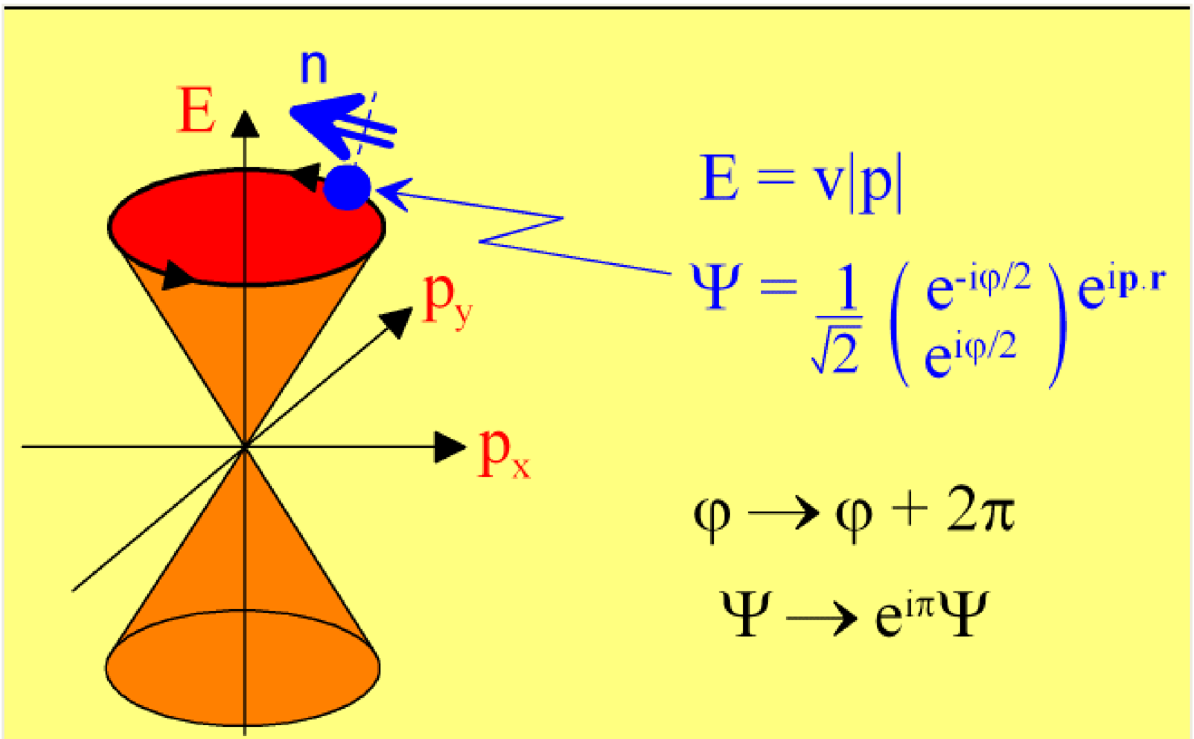
Berry's phase of π

Massless Dirac fermions with Berry's phase π

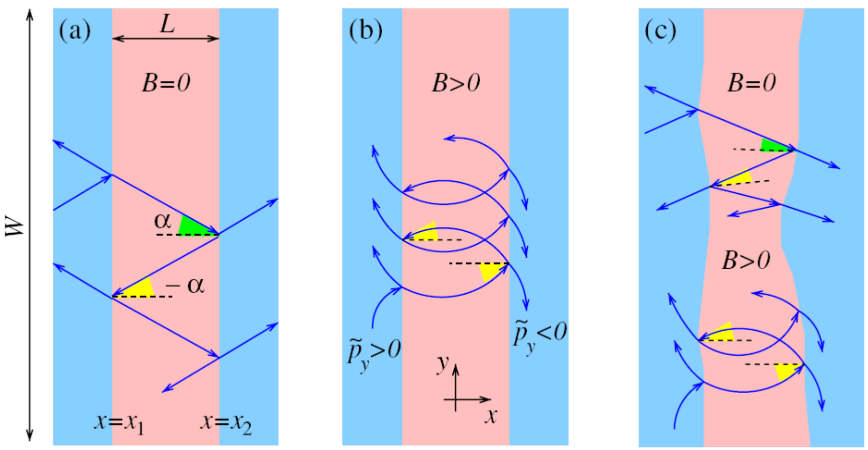
$$H = v\begin{pmatrix} 0 & \pi^+ \\ \pi & 0 \end{pmatrix} = vp\begin{pmatrix} 0 & e^{-i\varphi} \\ e^{i\varphi} & 0 \end{pmatrix};$$

Solution: $$E = vp \quad \Leftrightarrow \quad \psi(\varphi) = \tfrac{1}{\sqrt{2}}\begin{pmatrix} e^{-i\varphi/2} \\ e^{i\varphi/2} \end{pmatrix}$$



Making a loop around k=0 induces a phase shift of **π.**
Similar to the 360° rotation of an 1/2 e spin (SU(2) symmetry).

Klein backscattering & Fabry-Perot Interferences



Interferences on P-N-P junction

When incidence angle, α is varied from positive to negative, phase of the reflection amplitude (R) jumps π. Its sign changes. (At α=0, R=0).

If α<0 → R>0, several scatterings in P-N- P → interference pattern

Accumulated phase in one circle:

$$\Delta\theta = 2\theta_{WBK} + \Delta\theta_1 + \Delta\theta_2$$

where θ_{WBK} phase from travelling in N

$\Delta\theta_1, \Delta\theta_2$ Klein back reflection phase of the interfaces

At B=0 (see Fig. a) the incidence angles $\Delta\theta_{1(2)}$ at P-N and N-P have opposite signs → jumps in $\Delta\theta_1$, $\Delta\theta_2$ cancels

At B>0 (see Fig. b), trajectories are curved, → incidence angles at P-N and N-P can be equal

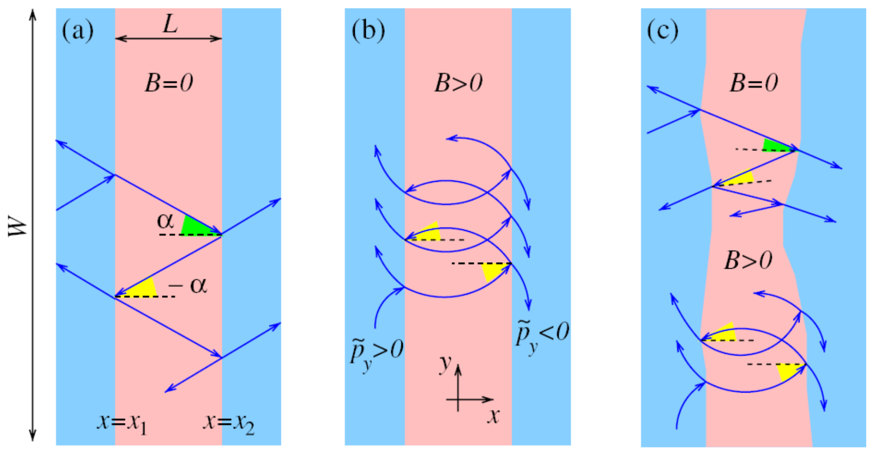
In this case one can show that $\Delta\theta_1 + \Delta\theta_2 = \pi$ (It is the Berry phase previously derived!)

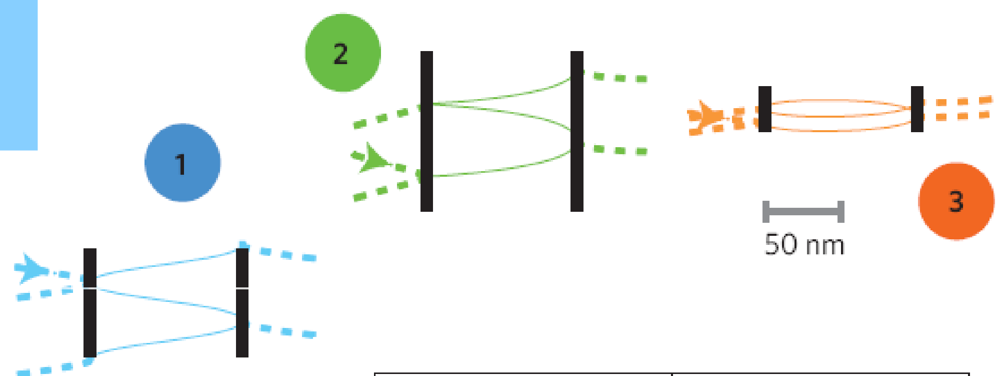
Thus for B=0 ↗ and trajectories with small p_y π shift is expected (i.e. sign change) in transmission amplitude

(Fig.c) one can show, it is robust against barrier roughness

Shytov et al. PRL 101, 156804 (2008)

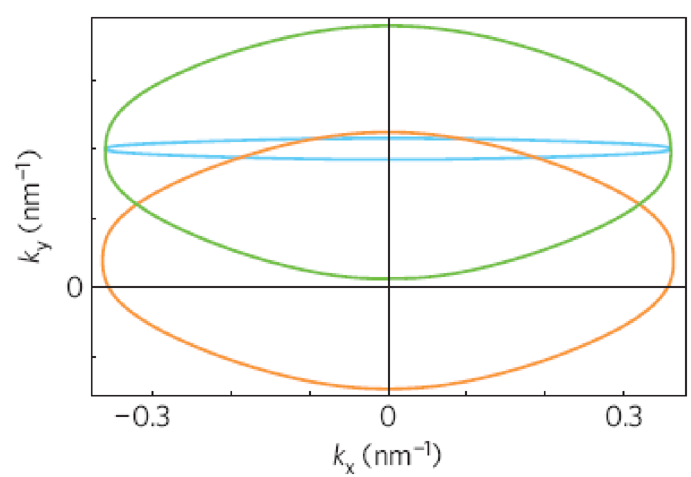
Klein backscattering & Fabry-Perot Interferences





Remark (Barry-phase):

Trajectory in Fig.a corresponds to **1**
Trajectory in Fig.b corresponds to **3**
The main difference that during one circle between P-N and N-P:
the k vector of **3** goes around k=0
while for **1** NOT.
This generates the Berry phase:
Due to the chiral symmetry, topological singularity at degeneracy point of the band structure k=0.

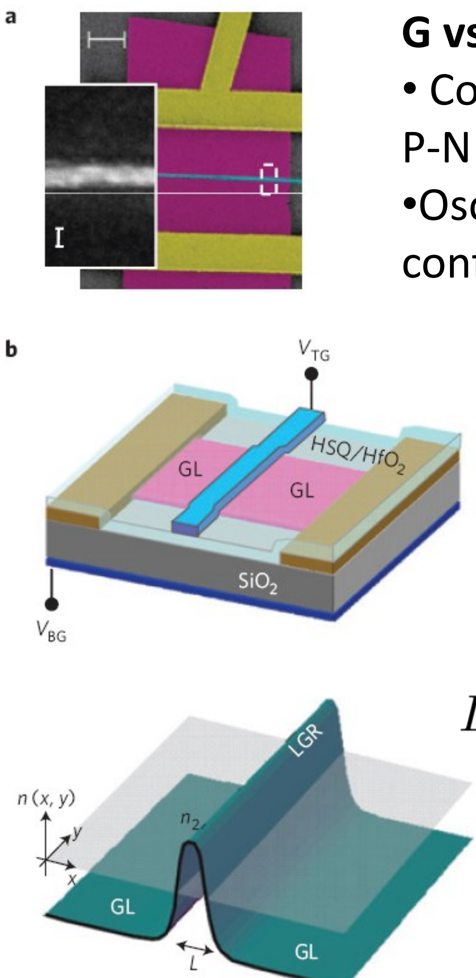


Shytov et al. PRL 101, 156804 (2008)

Klein backscattering & Fabry-Perot Interferences

N-P-N device

Separate gating by backgate and topgate
Topgate width=20nm! → ballistic

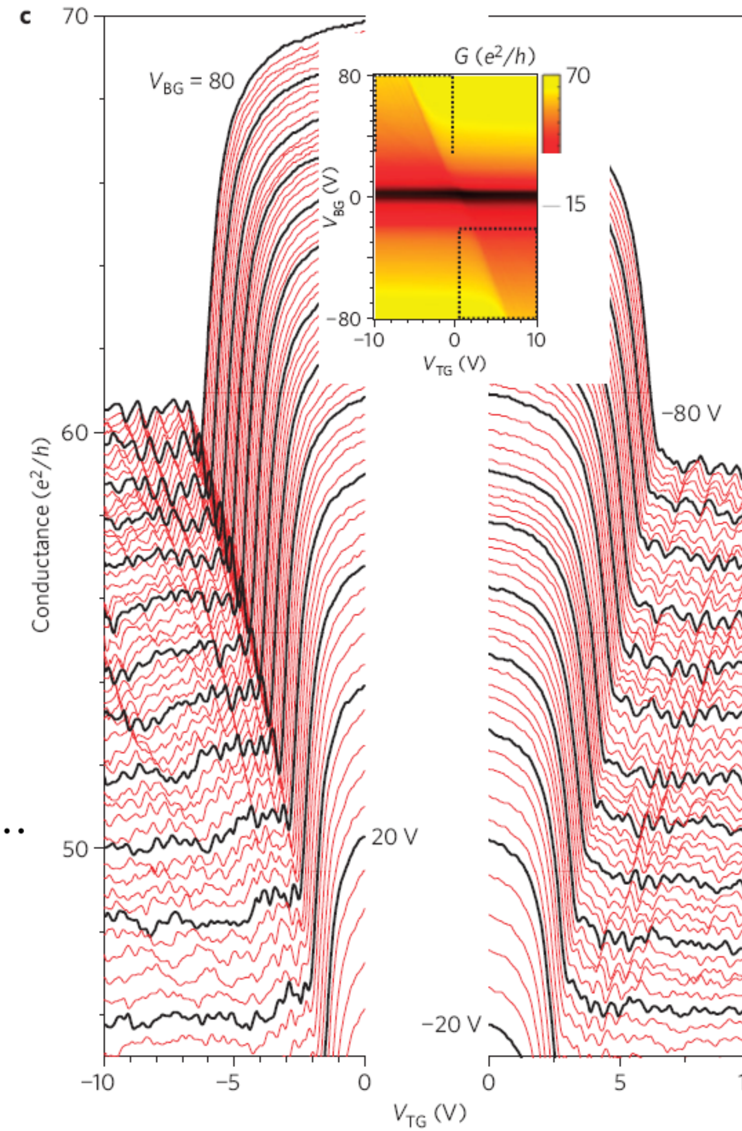


G vs. V_{TG} vs. V_{BG}

- Conductance is lower when N-P-N setting instead of N-N-N
- Oscillations at N-P-N configuration:
  - V_{TG} varies pot. barrier → $\delta\theta_{WBK}$ → oscillations
  - Oscillatory G is induced by trajectories with incident angle where neither T, nor R is large (i.e. α not too small)

$$L = j\frac{\lambda}{2} \quad \text{where } j = 1, 2, 3...$$

$$\lambda = \frac{2\pi}{\sqrt{n\pi}}$$



Young et al. Nature Physics 5, 222 (2009)

