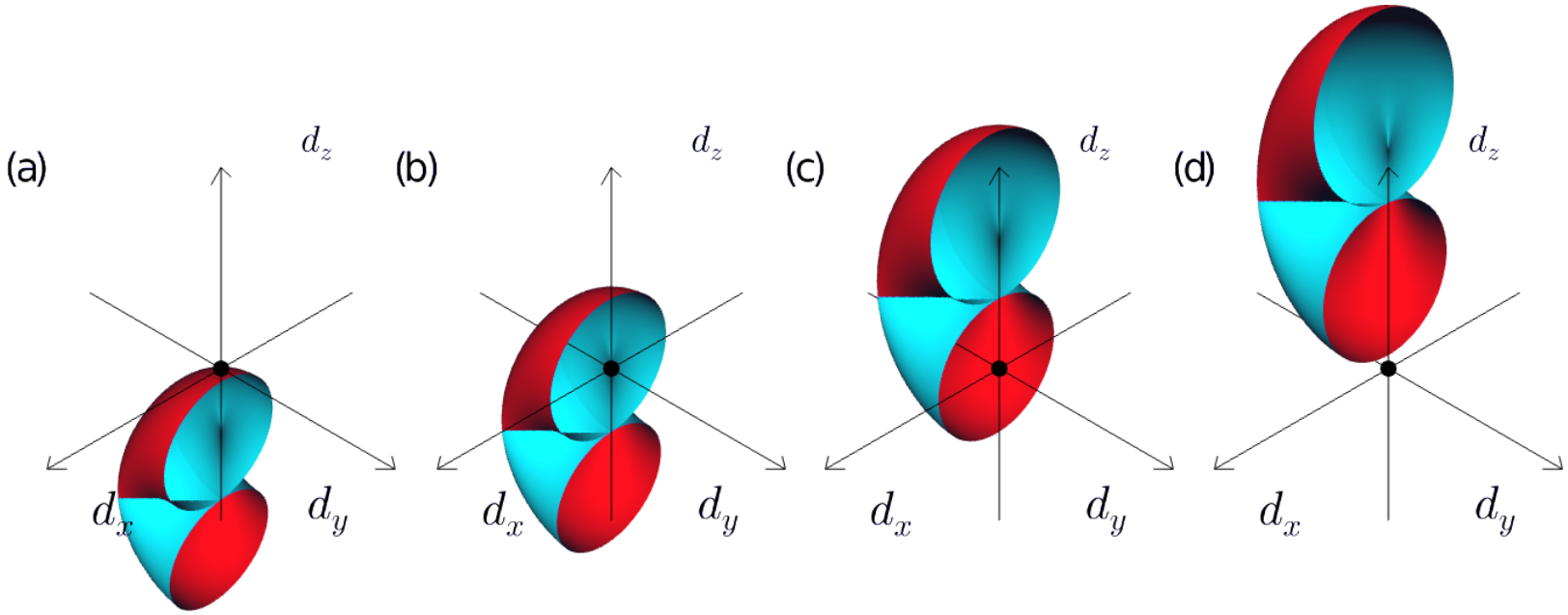
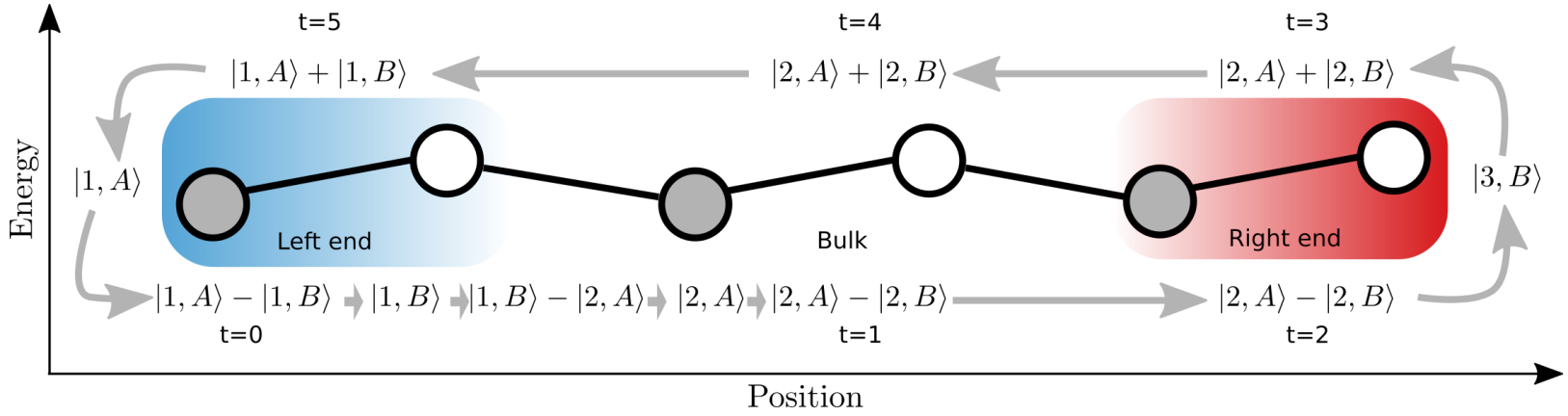
THE QWZ model

