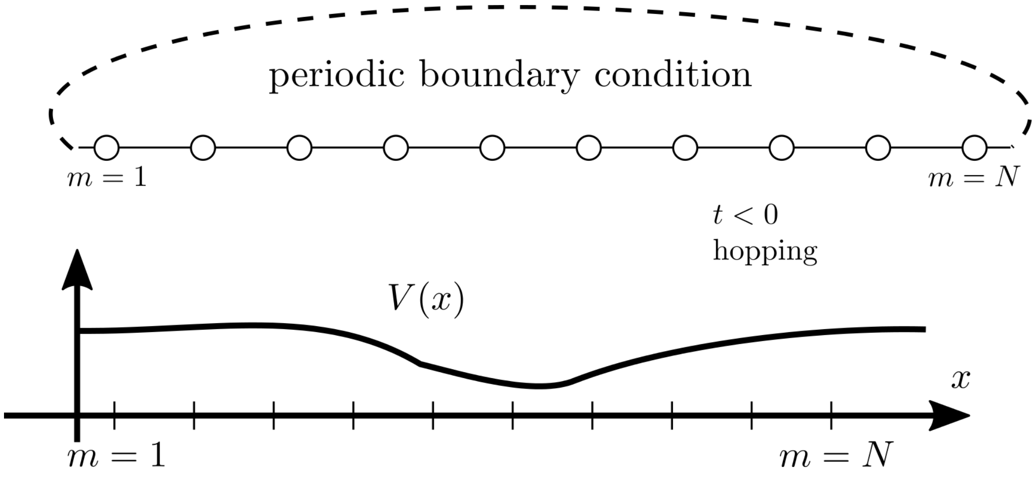
Continuum Model of Localized States at a Domain Wall

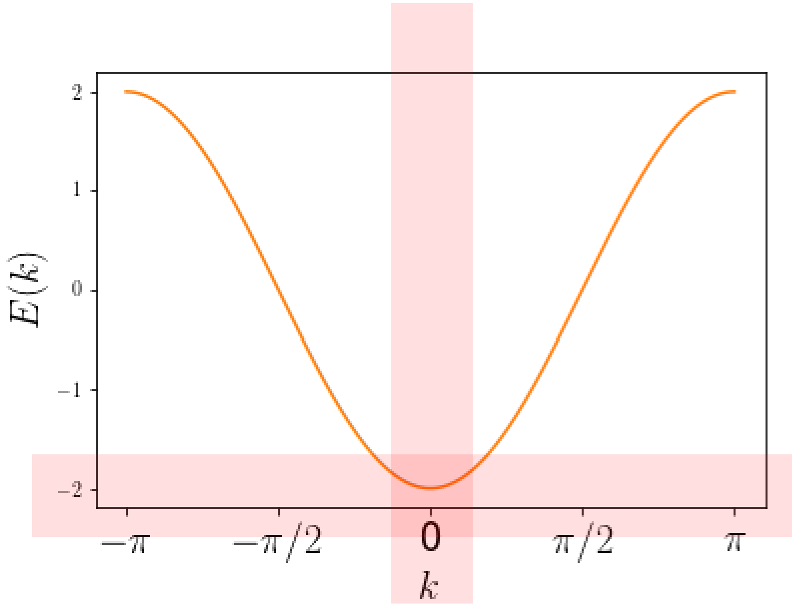
- Envelope-Function Approximation
- EFA for 1D chain
- EFA for metallic SSH and the masleess 1D Dirac equation
- EFA for gaped SSH and bound states at interfaces
- EFA for QWZ

Envelope-Function Approximation: Recipie

1. Rely on "spatially slowly varying" wave functions
2. Find relevant energy/momentum range
3. Expand the Hamiltonian around relevant momenta
4. Replace relevant momentum by derivatives