Klein backscattering & Fabry-Perot Interferences

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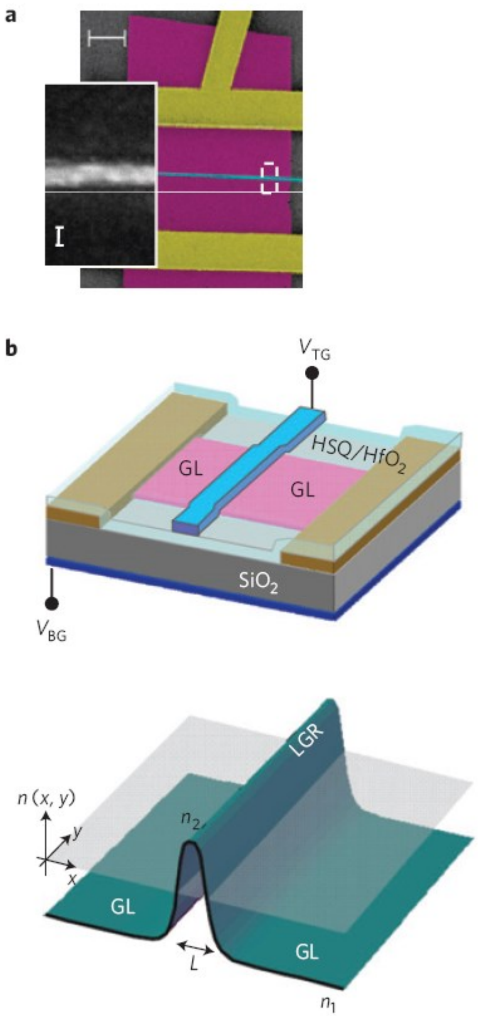
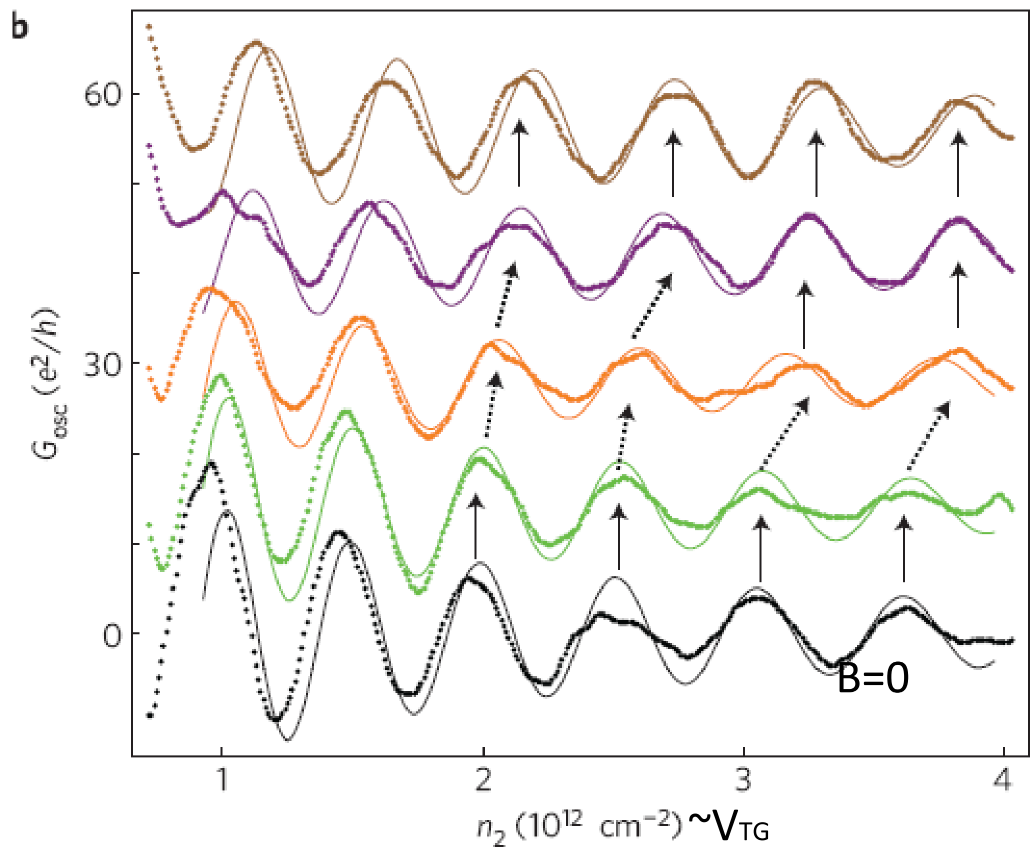


Young et al. Nature Physics 5, 222 (2009)

G oscillations vs. B *(Dots experiment, line theory)*

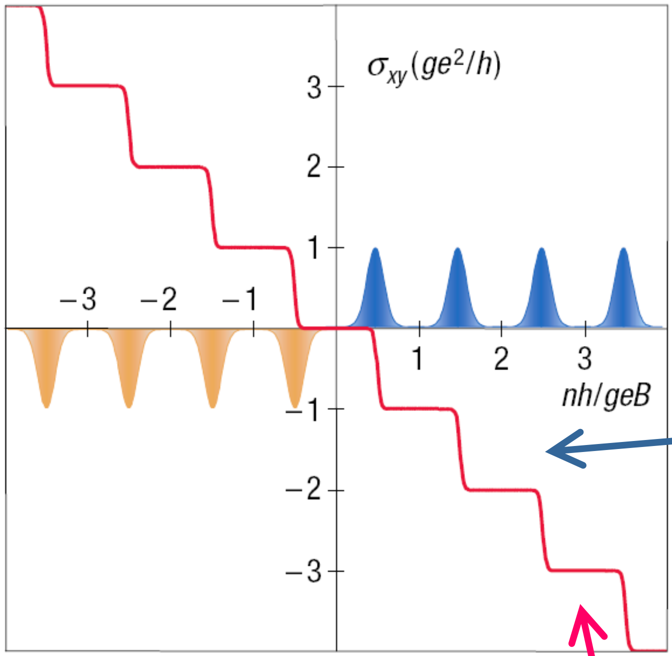
At different B fields (B=0, 200, 400, 600, 800mT from bottom to top) the oscillations of G.

In this B range ≈ π shift is induced in the interference pattern.

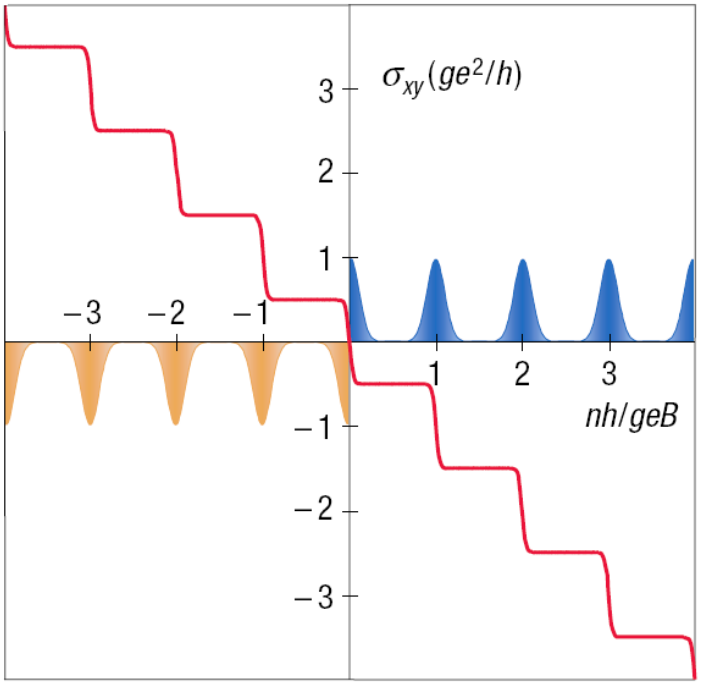


Quantum Hall effect in graphene

Classical 2DEG



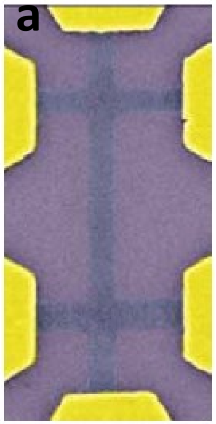
Graphene



Each filled Landau level with additional degeneracy g contributes conductance quantum ge^2/h towards the Hall conductivity

Half Integer Quantum Hall effect in graphene

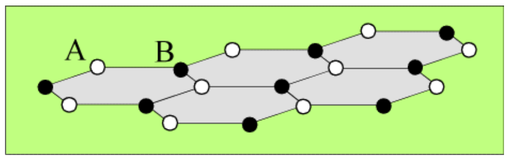
Novoselov *et al*, Nature 438, 197 (2005)

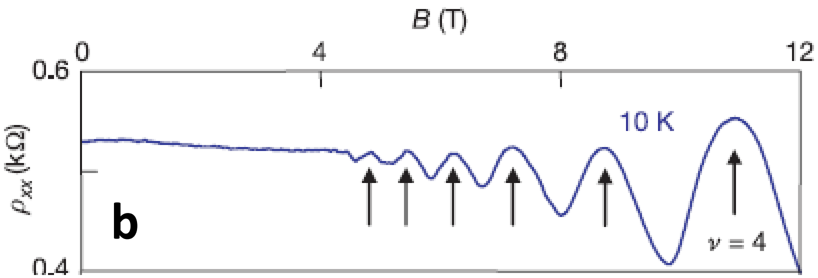


Graphene in Hall geometry

Sample width of 200nm

- Sample: Hall geometry is etched from graphene flakes by oxygen plasma (a)

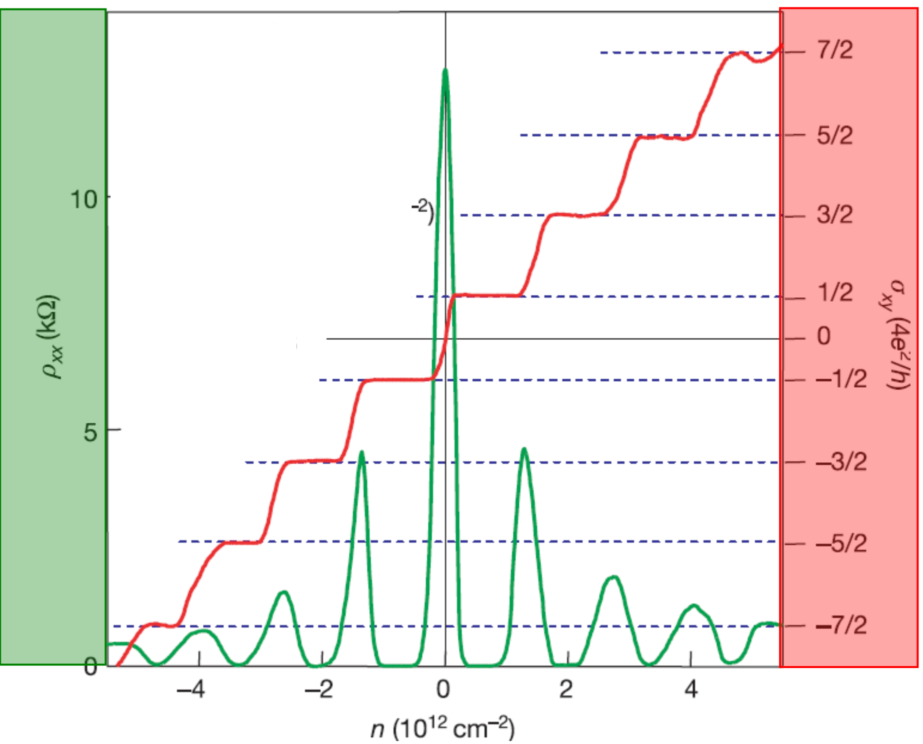




Longitudinal and Hall measurements vs B field

Conventional way of QHE measurement

QHE vs. density (gate voltage)



-In magnetic field Shubnikov-de Haas oscillations are present. (b) At large B field, ρ_{xx} gets zero as for QHE.

- Great advantage of graphene, that the charge density (n) can be varied by gate voltage. QHE effect can be studied as a function of n.

-Figure c: QHE measurement at 14T, 4K.

Half-Integer Quantum Hall effect

Properties:

- Height of the Hall plateaus is $4e^2/h$
- First e (h) plateau is at $2e^2/h$
- ρ_{xx} is zero at the place of the plateaus.

$$\sigma_{xy} = 4e^2/h(n+1/2)$$

ρ_{xx} has maximum at n=0 → There is Landau level at zero energy. Electrons or holes contribute?

Half Integer Quantum Hall effect in graphene

Solution of the graphene Hamiltonian in B field

Let us start with the effective Dirac Hamiltonian at the K point

$$H = v\begin{pmatrix} & \pi^+ \\ \pi & \end{pmatrix}, \qquad \pi = p_x + ip_y, \quad \pi^+ = p_x - ip_y.$$

Hint: Besides a constant π and $\pi^+$are the same operators as the raising and lowering operators of the harmonic oscillator Hamiltonian of the normal 2DEG in B field, i.e.

In case of magnetic field: $\vec{p} = \frac{\hbar}{i}\vec{\nabla} - \frac{e}{c}\vec{A}, \quad \vec{\nabla} \times \vec{A} = B\vec{e_z}$

Let us use a gauge of $\vec{A} = (-By, 0,0)$:

$$\pi = \frac{\hbar}{i}\partial_x + \frac{e}{c}By + \hbar\partial_y,$$

$$\pi^+ = \frac{\hbar}{i}\partial_x + \frac{e}{c}By - \hbar\partial_y.$$

Take the wave function ansatz, $\Psi(\vec{r}) = \begin{pmatrix} c_1\phi_n \\ c_2\phi_{n+1}\end{pmatrix}\frac{e^{ik_x x}}{\sqrt{L}}$:

$$\pi = \hbar k_x + \frac{e}{c}By + \hbar\partial_y,$$

$$\pi^+ = \hbar k_x + \frac{e}{c}By - \hbar\partial_y.$$

Replacing y by $y^{'}$, where $\hbar k_x + \frac{e}{c}By = \frac{e}{c}By^{'}$:

$$\pi = \frac{e}{c}By' + \hbar\partial_{y'},$$

$$\pi^+ = \frac{e}{c}By' - \hbar\partial_{y'}.$$

N.Peres et al., PRB 73, 125411 (2006)

Half Integer Quantum Hall effect in graphene

Solution of the graphene Hamiltonian in B field

Let us introduce a^+, a which fulfills the algebra of the raising and lowering operators of the harmonic oscillator: $a = \pi^+ \frac{c}{eB}\frac{1}{\sqrt{2}r_c}$, $a^+ = \pi \frac{c}{eB}\frac{1}{\sqrt{2}r_c}$, where r_c is the cyclotron radius $r_c{}^2 = \frac{\hbar c}{eB}$.