- Required:Thouless pumping
- New theory tool: Promoting time t → quasimomentum k
- Main results: Edge states in two-dimensional systems
  Bulk Chern number predicts edge states
  Topological protection
- Toy model: Qi-Wu-Zhang
  obtained from Thouless pump in Rice-Mele by promoting t → k



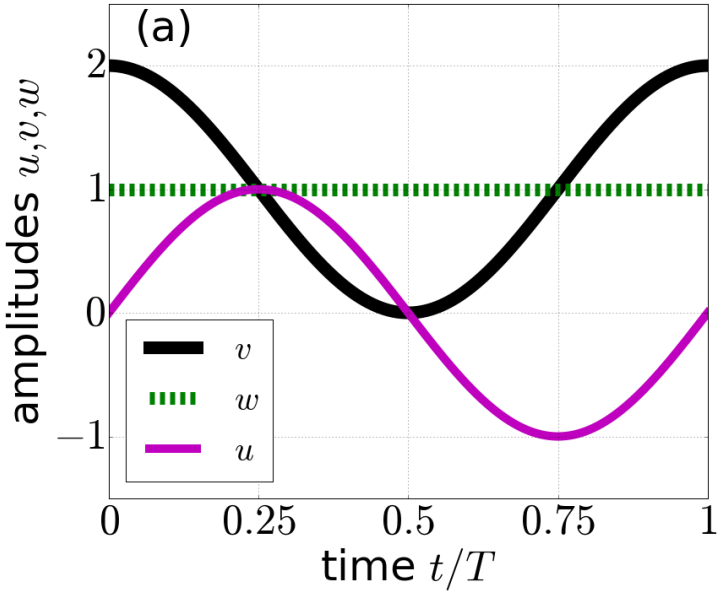
Reminder 1: Thouless pump sequence, Rice-Mele

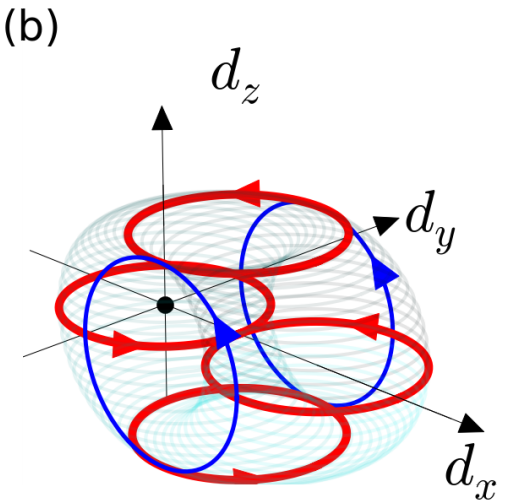


Pump charge along a dimerized chain using sublattice potential:

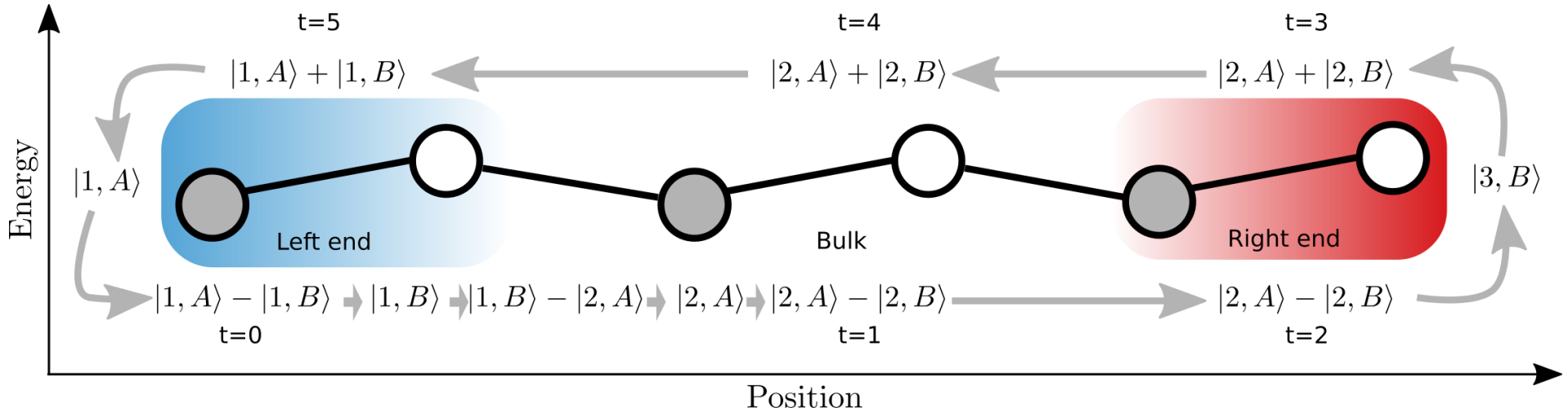
$$\hat{H}(k,t) = \mathbf{d}(k,t)\cdot\hat{\sigma} = (v(t) + w\cos(t))\,\hat{\sigma}_x + w(t)\sin(k)\hat{\sigma}_y + u(t)\hat{\sigma}_z$$

