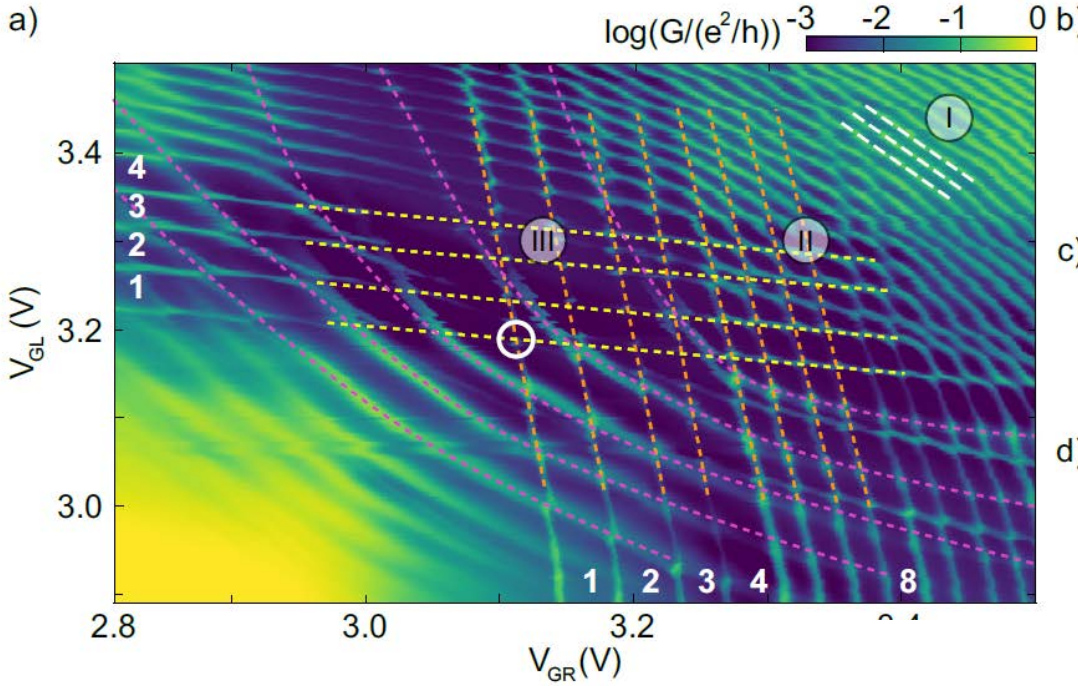
L. Banszerus, PhD Thesis

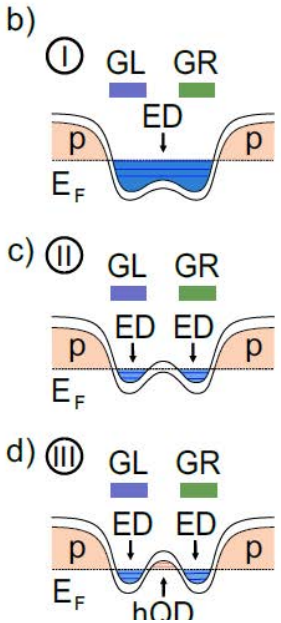


- Calculating orbital energies show expected band structure
- $g_s \sim 2, g_v \sim 30$ – valley g-factor is huge
- SOC can be extracted – matches expectation
- In parallel field (not shown) only spin splitting happens
- Can also investigate finite bias spectroscopy and go beyond single particle band structure
- Ideas to use this as qubit (valley?, spin?, Kramers)?

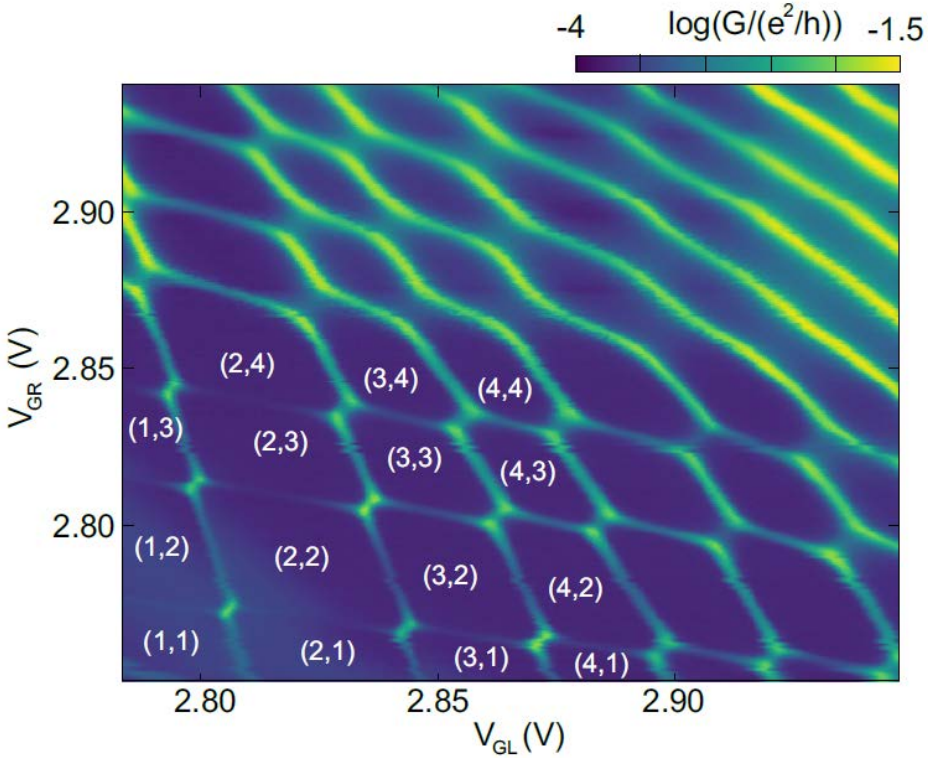


S. Möller et al., Phys. Rev. B 108, 125128 (2023)





- Double dots can be realized (e-e, h-h or even e-h)
- Can study all spin-qubit physics



L. Banszerus et al., Nano Lett. 20, 2005 (2020)