It gives

$$a = \frac{1}{\sqrt{2}r_c}\left(y' + r_c{}^2\partial_{y'}\right),$$

$$a^+ = \frac{1}{\sqrt{2}r_c}\left(y' - r_c{}^2\partial_{y'}\right).$$

These two operators fulfill: $[a, a^+] = 1$.

ϕ_n is the eigenfunction of the a related harmonic oscillator, i.e.

$$a|\phi_n\rangle = \sqrt{n}|\phi_{n-1}\rangle, \quad a^+|\phi_n\rangle = \sqrt{n+1}|\phi_{n+1}\rangle.$$

Returning to the Dirac Hamiltonian:

$$H = v\begin{pmatrix} & \pi^+ \\ \pi & \end{pmatrix} = -v\left(\frac{c}{eB}\frac{1}{\sqrt{2}r_c}\right)^{-1}\begin{pmatrix} & a \\ a^+ & \end{pmatrix} = -v\frac{\sqrt{2}\hbar}{r_c}\begin{pmatrix} & a \\ a^+ & \end{pmatrix}$$

N.Peres et al., PRB 73, 125411 (2006)

Half Integer Quantum Hall effect in graphene

Solution of the Hamiltonian of Dirac electrons in B field

Let us start with the wavefunction $\Psi_n(\vec{r}) = \begin{pmatrix} \phi_n \\ \alpha\phi_{n+1} \end{pmatrix} \frac{e^{ik_x x}}{\sqrt{L}}$ where $\alpha = \pm 1$.

$$H\Psi_n \to \begin{pmatrix} & a \\ a^+ & \end{pmatrix} \begin{pmatrix} \phi_n \\ \alpha\phi_{n+1} \end{pmatrix} = \begin{pmatrix} \sqrt{n+1}\alpha\phi_n \\ \sqrt{n+1}\,\phi_{n+1} \end{pmatrix} = \sqrt{n+1}\alpha \begin{pmatrix} \phi_n \\ \alpha\phi_{n+1} \end{pmatrix}$$

$$H\Psi_n = -v\frac{\sqrt{2}\hbar}{r_c}\sqrt{n+1}\alpha\Psi_n$$

Landau levels in graphene: $\boldsymbol{E_n = \pm v\frac{\sqrt{2}\hbar}{r_c}\sqrt{n+1}, \quad n = 0, 1, 2, \ldots}$

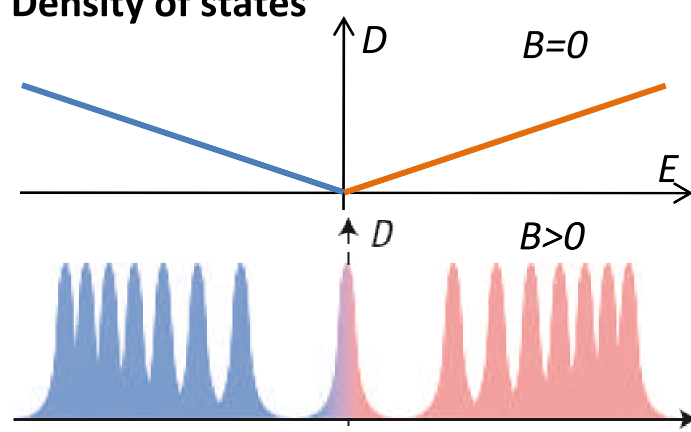
There is an extra solution as well: $\Psi_0 = \begin{pmatrix} 0 \\ \phi_0 \end{pmatrix} \frac{e^{ik_x x}}{\sqrt{L}}$. $\qquad H\Psi_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} = E\Psi_0 \to \boldsymbol{E_0 = 0}$.

Degeneracy of the levels:

Similar to normal Landau Levels. $L > y > 0 \to L > \frac{\hbar c}{eB}k_x > 0$ and $k_x = \frac{2\pi}{L}n$ where n is integer.

→The degeneracy: $N = \frac{L^2 B/c}{h/e}$ i.e. number of flux quantum penetrating the sample.

Density of states



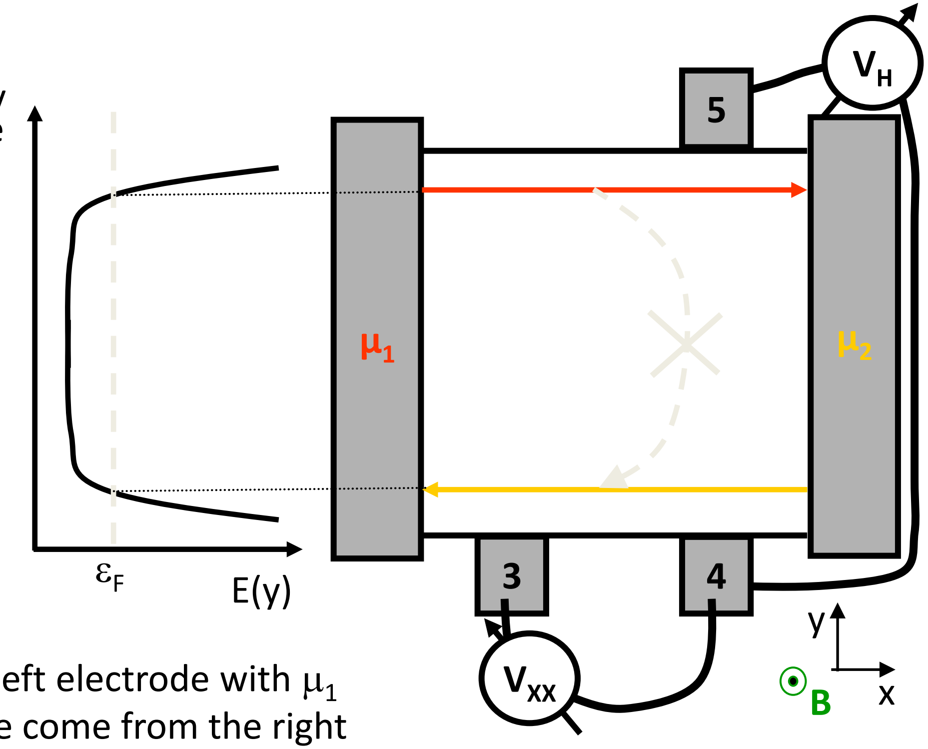
Solving the problem for the K' effective Hamiltonian gives the same spectrum as the one for K. Therefore each E_n energy level has a degeneracy of $N * 2 * 2$. 2 from the two valleys, 2 from the real spin of the electrons.

N.Peres et al., PRB 73, 125411 (2006)

Single Landau level

At the Fermi energy electron states are only available at the sample edges -> the current flows along the edges

At the upper edge: $\dot{x}_0 = \dfrac{1}{eB}\dfrac{\partial U_{\text{enclosing}}}{\partial y_0} > 0$ -> motion in positive *x* direction

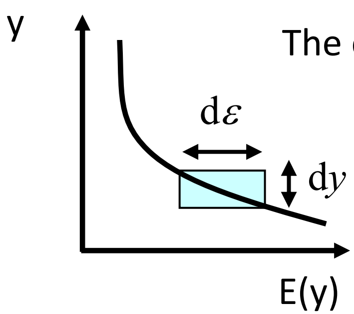


Similarly: at the lower edge the electrons move along the negative *x* direction

Inside the sample no states are available at the Fermi energy, so the upper „**edge states**" cannot be scattered to the lower edge states and vice versa.

The electrons moving at the upper edge all come from the left electrode with μ_1 chemical potential, whereas the electrons at the lower edge come from the right electrode with μ_2 chemical potential!

$$\boxed{V_{xx} = 0, \quad V_H = (\mu_1 - \mu_2)/e}$$



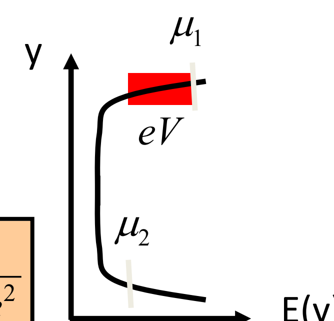
The contribution of a dε interval of an edge state to the current:

$$I = j \cdot \mathrm{d}y = \underbrace{n}_{\frac{2eB}{h}} \cdot e \cdot \underbrace{v}_{\frac{1}{eB}\frac{\mathrm{d}\varepsilon}{\mathrm{d}y}} \cdot \mathrm{d}y = \frac{2e}{h}\mathrm{d}\varepsilon$$

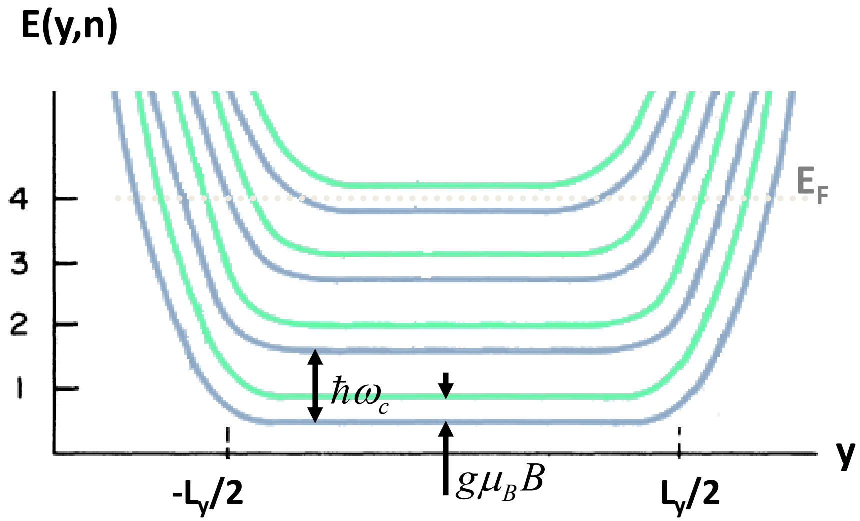
At μ_1-μ_2=eV the upper edge state is occupied to an energy higher by eV than the lower edge state, thus the net current:

$$I = \frac{2e}{h}eV \Rightarrow G_H = \frac{I}{V_H} = \frac{2e^2}{h}$$

$$\boxed{R_H = \frac{V_H}{I} = \frac{h}{2e^2}}$$



Multiple Landau levels, Zeeman splitting



So far the spin was not considered. In a B field the Landau levels are split for ↓, ↑ electrons (Zeeman splitting)

$$\mathrm{E} = \hbar\omega_C\left(\mathrm{n}+\frac{1}{2}\right) + U_{\text{confinement}} + \boldsymbol{g\mu_B B s_z}$$

In semiconductors $\hbar\omega_C >> g\mu_B B$,

($\hbar\omega_C[K] = 20 \cdot B[T]$, $g\mu_B B[K] = 0.3 \cdot B[T]$)

At large enough B the ↓ and ↑ spin electrons form well-separated energy levels, the so-called **spin polarized Landau levels**.

In case of spin splitting the previous considerations for the edge states are not affected, only the 2x spin degeneracy factor should be omitted!

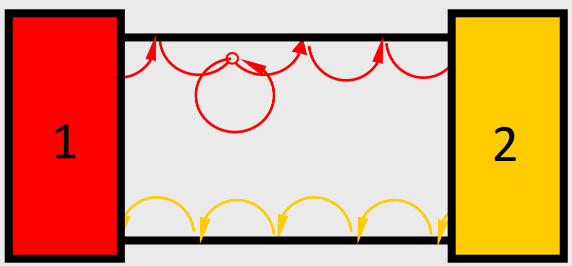
With M spin polarized Landau levels crossing the Fermi energy at the edges (but non of them is close to the Fermi energy inside the sample):

$$G_H = \frac{I}{V_H} = \frac{e^2}{h} M \qquad \boxed{R_H = \frac{h}{e^2}\frac{1}{M}}$$

This is observed in the measurements!

The relative accuracy of R_H is ~10^{-7}

→ This demonstrates the perfect absence of backscattering



Classical picture: an electron from the left electrode will always arrive at the right electrode, even if it scatters on impurities

Half Integer Quantum Hall effect in graphene

Solution of the Hamiltonian of Dirac electrons in B field

Remark:

The edge states behave similar to the ones of QHE of normal 2DEGs.

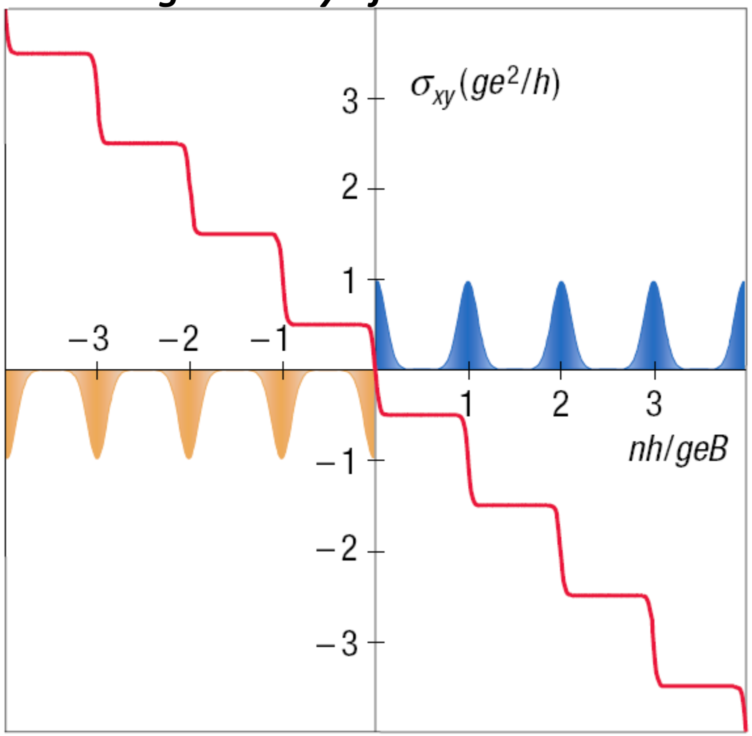
On the two sides of the sample, they propagate to opposite direction

$$v_x = \frac{1}{\hbar}\frac{\partial E}{\partial k_x} = \frac{1}{\hbar}\frac{\partial E}{\partial y}\frac{\partial y}{\partial k_x} = \frac{1}{\hbar}\frac{\partial E}{\partial y}\frac{1}{eB/c}$$

Half-integer quantum Hall-effect:

Due to the 2 spin and 2 valley, there are 4-fold degenerate Landau levels. Each degeneracy provides a conductance channel with $G = \frac{e^2}{h}$. Therefore, each filled LL enhance the Hall conductance by $G = \frac{2 \cdot 2 \cdot e^2}{h}$. When E_F is placed on a LL, the Hall conductance changes from a quantized plateau to the next one. Since there is a LL at ZERO ENERGY the first electron like Hall plateau is at $G = \frac{2 \cdot e^2}{h}$ and the rest are at $G = \frac{2 \cdot 2 \cdot e^2}{h}\left(n + \frac{1}{2}\right)$. The zero energy LL makes the QHE of graphene special. It consist e and hole states as well.