EFA for 1D chain





$$H_i = H_{per} + V \rightarrow \frac{p^2}{2m} + V(x)$$

EFA for metallic SSH

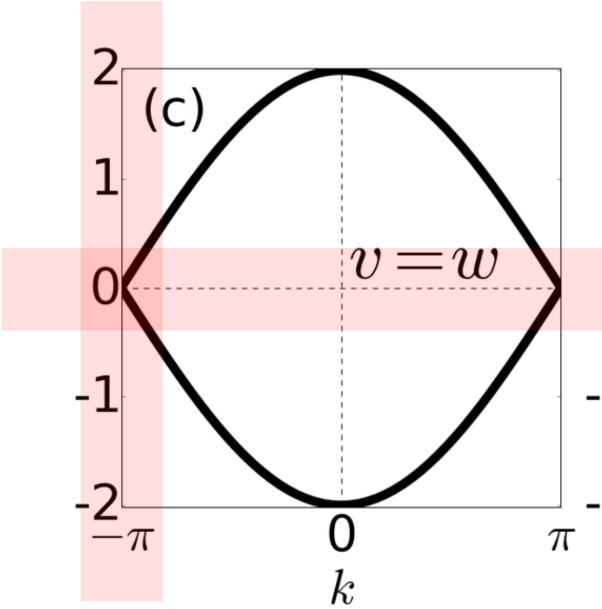
$$H(k) = [v + w\cos(k)]\,\sigma_x + w\sin(k)\sigma_y$$

-1

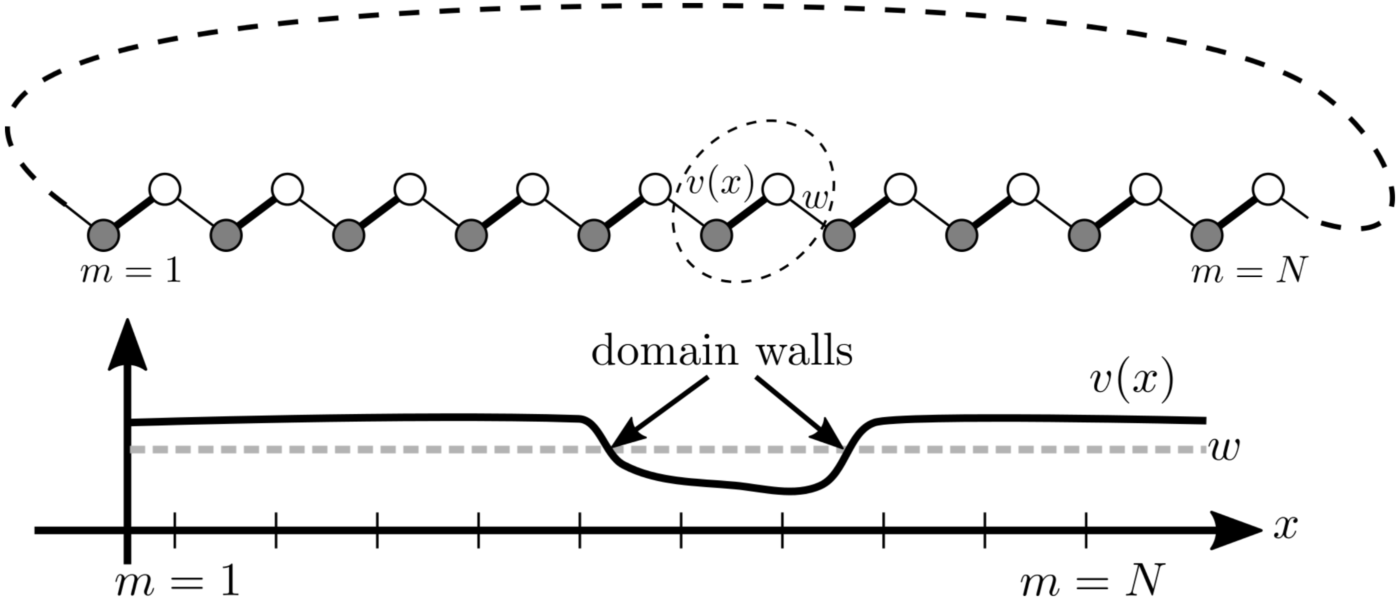
-q

$$H(k_0 + q) \approx -wq\sigma_y$$

"masless Dirac fermion"



