Term Assignments: Rules, Examples

Topological insulators 2 (Topological superconductors)

BME/ELTE 2020 Spring Semester

2020/04/29

To get a grade for this course, you could (i) take an **oral exam** (most probably online), or you could complete a **term assignment**: either (ii) a **project**, or (iii) a **literature review**.

For the **project** and the **literature review**,

1) The outcome is a written documentation, a **term paper**, preferably done in tex. The volume should be between 4-6 pages, two-column Phys. Rev. format. In case of a numerical project, the (thoroughly commented) numerical code should also be submitted.

2) You should spend ~ 40 hours (~5 working days) on the assignment, in accordance with the course data sheet http://ttk.bme.hu/BMETE11MF35.

3) Ideally, your assignment allows you to gain experience with the concepts and tools learned in this semester, or to learn extra things.

4) The specific tasks of the assignment are decided through a negotiation process between you and your **mentor** (either Janos, Laci, or Andras; we will direct students to mentors). You are welcome to suggest a project topic. Your term paper should start with the assignment description (at most one page). This assignment description might evolve during the work, but only with the mentor's agreement; the term paper should contain the final version. Some example **project** descriptions are found below - you could pick one of those if you like (with the caveat that two students are not allowed to do the same term assignment).

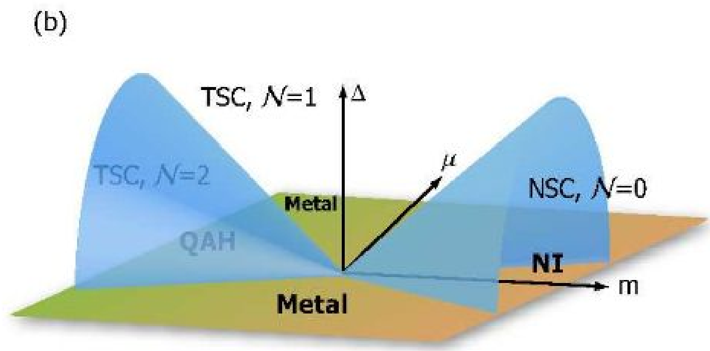
5) It is encouraged to define an assignment that is related to and/or beneficial for your own research.

Deadlines: If you don't want to take an oral exam, but you want to do a term assignment, then you have to contact Andras (in email, palyi at mail.bme.hu), no later than the beginning of the exam period (**May 25 Monday**), and you have to hand in the term paper (upload the pdf to Teams > General channel > Files AND email it to your mentor) no later than one week before the end of the exam period (**June 22 Monday**).

Please send your questions, comments related to this to Andras. Below you'll find a few example project descriptions.

1. Topological superconductors by proximitizing Chern insulators [J]

Qi, Hughes and Zhang had the idea that you can obtain topological superconductors by proximitizing a Chern insulator (Chern insulators are also called Quantum Anomalous Hall insulators, QAH).
https://arxiv.org/pdf/1003.5448.pdf
The linearized description is:



$$H_{QAH} = \sum_{\mathbf{p}} \psi_{\mathbf{p}}^\dagger h_{QAH}(\mathbf{p}) \psi_{\mathbf{p}}$$

$$h_{QAH}(\mathbf{p}) = \begin{pmatrix} m(p) & A(p_x - ip_y) \\ A(p_x + ip_y) & -m(p) \end{pmatrix}$$

$$H_{BdG} = \frac{1}{2} \sum_{\mathbf{p}} \Psi_{\mathbf{p}}^\dagger \begin{pmatrix} h_{QAH}(\mathbf{p}) - \mu & i\Delta\sigma^y \\ -i\Delta^*\sigma^y & -h_{QAH}^*(-\mathbf{p}) + \mu \end{pmatrix} \Psi_{\mathbf{p}}$$

You can expect that if the Chern number was ±1, you will obtain a topological superconductor, with 2 Majorana modes at the edges. It is perhaps more surprising that a superconductor with a single Majorana mode at the edges is also possible to realize in this way (see plot above).

Try this numerically, without linearizing, calculating the spectrum of the proximitized the Qi-Wu-Zhang model (QWZ) on a ribbon.
1) Plot the dispersion relation of the BdG Hamiltonian, in the style of Fig. 6.5 from our lecture notes (1st semester), indicating the Majorana edge modes, in a regime where there are 2 of these on each edge.
2) Plot the dispersion relation, with parameters fixed so that there is only 1 Majorana mode on each edge.
3) In the regime with a single Majorana mode on each edge, obtain numerical evidence that it is indeed a Majorana mode.