Charge density of Landau levels



N.Peres et al., PRB 73, 125411 (2006)

Room-temperature Quantum Hall effect in graphene

Landau-levels

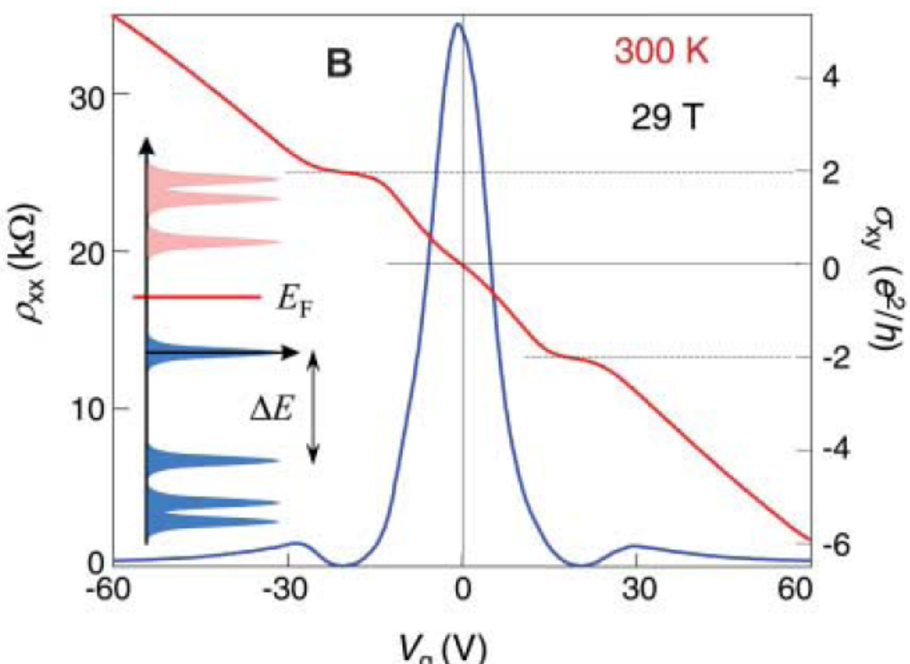
$$E_n = \pm\sqrt{2e\hbar v^2 |n| B}$$ 2D Dirac fermions (m=0)

$$E_n = \hbar\omega_C (n + 1/2)$$ 2D free electrons

Comparing to GaAs based 2DEGs

Graphene:	GaAs/AlGaAs:
E_1(B=1T)≈350K	ħω(B=1T)≈20K
E_1(B=10T)≈10^3K	ħω(B=10T)≈200K
μ≈10^4 cm²/Vs (2006) @4K	μ≈10^5 cm²/Vs (1980)
μ≈10^6 cm²/Vs (2010) @4K	μ≈10^7 cm²/Vs (2004)

Experiment

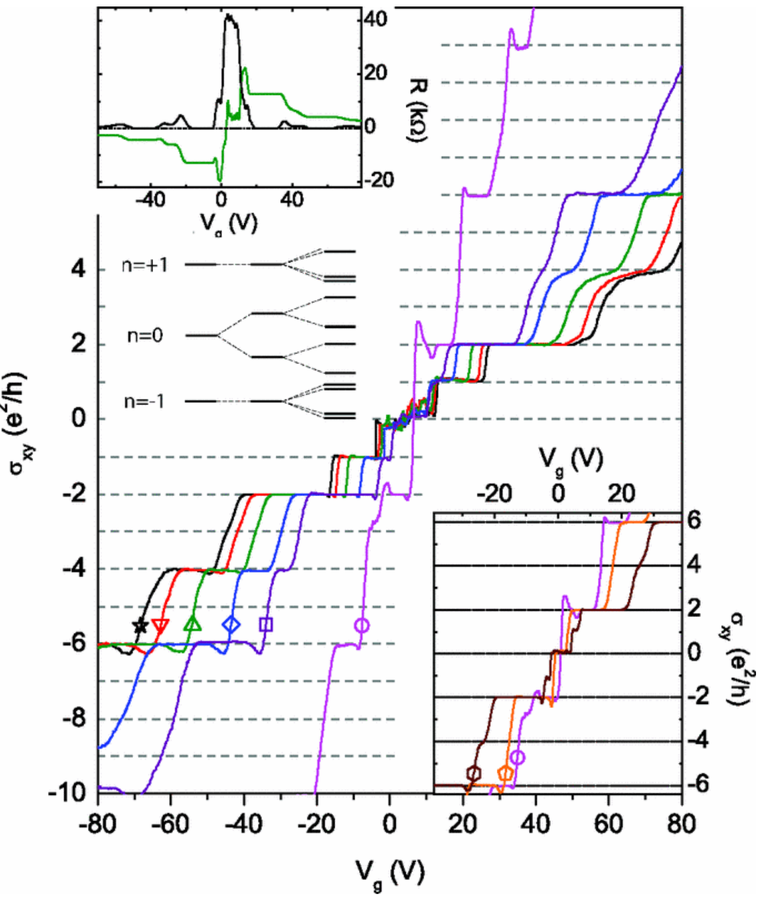


Novoselov, Science **315**, 1379 (2007)

E_1(29T)≈1800K >>kT

μ≈10^4 cm²/Vs @RT (weak T dependence)

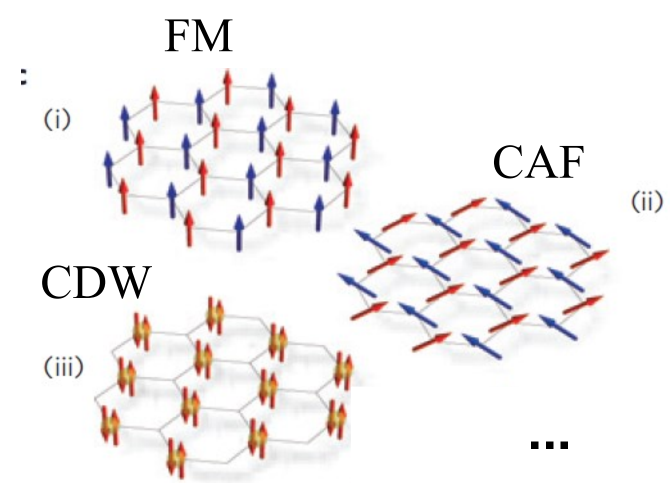
Limitation of B, that $\omega_C\tau$>>1 (τ elastic mean free path). If the amount of scattering can be further decreased, QHE gets visible at lower B fields. → New possibilities for current standard, quantum circuits at room temperature



Broken symmetries in QHE

Landau level degeneracies split up in high magnetic fields
4-fold degeneracy: spin and valley

Interactions:
- Cyclotron gap
- Coulomb (Exchange interactions)
- Zeeman – energy
- Disorder scale

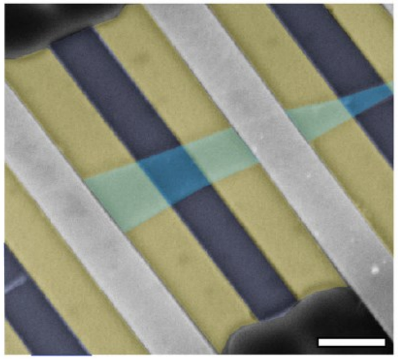


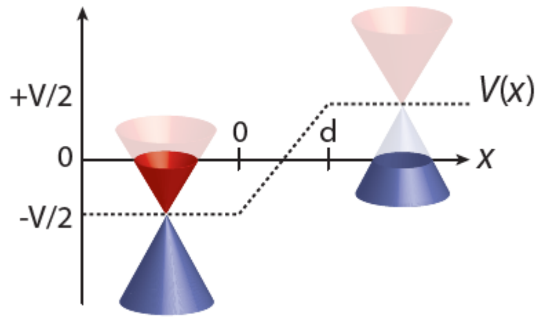
Complex phases appear (e.g. half filling, n=0):
Many possible ground states (e.g. Ferromagnet (FM), canted antiferromagnet (CAF), charge density wave)
Use tilted field measurements (only acts on Zeeman-term, no orbital contribution)
-N=0, n=0 (half filling) – not spin polarized
Canted antiferromagnetic state

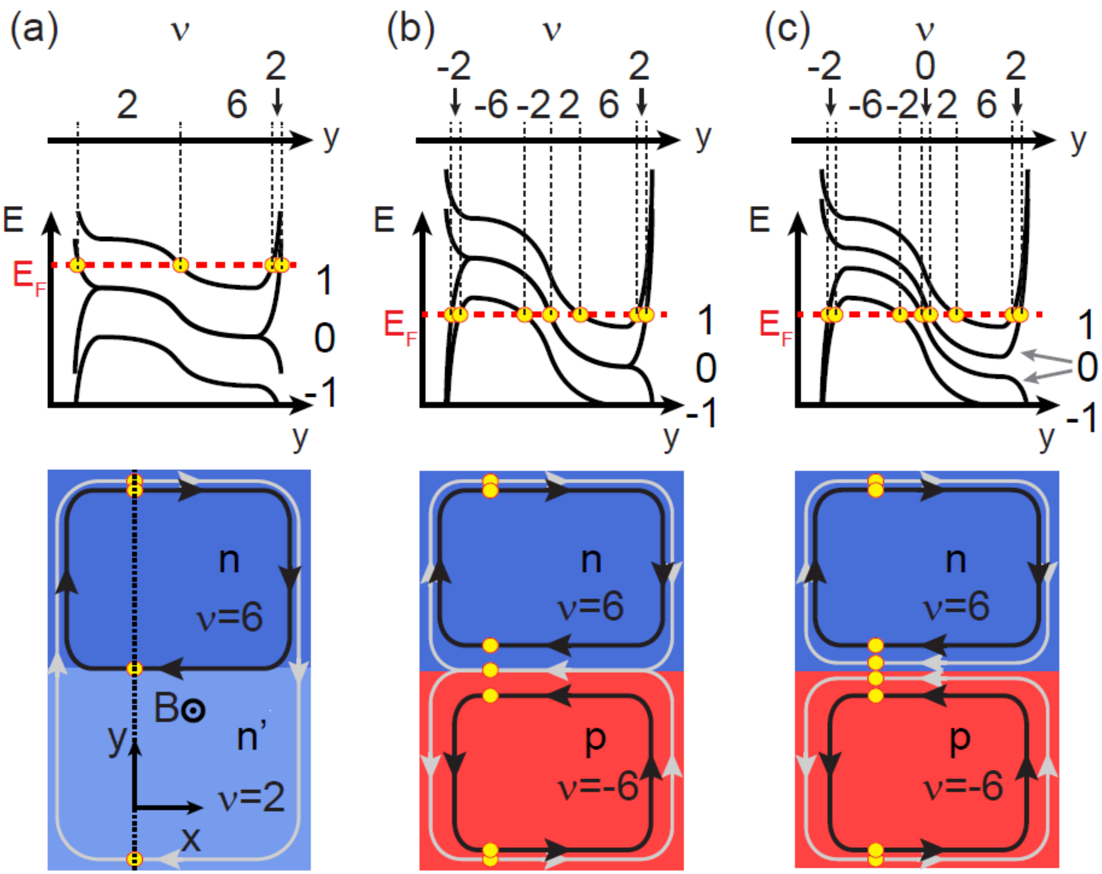
A. F. Young et al., Nat. Phys. 8, 550 (2012)

Y. Zhang et al., PRL 96, 136806 (2006)

QHE with non-uniform doping





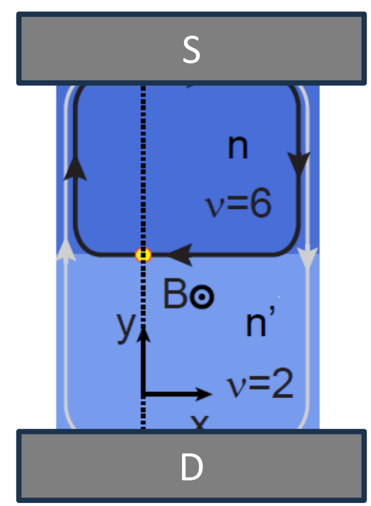


Endre Tóvári PhD Thesis

In n-n' or p-n junctions Quantum Hall channels flow in the bulk

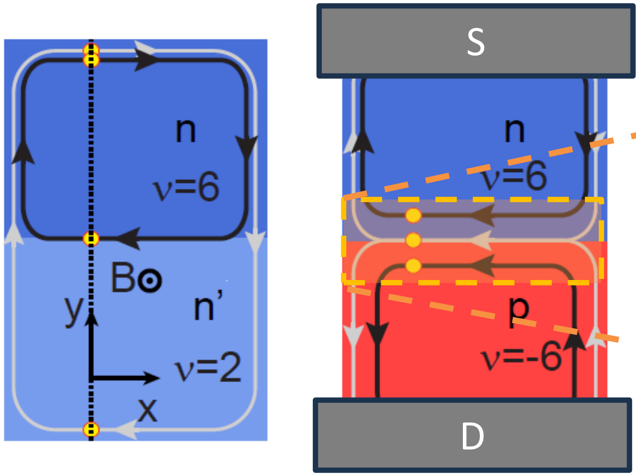
Where bands meet E_f, quantum channels form in the bulk

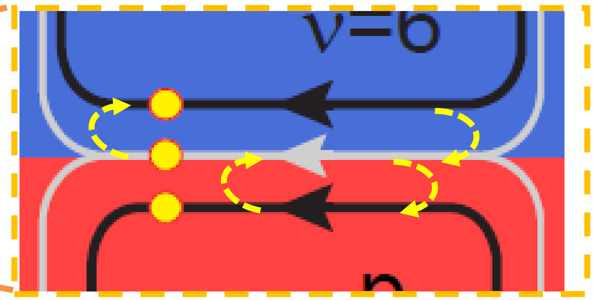
$$\nu = \frac{h}{eB}\alpha V_{BG}$$



Edge state equilibration

What is the conductance in p,n a device?