EFA for gapped SSH



$$H(k) = [v + w\cos(k)]\,\sigma_x + w\sin(k)\sigma_y$$

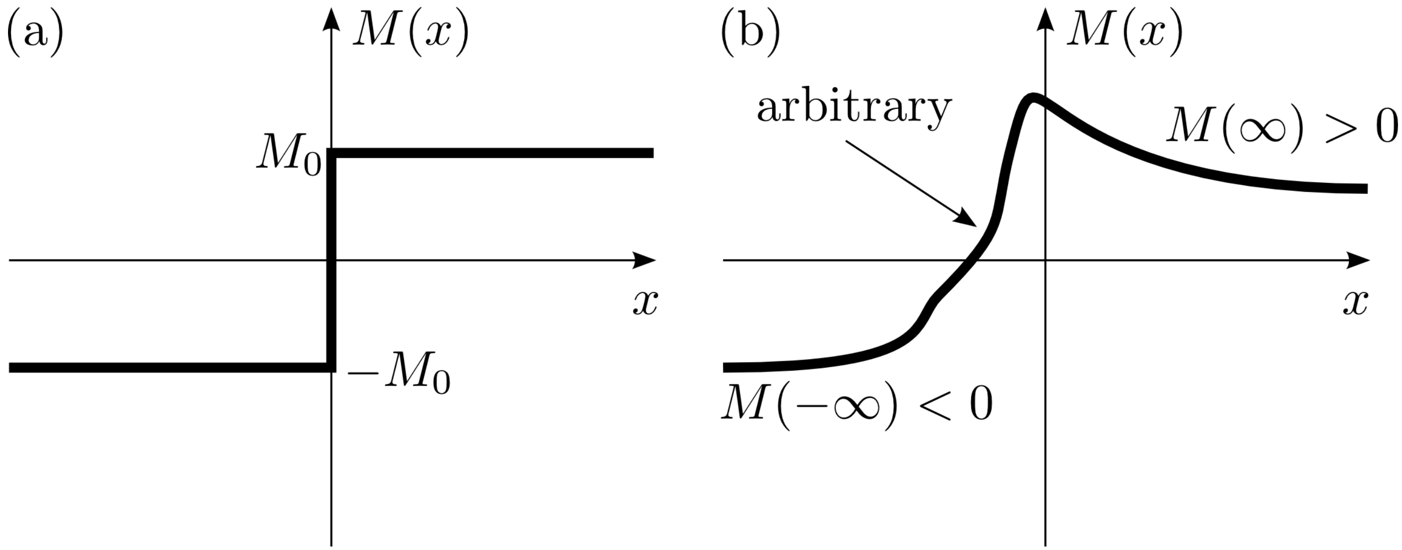
-1

-q

$$H(k_0 + q) \approx \overbrace{(v - w)}^{M}\sigma_x - wq\sigma_y$$

"massive Dirac Hamiltonian"