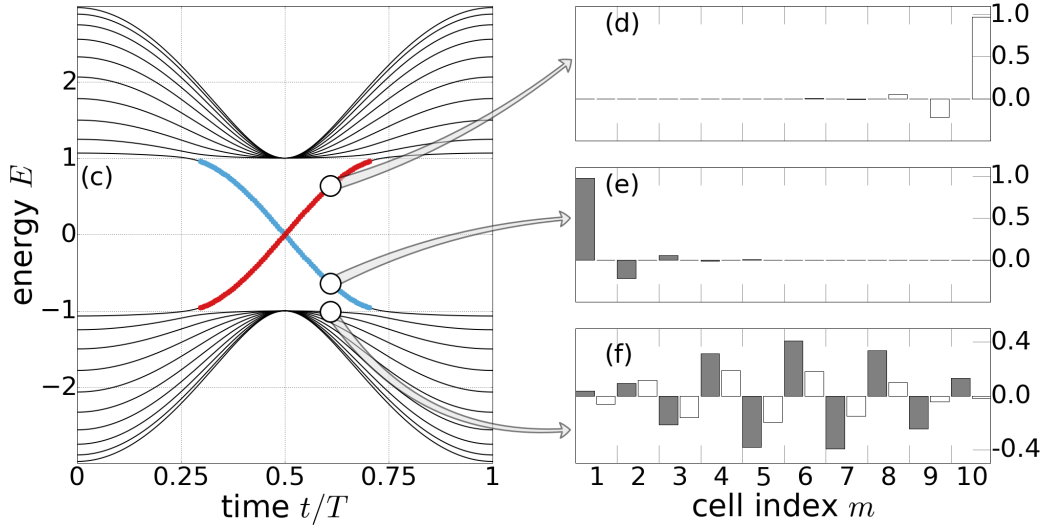
$$u(t) = \sin(\Omega t)$$
$$v(t) = \bar{v} + \cos(\Omega t)$$
$$w(t) = 1$$





Reminder 2: Protected Edge States in Thouless pump



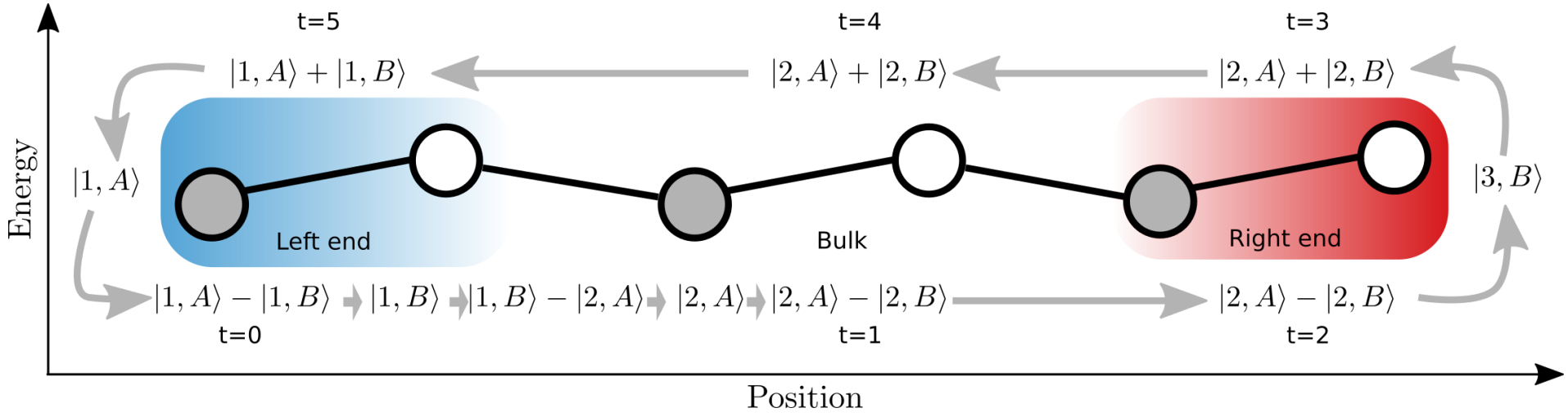


Topologically protected = robust:

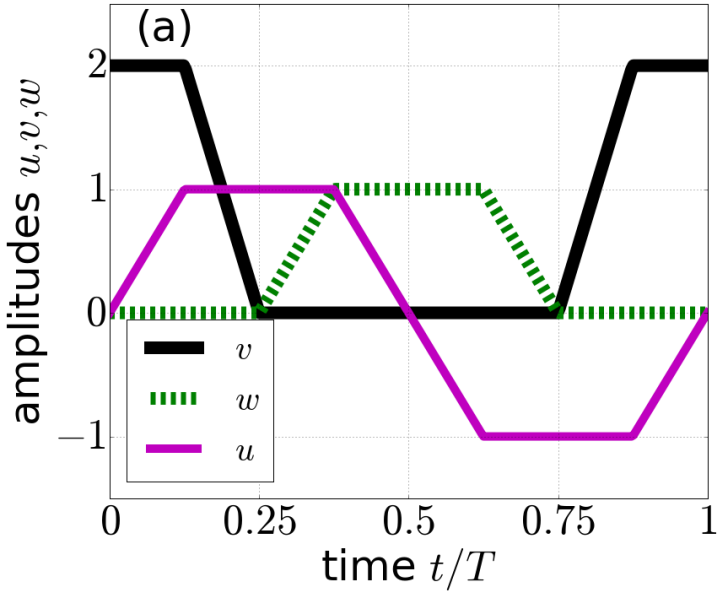
- Time - Periodic drive
- No long range hopping

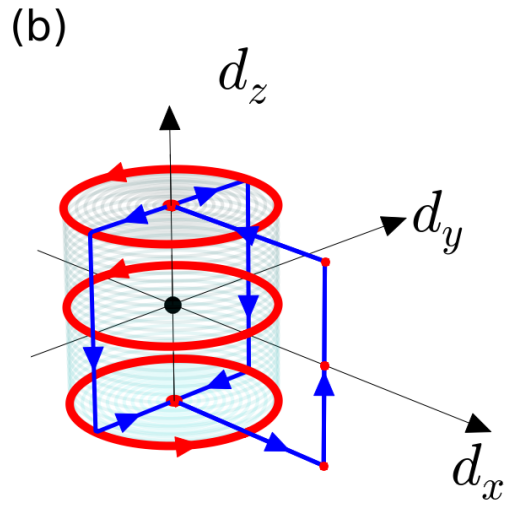
1. spectrum time-periodic
2. spectrum continuous
3. bulk gap separates two edges
4. → no direct coupling,
5. → crossing, not anticrossing

Reminder 3: Thouless pump in the bulk in d-space:
times origin in torus = # charge pumped = Chern

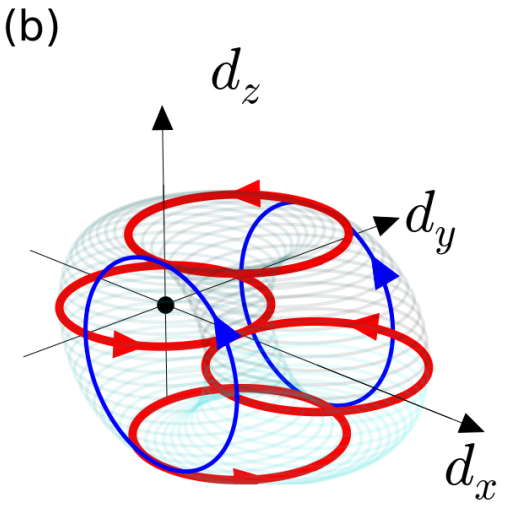


control freak sequence:

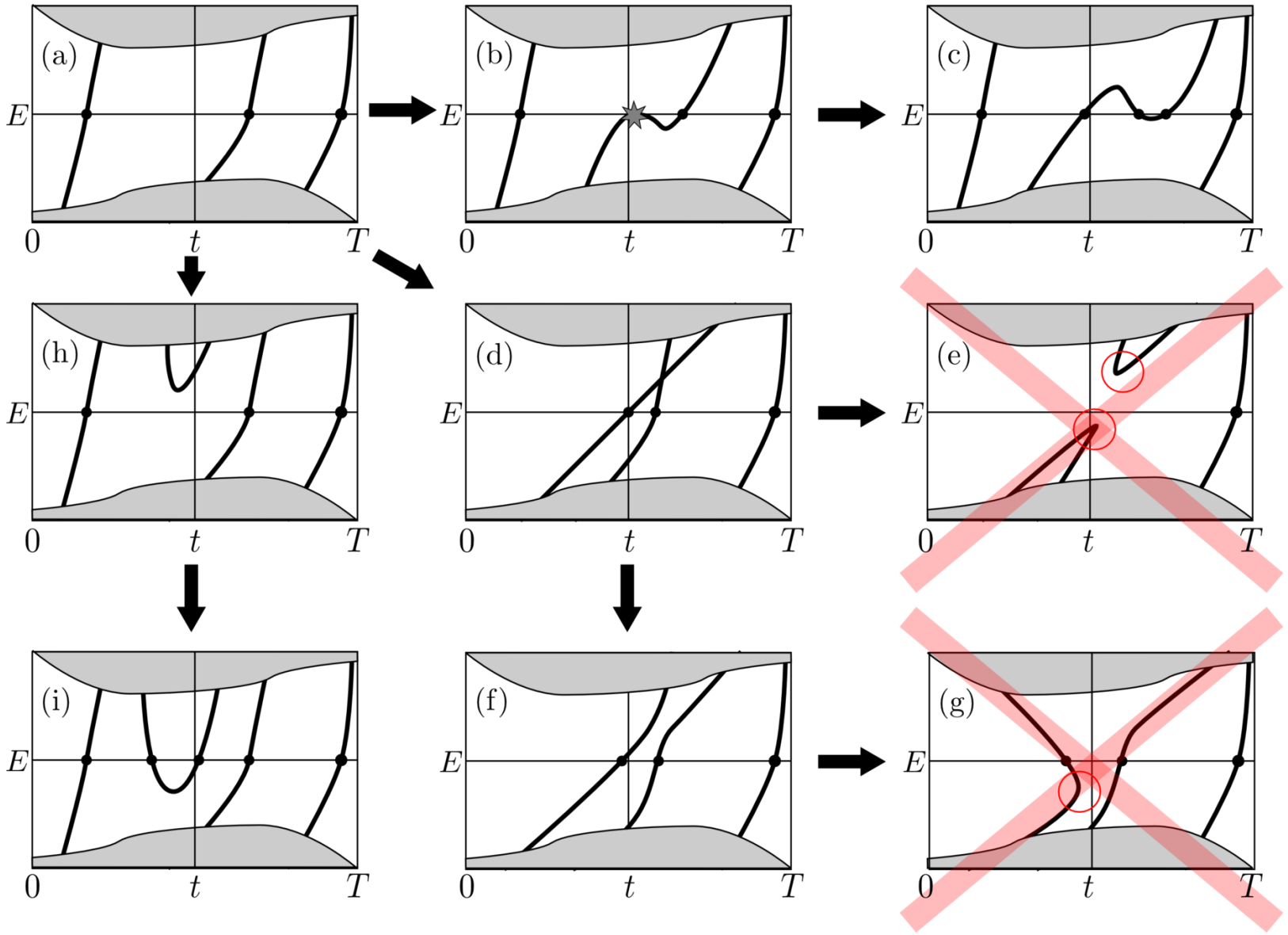




smooth sequence:



Reminder 4: Net number of charge pumped up in energy at an edge is protected against continuous deformations