In bipolar regime electrons can scatter between edge states and current can be distributed between available channels.

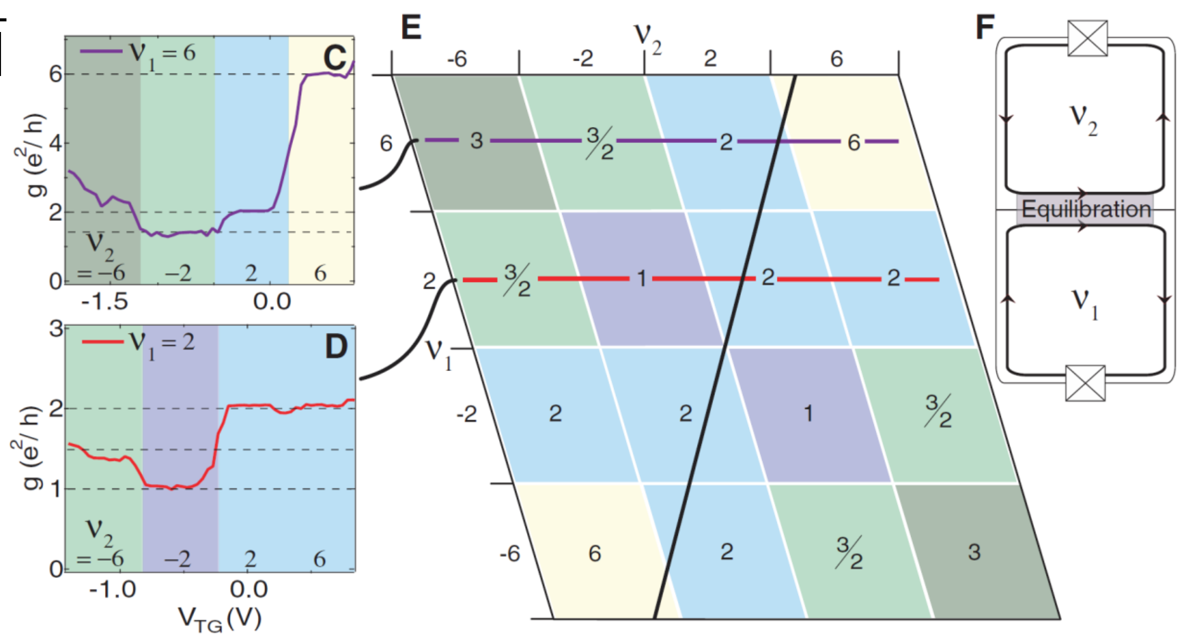
$$G_{unip} = e^2/h \min(\nu, \nu')$$

$$G_{bip} = \frac{e^2}{h}\frac{|\nu * \nu'|}{|\nu| + |\nu'|}$$

E.g. source is biased, states from source are filled, but can be scattered to the states on the p –side (black channel)
Special state at zero energy (electron-hole state)

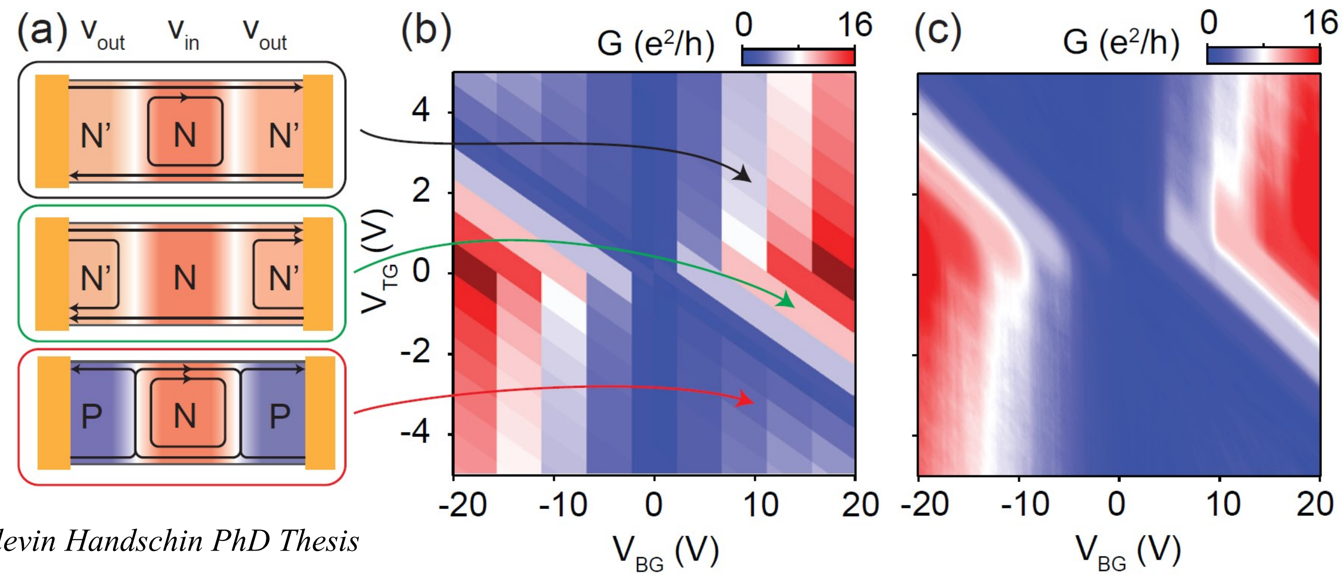
Sample on SiO_2 (low quality) shows full equilibration – fractional plateaus.

What happens in higher quality samples?



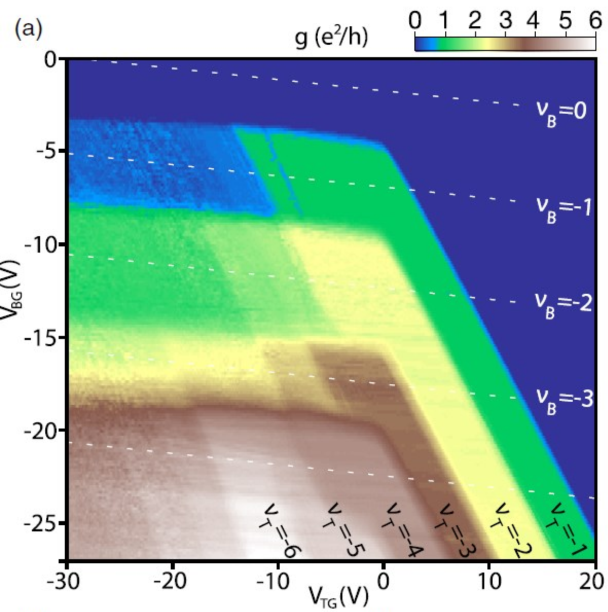
J. R. Williams et al., Science 317, 638 (2007)

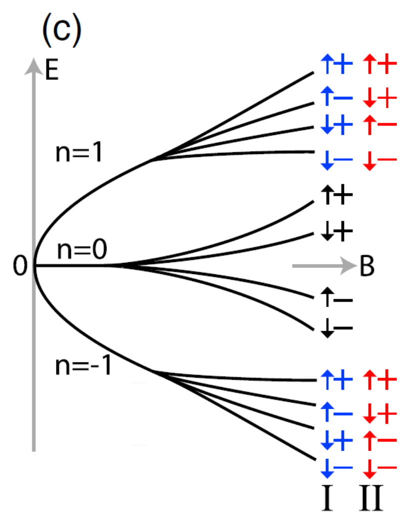
Edge state equilibration



Clevin Handschin PhD Thesis

Similar formulas for p-n-p type junctions can be made
For unipolar: equilibration along the edge (rough)
For bipolar: equilibration along the p-n interface (smooth)
Seems unipolar works better for good samples, more equilibration along the edges





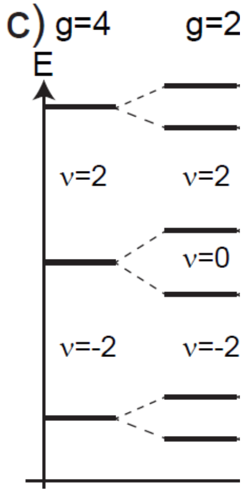
High quality device: the all the degeraciess are split. It seems along the edge valley states equilibrate, spin not. Can be used also to figure out LL scenarios (e.g. here I.)

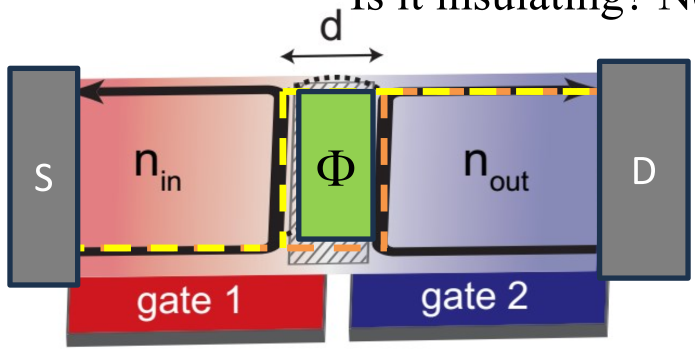
ν_B	ν_T	$g_{\rm full}$	$g_{\rm partial}$	$g_{\rm exp}$	Edge state polarization
1	2	2/3	1	0.98 ± 0.04	ν=1 ν=2 ν=1
1	3	3/5	2/3	0.66 ± 0.005	ν=1 ν=3 ν=1
2	3	1.5	5/3	1.68 ± 0.01	ν=2 ν=3 ν=2

F. Amet et al., PRL., 112, 196601 (2014)

Edge state interferometers

What happens for the p-n regime?
For clean graphene, **gapped region** forms at the pn interface.
Is it insulating? Not fully.



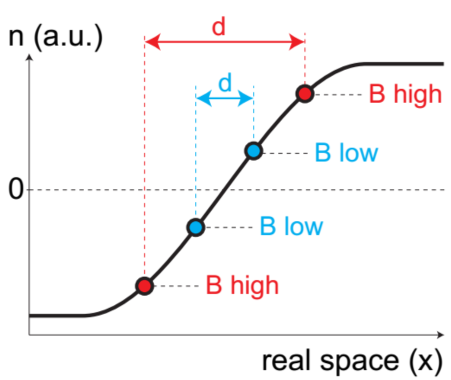


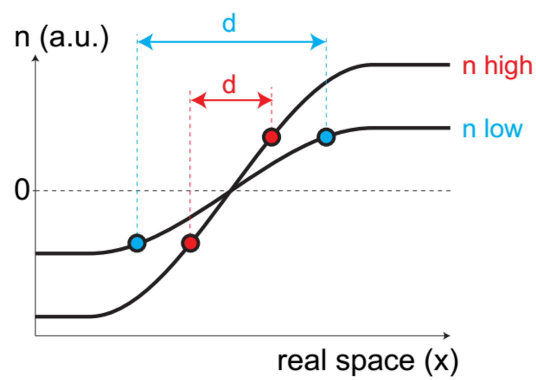
Spin and valley is conserved in the bulk, but scattering can happen at the edge (valley)
Works as an **Aharonov Bohm interferometer**

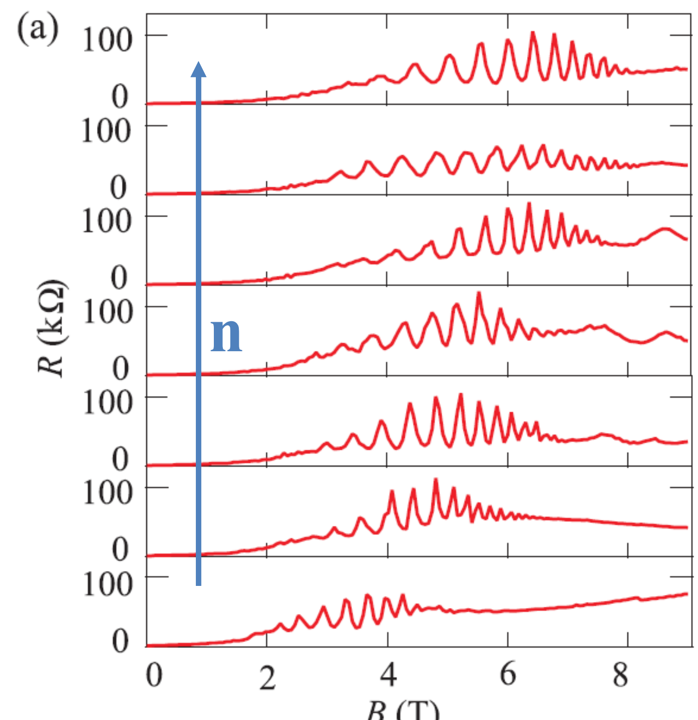
$$G \sim \cos(\Phi)$$

Tunable AB interferometer

- Gate tune pn junction potential slope – position of edge states
- B field also tunes edge state position