EFA for gapped SSH

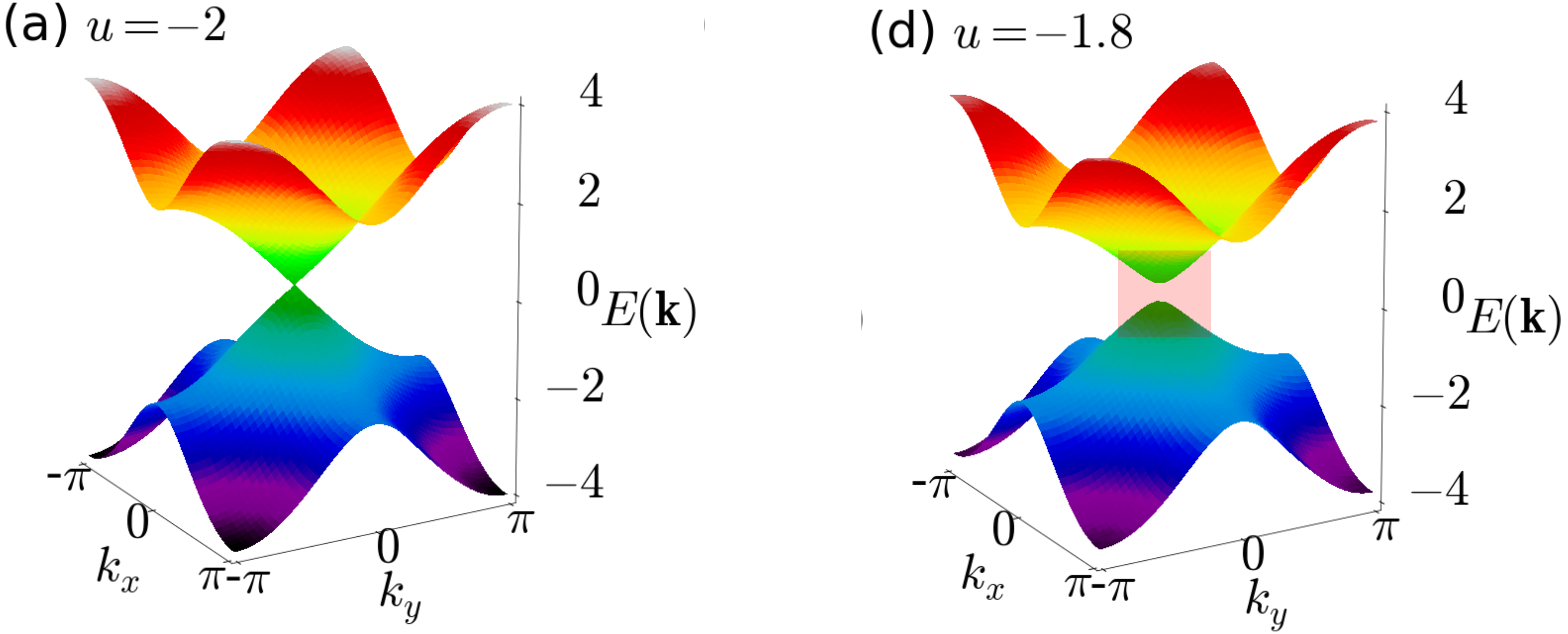


This language gives a single, sublattice polarized zero energy state localized on the interface also!

$$\varphi(x) = \begin{pmatrix} 0 \\ 1 \end{pmatrix} f(x)$$

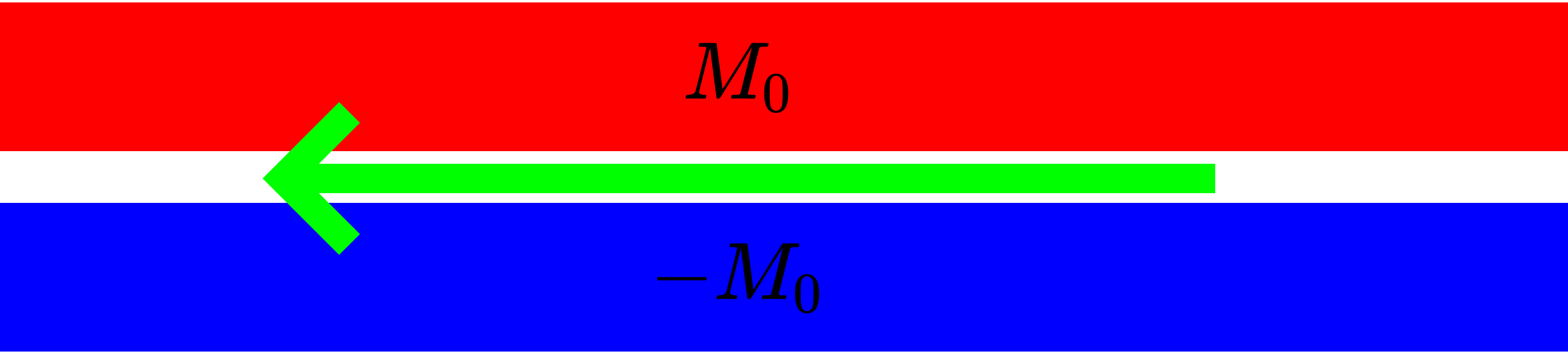
$$f(x) \propto e^{-\frac{1}{w}\int_0^x dx' M(x')}$$

EFA for QWZ



$$\hat{H}(\mathbf{k}_0 + \mathbf{q}) \approx \underbrace{(u+2)}_{M}\sigma_z + q_x\sigma_x + q_y\sigma_y$$

EFA for QWZ



$$\varphi(x,y) = \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{iq_x x} f(y)$$

$$f(y) = e^{-\int_0^y dy' M(y')}$$