New material:
From Thouless pump to Chern insulator

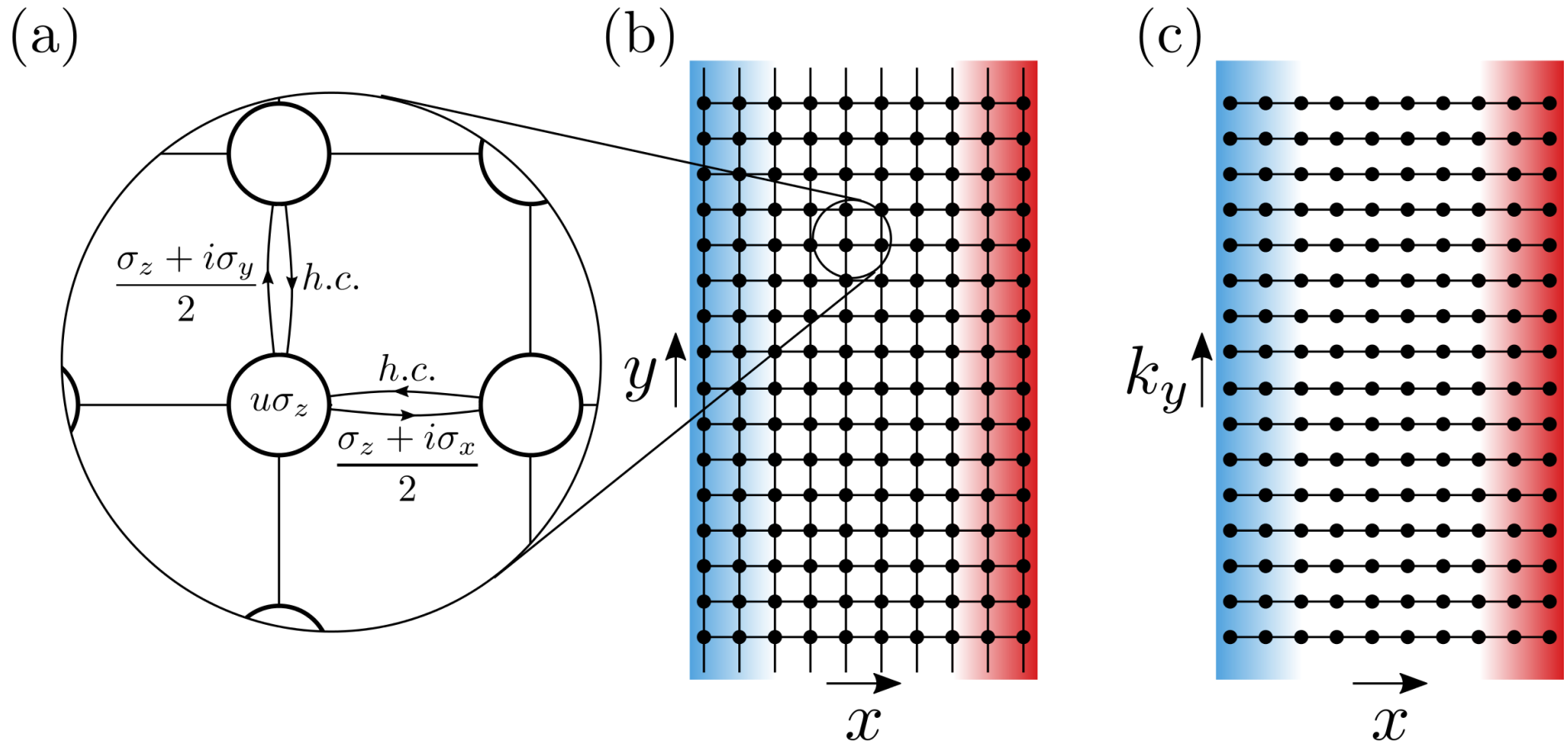
Promote time t → wavenumber k
1D time-periodic Rice-Mele → 2D Qi-Wu-Zhang

$$\hat{H}_{\mathrm{RM}}(k,t) = \sin(k)\hat{\sigma}_y + \sin(\Omega t)\hat{\sigma}_z + \left(\bar{v} + \cos(k) + \cos(\Omega t)\right)\hat{\sigma}_z$$

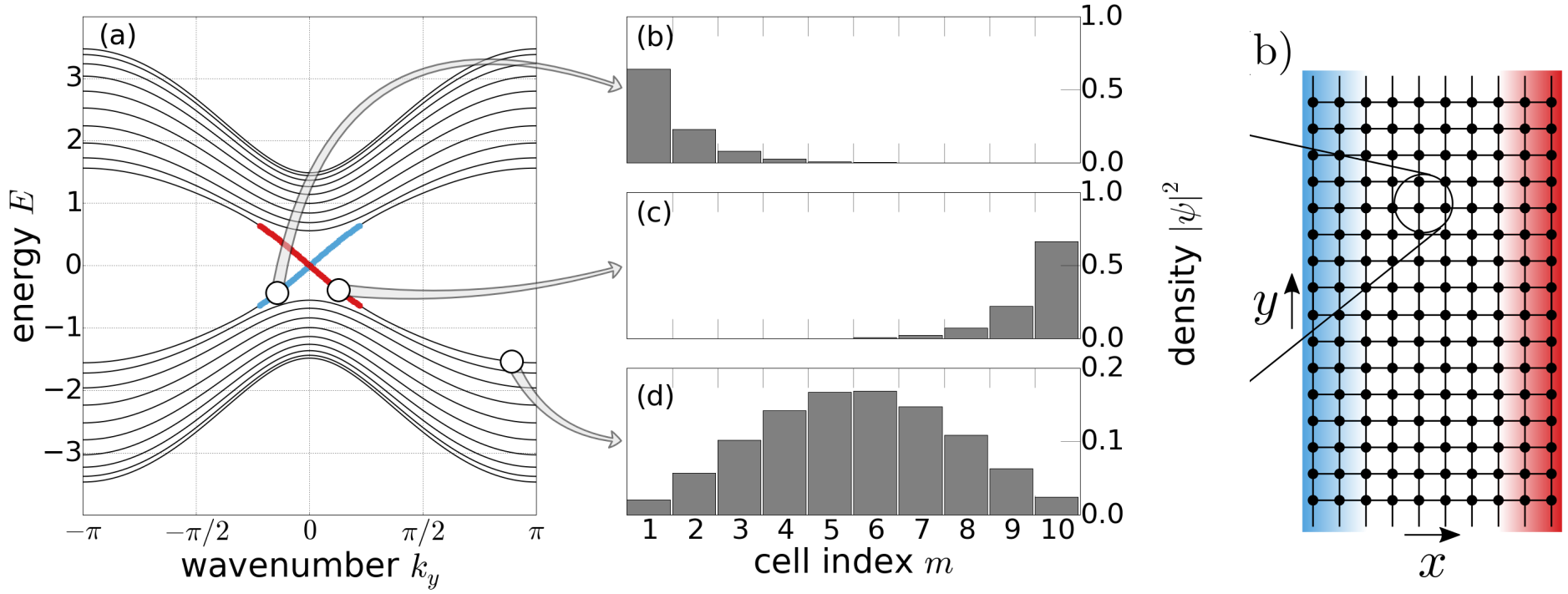
$$\Omega t \to k_y \qquad \hat{\sigma}_y \to \hat{\sigma}_x$$
$$k \to k_x \qquad \hat{\sigma}_z \to \hat{\sigma}_y$$
$$\bar{v} \to u \qquad \hat{\sigma}_x \to \hat{\sigma}_z$$