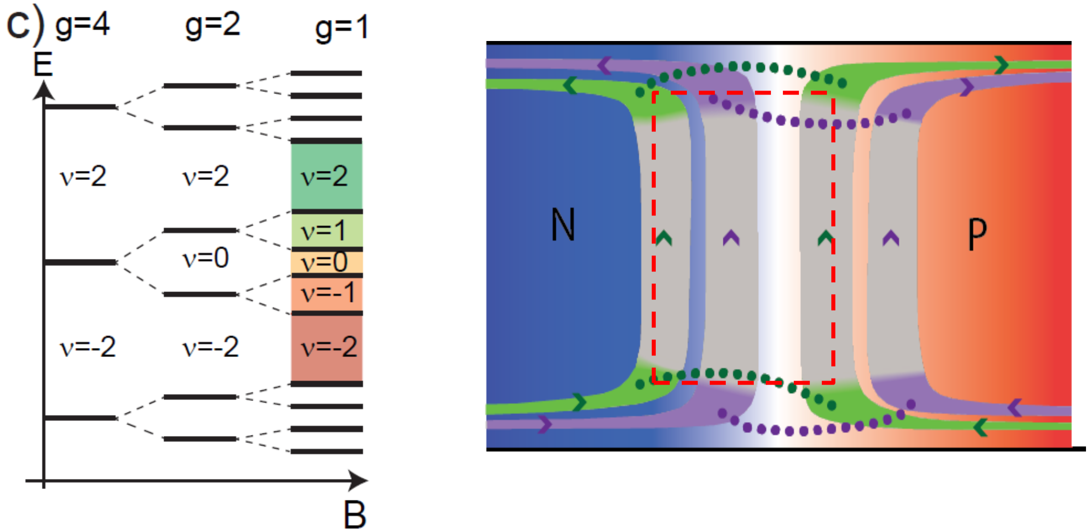


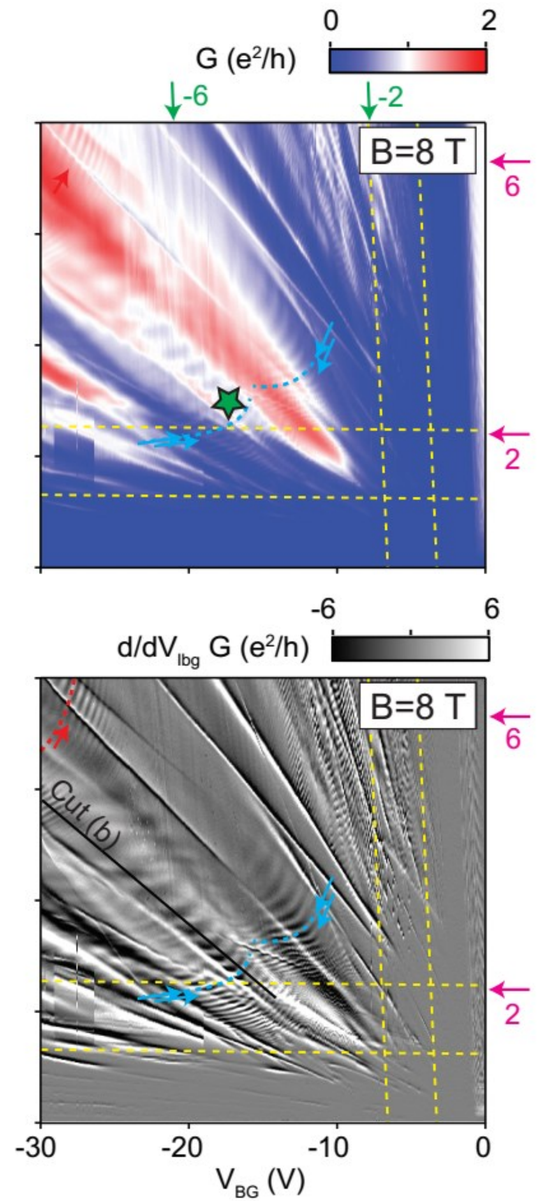


Clevin Handschin PhD Thesis

S. Morikawa et al., Applied. Phys. Lett. 106, 183101 (2015)

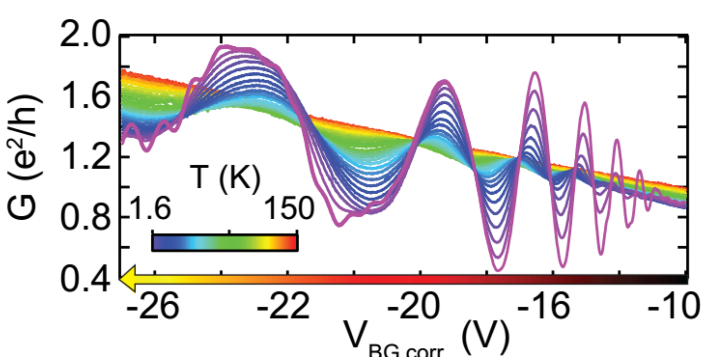
Edge state interferometers





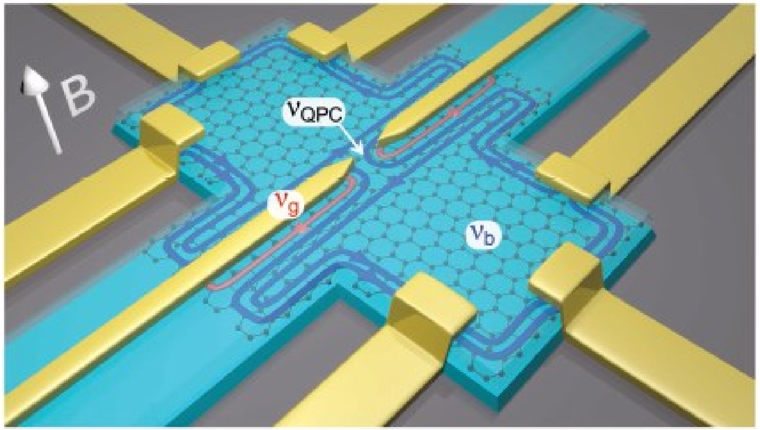
Full lifting of degeneracy – only same spins can mix
Visible on gate-gate maps at large field

Temperature dependence – dephasing and/or relaxation



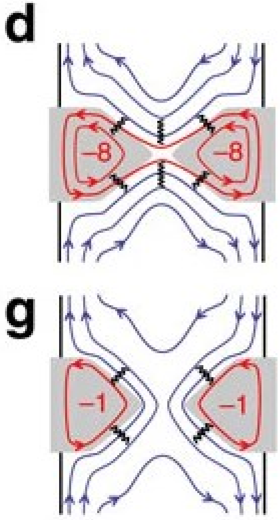
P. M. et al., PRB 2017

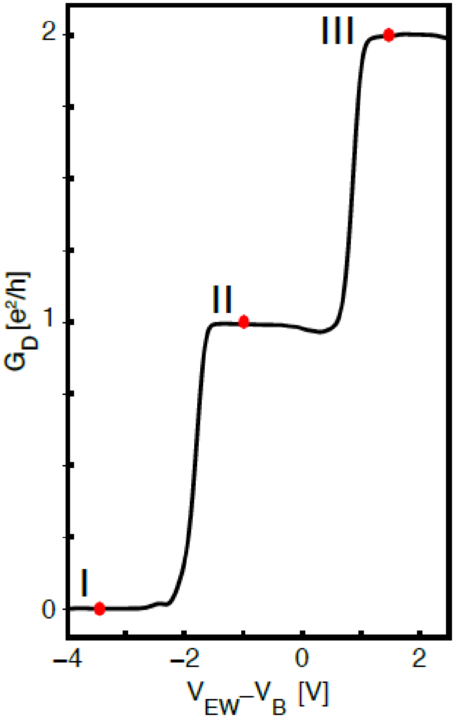
Fabry-Perot interferometers

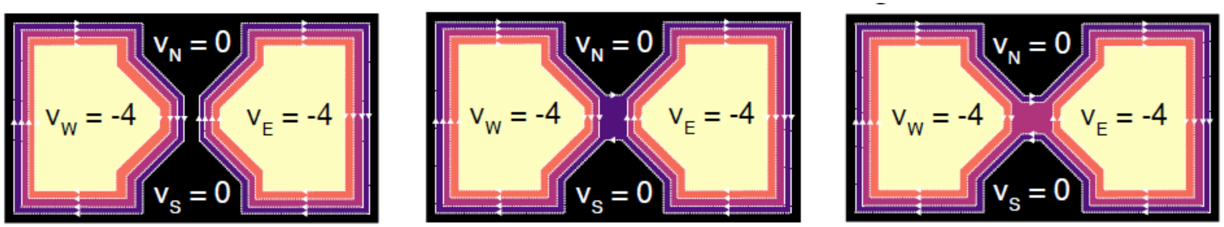


Idea: use split gates to make reflectors for edge states
Problem: graphene is not gapped
Equilibration physics appear with edge state below the gates.
Solution: use the gap at n=0

K. Zimmerman et al., Nat. Comm. 8, 14893 (2017)



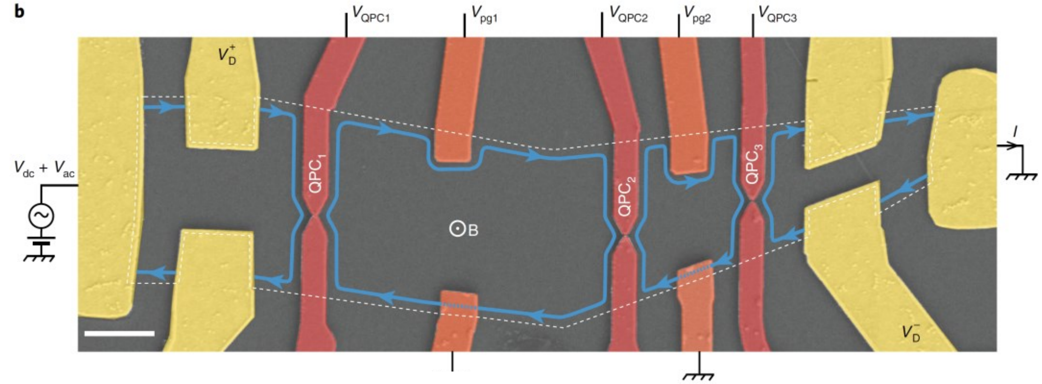


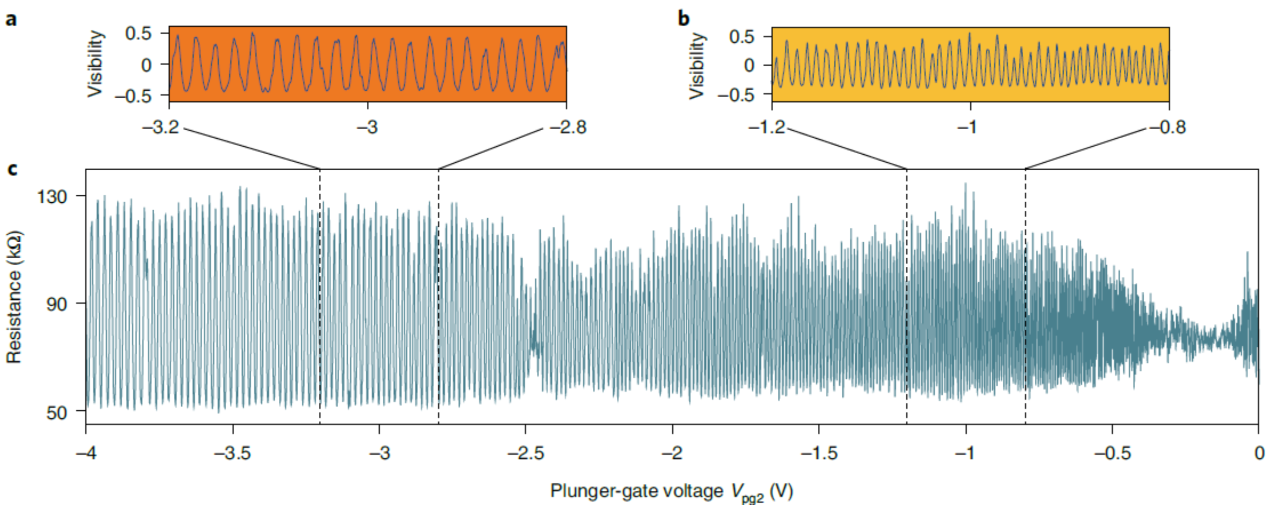


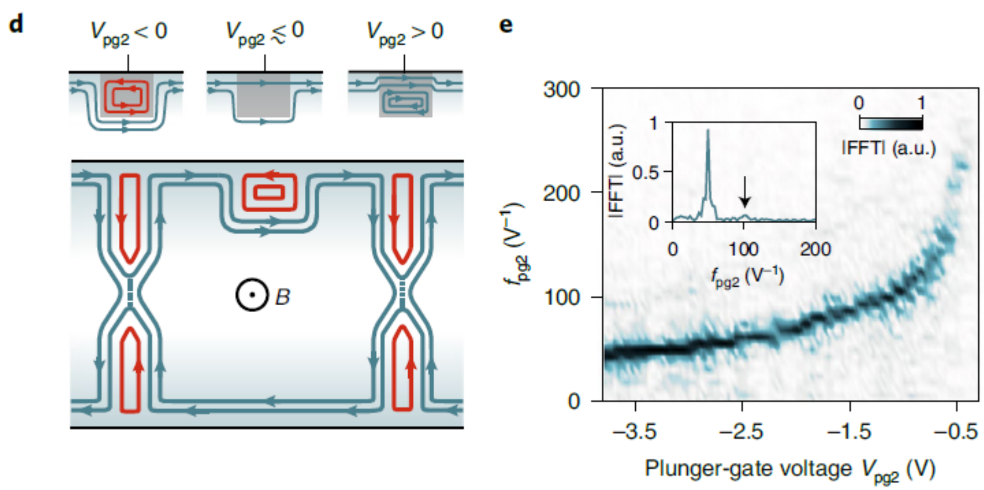
Can control number of channels one-by-one

L. A. Cohen et al., Nat. Phys. 19, 1502 (2023)

Fabry-Perot interferometers



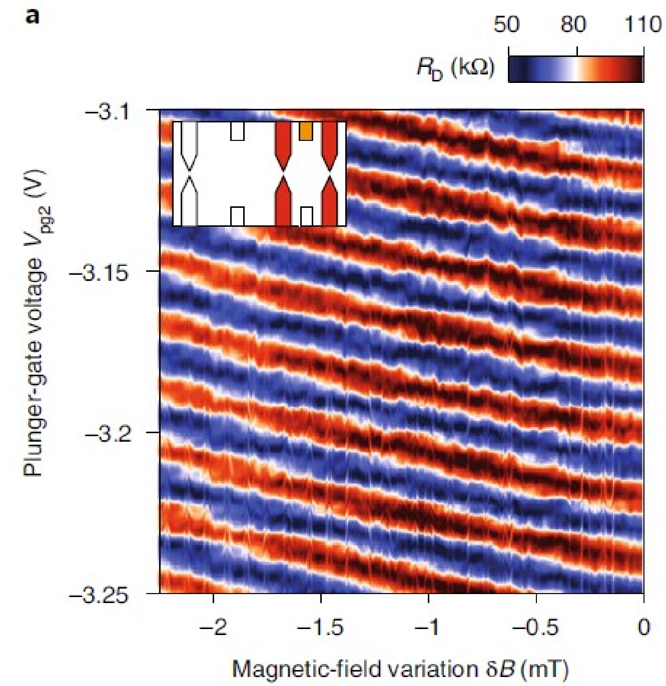




FP interferometer with QPCs
Different length of interferometers
Oscillation period is tuned by V_{PG2} – changes the position of the edge states, hence it changes the AB phase
No equilibration – there is a gapped region and good enough sample