2. Majorana zero mode at a dislocation [J]

A stack of coupled Kitaev wires can be a weak topological superconductor, much like how - as we saw in the 1st semester - a stack of coupled SSH chains can be a weak topological insulator. Build a numerical model for this, from N Kitaev chains, each of which are on N sites, i.e., an NxN lattice of fermionic sites. What kind of couplings can you introduce between the wires that preserve not only the particle-hole symmetry, but also the chiral symmetry? How can you break this chiral symmetry? As you turn on these two types of coupling between the chains, what happens to the Majorana zero modes? Provide numerical evidence for your answers.

A dislocation in such a system can host a Majorana zero mode. Provide a numerical model for this, find the Majorana zero mode.

3. ZBCP without Majorana fermions: the "Pikulin" peak [L]

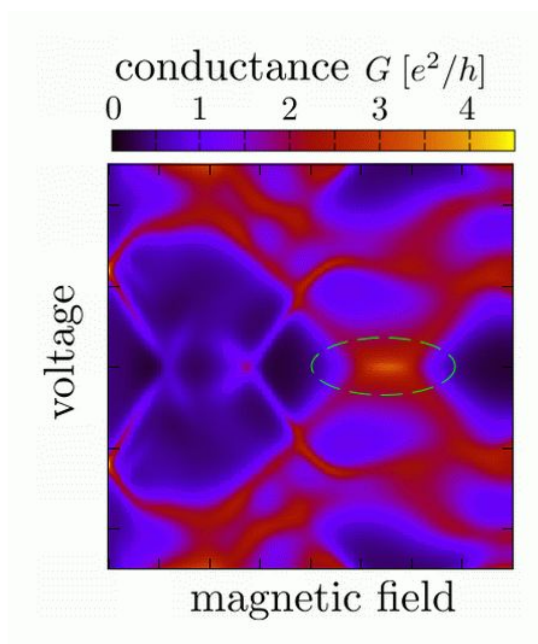
We have seen that the possible signature of Majorana fermions is a peak in the differential conductance dI/dV at zero bias voltage V. At zero temperature the height of this peak must be e^2/h for topological superconductors. We have also seen that in a simple model trivial superconductors have a zero-bias conductance that depends on the particular details of the system. One might ask whether an (un)fortunate set of circumstances could lead to the formation of robust ZBCPs due to mechanisms not of topological origin.

Weak localization (or antilocalization) is the systematic constructive (or destructive) interference of phase conjugate series of scattering events. In disordered metals, it is time-reversal symmetry that provides for phase conjugation of backscattered electrons and protects their interference from averaging out to zero. A magnetic field breaks time-reversal symmetry, changing the disorder-averaged conductance by an amount δG of order e^2/h. The sign of δG distinguishes weak localization ($\delta G < 0$, conductance dip) from weak antilocalization ($\delta G > 0$, conductance peak). Andreev reflection at a superconductor provides an alternative mechanism for phase conjugation due to particle-hole symmetry. No time-reversal symmetry is needed, so weak (anti)localization can coexist with a magnetic field and is only destroyed by a bias voltage. In this project, you will explore how the peak generated by this mechanism can be distinguished from that given by Majorana zero modes.

Your tasks:

Reproduce the findings of this:https://arxiv.org/abs/1206.6687 paper.

1. Familiarize yourself with the kwant package.
2. Implement the Lutchyn wire in a kwant simulation and show that ZBCP appears in the topological regime and that it is absent in the trivial.
3. Implement a wide disordered SN junction and reproduce microscopic predictions from the paper. For example, reproduce a figure similar to Fig 4. of the paper. (Note that this involves a random sample, so the position of key features might be different.)



5. Interference effects in Josephson junctions [A]

Read our lecture notes on the Josephson effect:

http://eik.bme.hu/~palyi/TopologicalInsulators2-2018Spring/TopSup-Lecture08-JosephsonEffect-v2.pdf

1. Reproduce the numerical results shown on the plots. As seen in the notes, those results correspond to simple tunnel junctions as shown in Fig. (a) below.
2. Extend the s-wave calculation to an Aharonov-Bohm Josephson interferometer (see Fig. (b) below), where instead of a tunnel junction, you have two single-site paths connecting the leads. Assume zero on-site energies for those newly added sites (circles), and set all 4 hoppings to the same absolute value t' > 0. A magnetic flux Phi piercing the loop of the four sites can be described by making one of the hoppings complex with a Phi-dependent phase, as shown in Fig. (b). For Delta = 1 and all tunnelings set to t' = 0.2, plot the ground-state energy of the junction as the function of phase bias ϕ and Aharonov-Bohm flux Phi. Plot the Josephson current as a function of ϕ and Phi.
3. Try to interpret the results you get, e.g., in terms of perturbation theory.
4. Repeat the exercise for an interferometer where the superconducting leads are topological, and modelled by fully dimerized two-site Kitaev chains.