C. Deprez et al., Nat. Nano 16, 555 (2021)

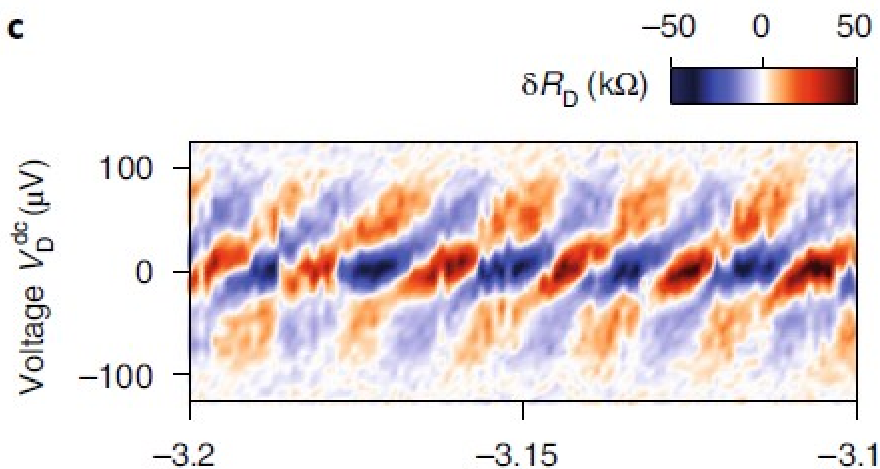
Fabry-Perot interferometers



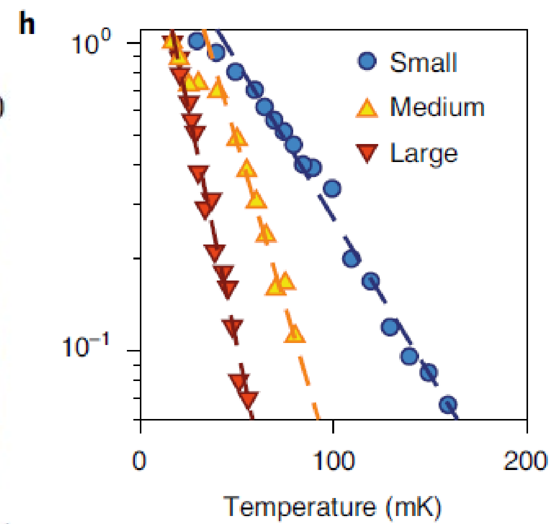
With changing magnetic field or gate voltage oscillations appear. Line follow constant flux. By applying a source-drain bias, the energy of the electrons are changed – lead to an oscillation pattern as well. From bias dependence the edge state velocity/Thouless energy can be obtained.

$$\varphi_{din} = \frac{2\pi e V_{DC} 2L}{hv} = 4\pi e\, V_{DC}/E_{TH}$$

$$\delta R_D \sim \cos\left(\frac{2\pi\Phi}{\Phi_0}\right)\cos(2\pi e V_{DC}/E_{TH})$$



V_{PG2}

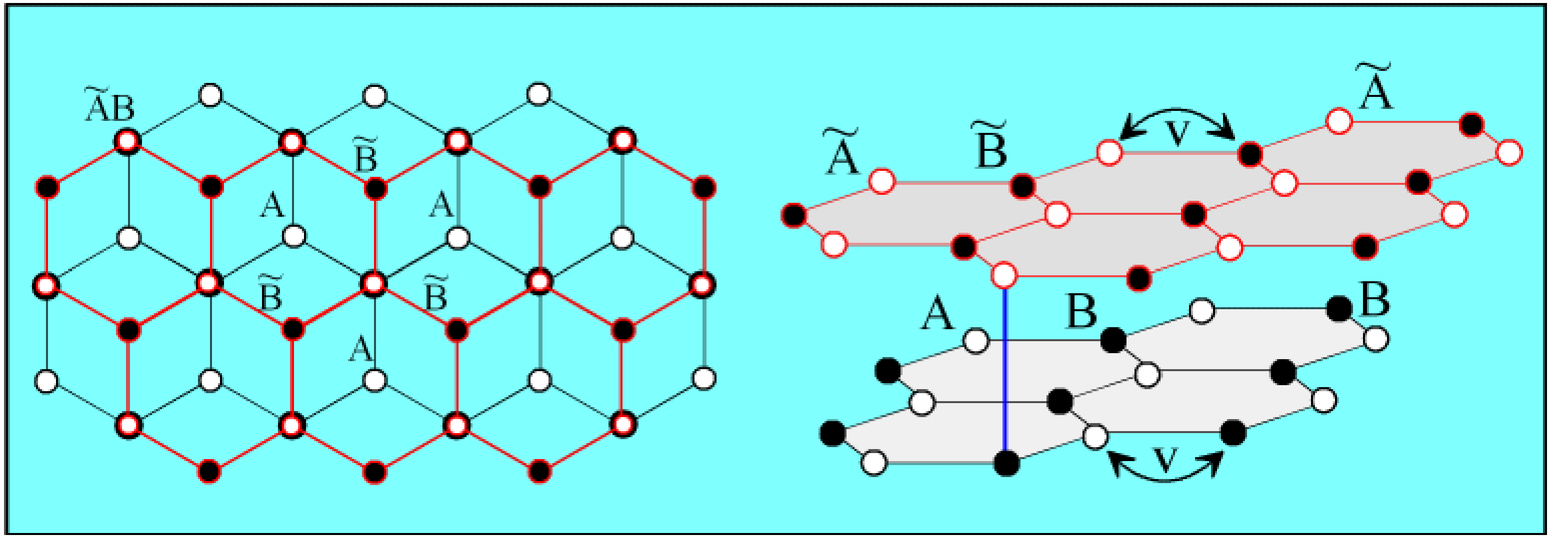


From temperature dependence the Thouless energy can be obtained. Longer trajectories diphase faster

$$\delta R_D \sim \exp(-4\pi^2 k_B T/E_{TH})$$

C. Deprez et al., Nat. Nano 16, 555 (2021)

Bilayer graphene

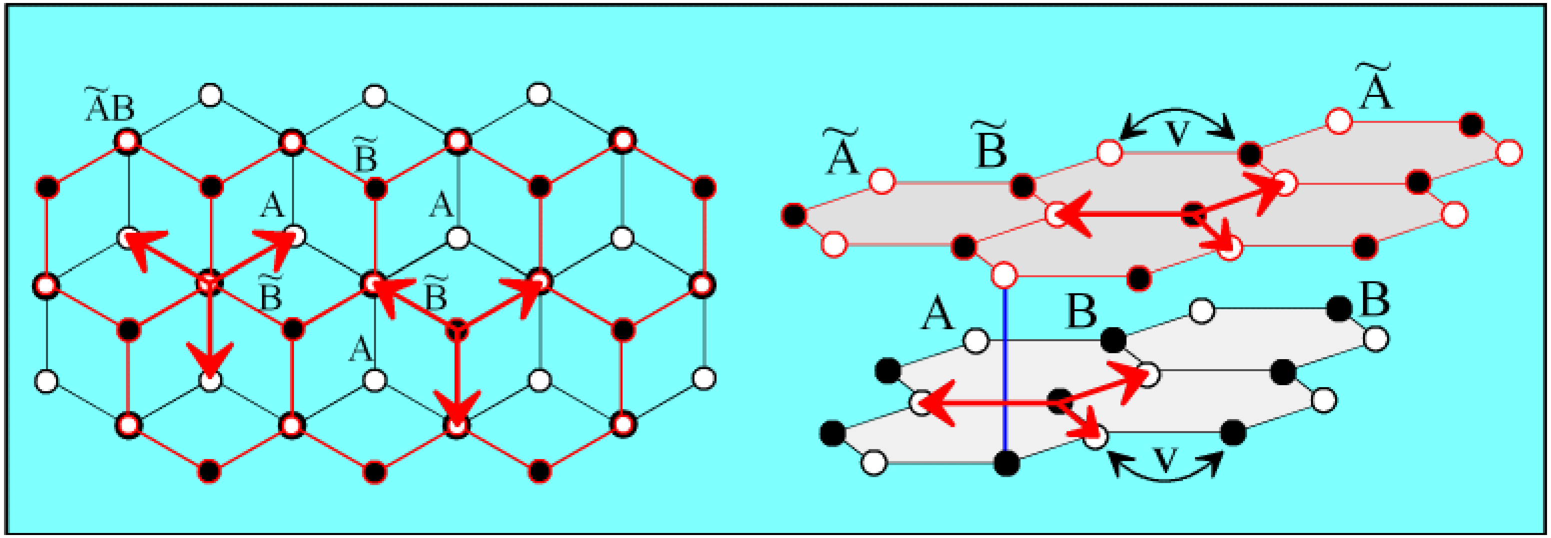


4 atoms
per unit cell

$$\mathcal{H}=\begin{array}{c} \begin{matrix} A & \tilde{B} & \tilde{A} & B \end{matrix} \\ \left(\begin{matrix} \quad & \quad & \quad & \quad \\ & & & \\ & & & \\ & & & \end{matrix} \right) \end{array} \begin{matrix} A \\ \tilde{B} \\ \tilde{A} \\ B \end{matrix}$$

Images: V. Falko, Lecture notes

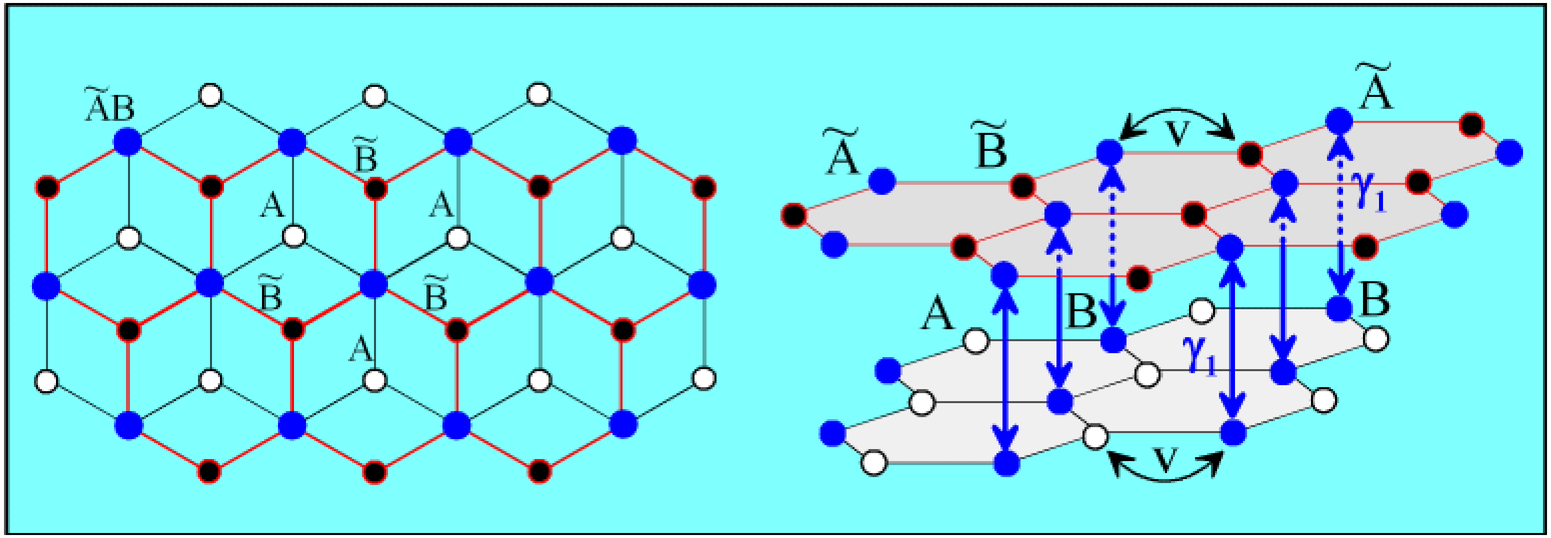
Bilayer graphene



(B to A) and ($\tilde{B}$ to $\tilde{A}$)
hopping
given by
$\pi^+ = p_x - ip_y$

$$H = \begin{array}{c} \begin{array}{cccc} A & \tilde{B} & \tilde{A} & B \end{array} \\ \begin{pmatrix} & & & v\pi^+ \\ & & v\pi & \\ & v\pi^+ & & \\ v\pi & & & \end{pmatrix} \end{array} \begin{array}{c} \\ A \\ \tilde{B} \\ \tilde{A} \\ B \end{array}$$

Bilayer graphene



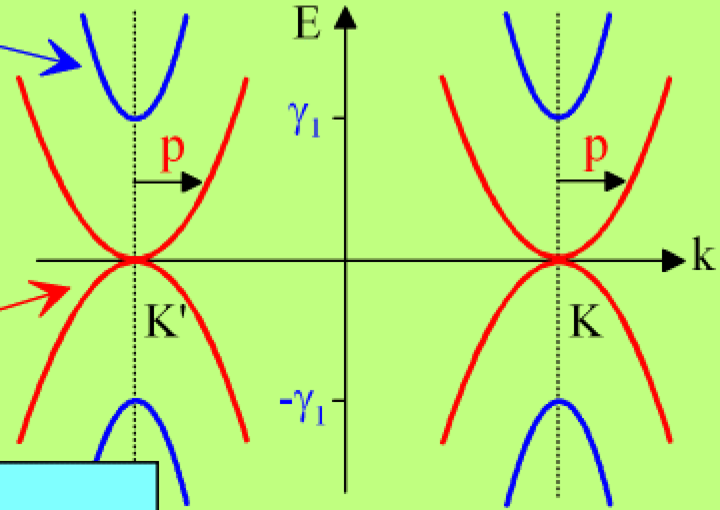
Bilayer Hamiltonian

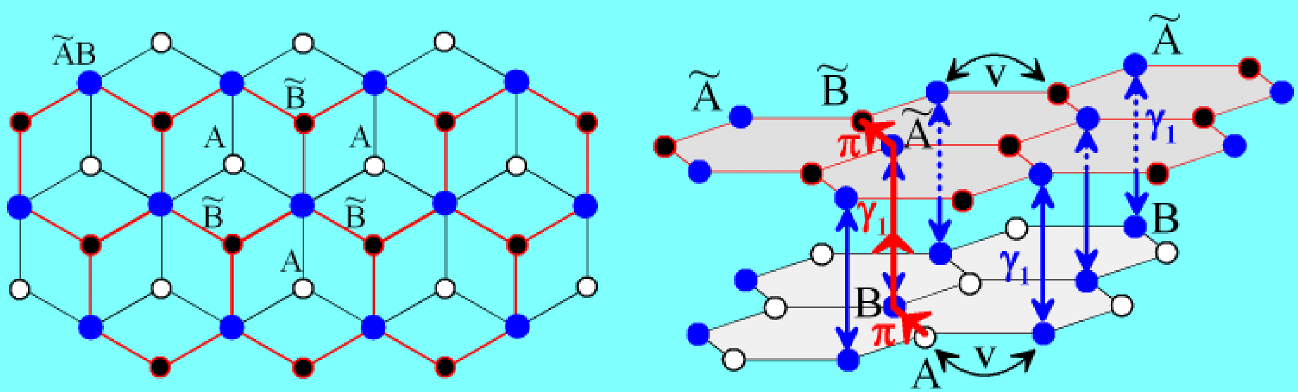
$$
H = \begin{array}{c}
\begin{array}{cccc} A & \tilde{B} & \tilde{A} & B \end{array} \\
\begin{pmatrix} 0 & 0 & 0 & v\pi^{+} \\ 0 & 0 & v\pi & 0 \\ 0 & v\pi^{+} & 0 & \gamma_1 \\ v\pi & 0 & \gamma_1 & 0 \end{pmatrix}
\end{array}
\begin{array}{c} A \\ \tilde{B} \\ \tilde{A} \\ B \end{array}
$$

$\tilde{A}B$ orbitals form dimers with energy $|E| \gtrsim \gamma_1$

Quadratic dispersion at low energy:

$$E = \pm \frac{p^2}{2m}$$





Bilayer Hamiltonian written in a 2 component basis of A and $\tilde{B}$ sites

$$H = \frac{-1}{2m}\begin{pmatrix} 0 & (\pi^+)^2 \\ \pi^2 & 0 \end{pmatrix}$$

mass

$m = \gamma_1 / v^2$

<u>A to $\tilde{B}$ hopping</u>

- bottom layer A $\rightarrow$ B (factor π)
- switch layers via dimer $B\tilde{A}$ (γ_1^{-1})
- top layer $\tilde{A} \rightarrow \tilde{B}$ (factor π)

$$\pi = p_x + ip_y$$

Backscattering?

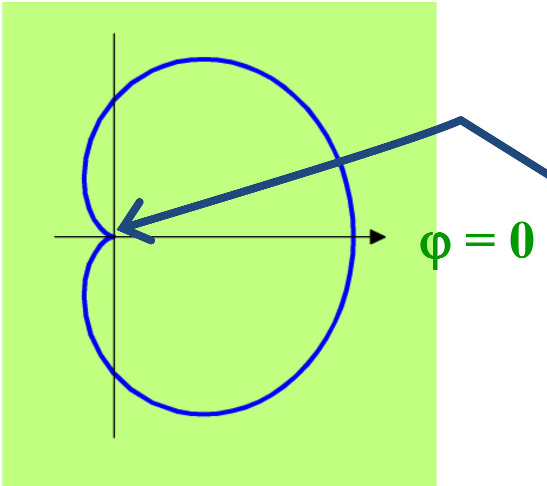
Single layer:

$$H = v\begin{pmatrix} 0 & \pi^+ \\ \pi & 0 \end{pmatrix} = vp\begin{pmatrix} 0 & e^{-i\varphi} \\ e^{i\varphi} & 0 \end{pmatrix}; \qquad E = vp \quad \Leftrightarrow \quad \psi(\varphi) = \tfrac{1}{\sqrt{2}}\begin{pmatrix} e^{-i\varphi/2} \\ e^{i\varphi/2} \end{pmatrix}$$

Bilayer:

$$H = -\frac{1}{2m}\begin{pmatrix} 0 & (\pi^+)^2 \\ \pi^2 & 0 \end{pmatrix} = -\frac{p^2}{2m}\begin{pmatrix} 0 & e^{-2i\varphi} \\ e^{2i\varphi} & 0 \end{pmatrix}; \qquad E = \frac{p^2}{2m} \quad \Leftrightarrow \quad \psi(\varphi) = \tfrac{1}{\sqrt{2}}\begin{pmatrix} e^{-i\varphi} \\ e^{i\varphi} \end{pmatrix}$$

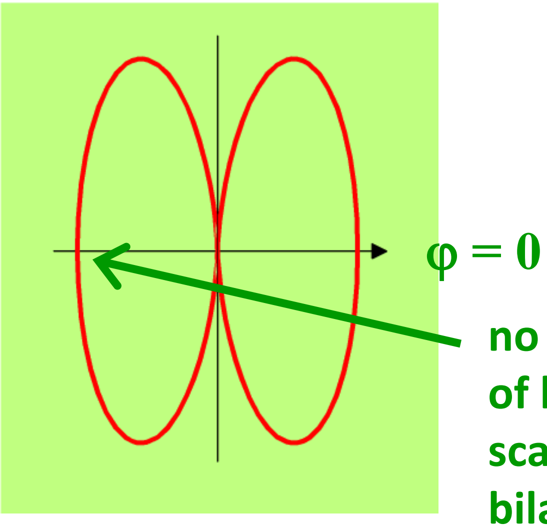
Single layer:



under pseudospin conservation, helicity suppresses backscattering in a monolayer

$$\left|\langle\psi(\varphi)|\psi(\varphi=0)\rangle\right|^2 = \cos^2(\varphi/2)$$

Bilayer:



no suppression of back-scattering in a bilayer

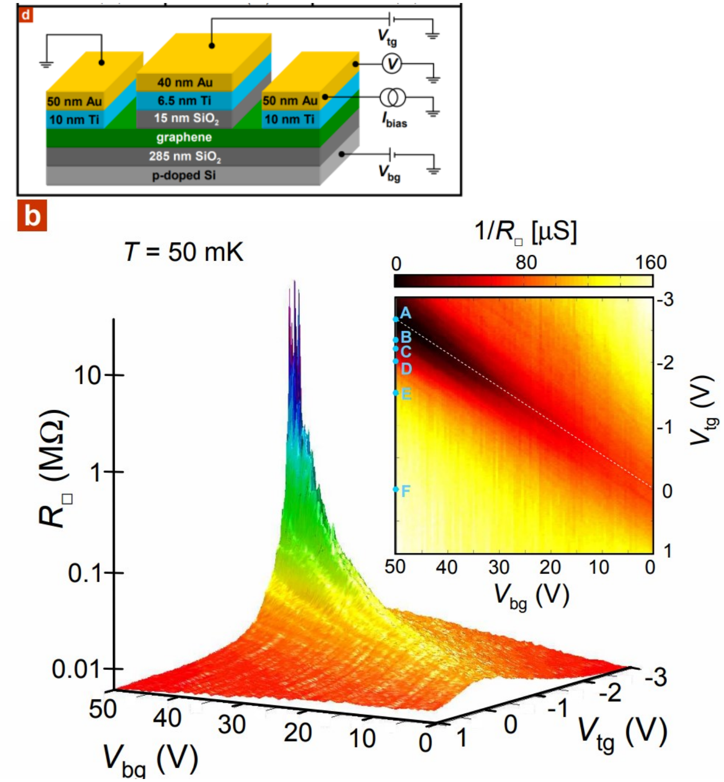
$$\left|\langle\psi(\varphi)|\psi(\varphi=0)\rangle\right|^2 = \cos^2(\varphi)$$

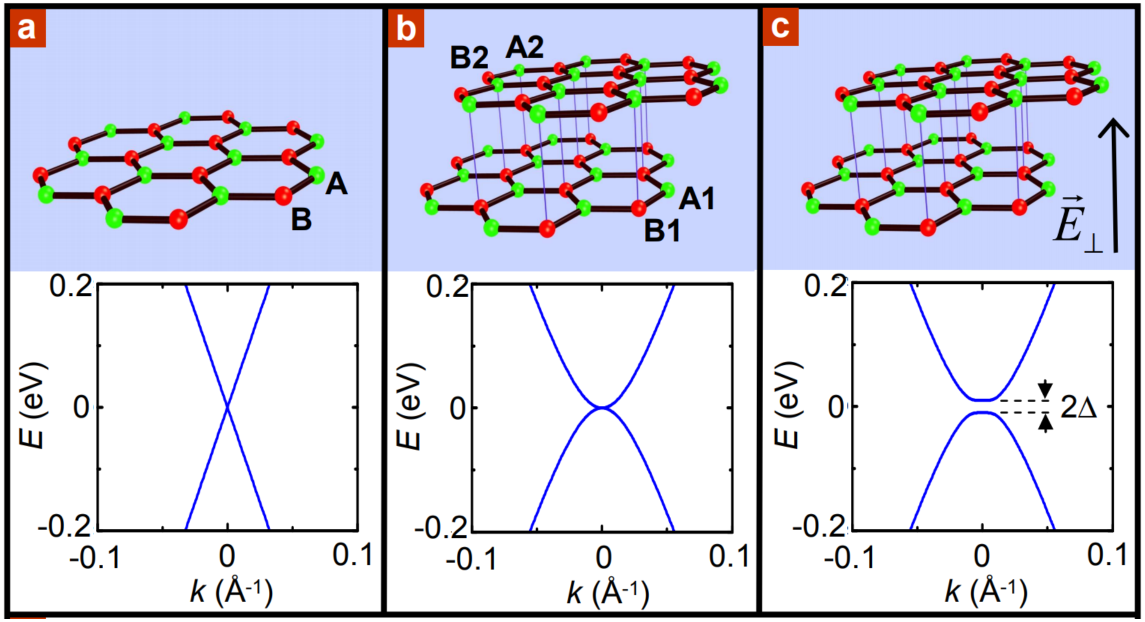
Bilayer graphene in electric fields

Effective 2x2 bilayer Hamiltonian acts on A1 and B2 subblattices, where 1 and 2 are on different layers.
-> Applying an electric field perpendicular to carbon plane generates a finite Δ term.
→ Band gap opens in K and K' with size of 2Δ

$$H = \begin{pmatrix} \Delta & -\frac{\hbar^2}{2m}(k_x - ik_y)^2 \\ -\frac{\hbar^2}{2m}(k_x + ik_y)^2 & -\Delta \end{pmatrix}$$

$$\psi = \begin{pmatrix} \phi_{A1} \\ \phi_{B2} \end{pmatrix}$$



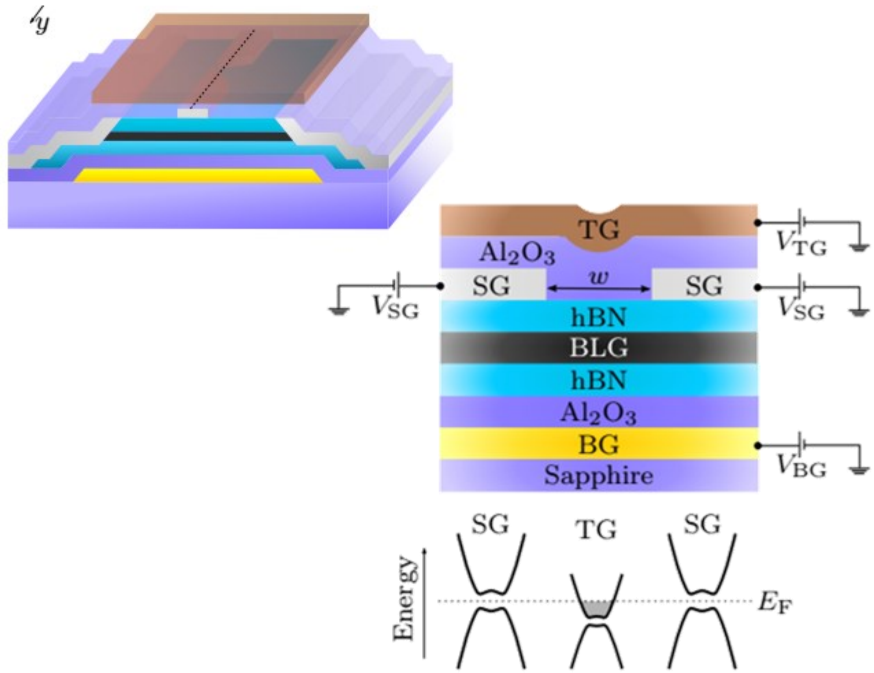


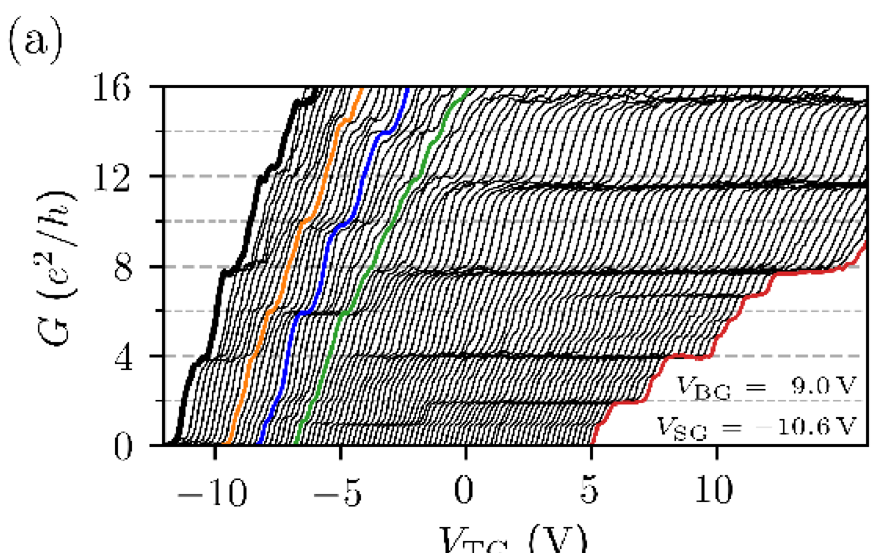
J. B. Oostinga, Nature Mat., 7, 151 (2008)

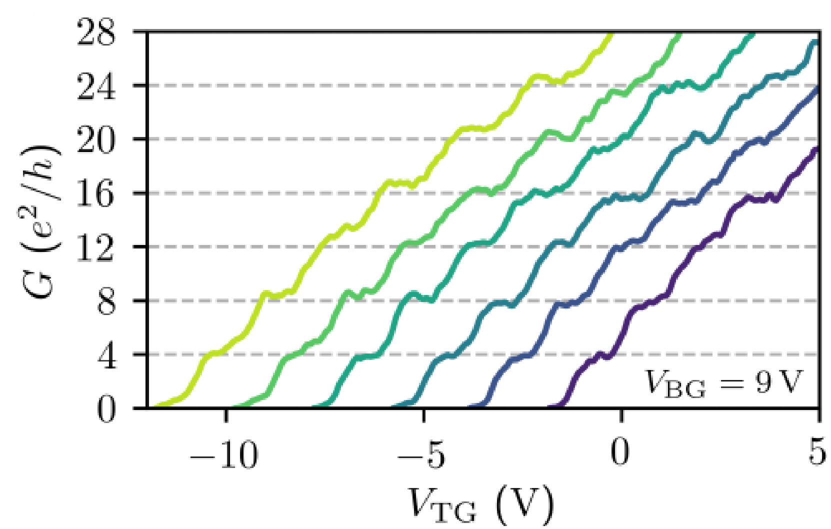
QPCs in bilayer graphene

- High quality BLG due to hBN stack
- Electric field is defined by BG, TG and SG.
- SG and BG generate perpendicular E field and open gap and define confinement
- TG is used to set Fermi level

- Conductance quantization 4e2/h
2spin x 2valley degree of freedom
- In B field 4 fold degeneracy changes.







R. Kraft Phys. Rev. Lett. 121, 257703 (2018)