$$\hat{H}_{\mathrm{QWZ}}(k_x,k_y) = \sin(k_x)\hat{\sigma}_x + \sin(k_y)\hat{\sigma}_y + \left(\bar{v} + \cos(k_x) + \cos(k_y)\right)\hat{\sigma}_z$$

Promote time t → wavenumber k
1D time-periodic Rice-Mele → 2D Qi-Wu-Zhang



Edge states rising/falling in Thouless pump
→ unidirectional edge modes in Chern insulators



Topologically protected = robust:

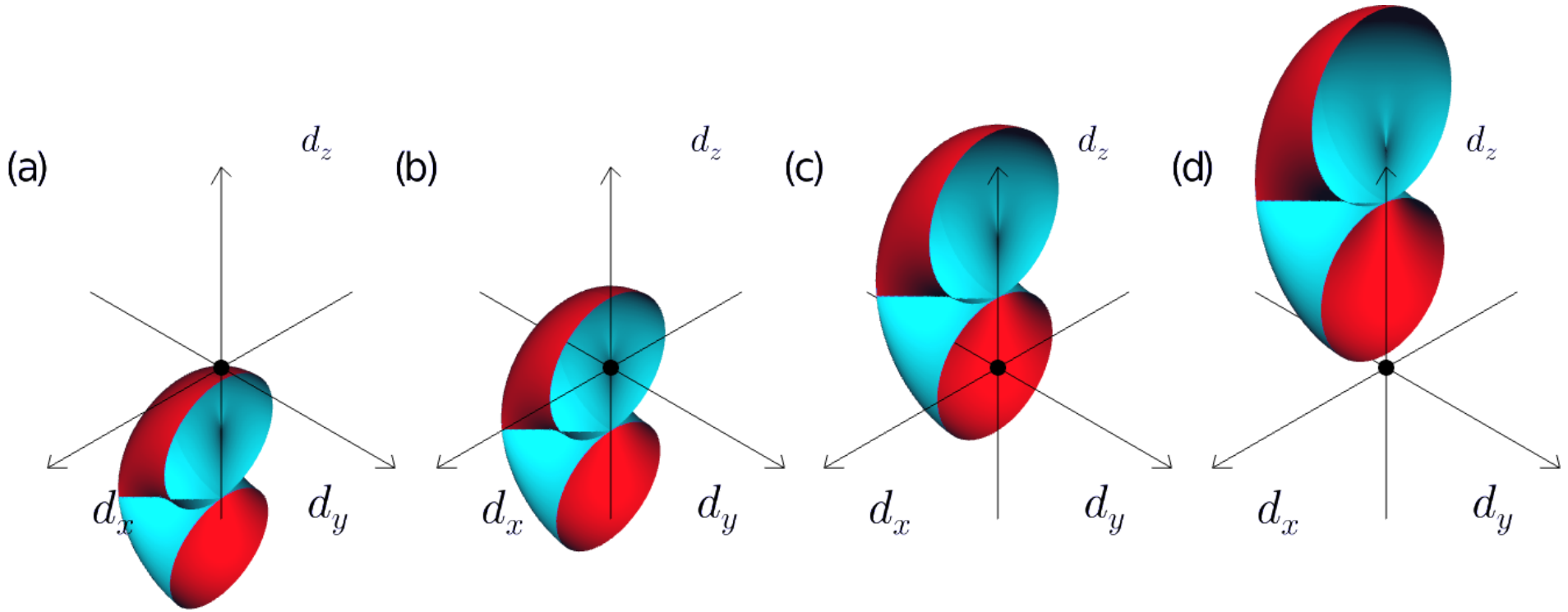
- No long range hopping

→ spectrum periodic & smooth

→ bulk gap separates two edges → no direct coupling → crossing, not anticrossing

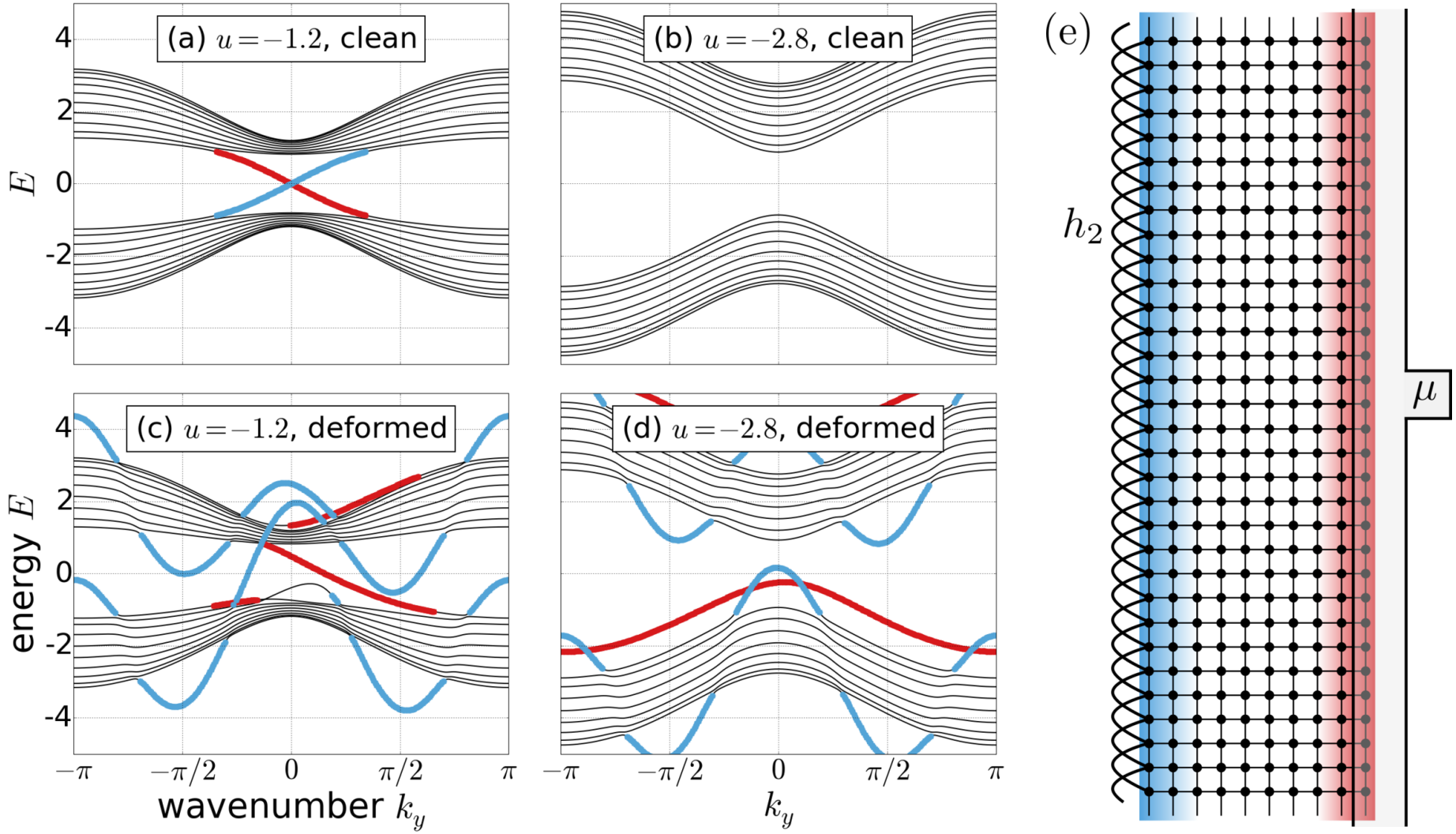
Presence, net # of edge state modes seen in bulk:
times origin in torus = # edge state modes = Chern

u<-2 | -2<u<0 | 0<u<2 | u>2

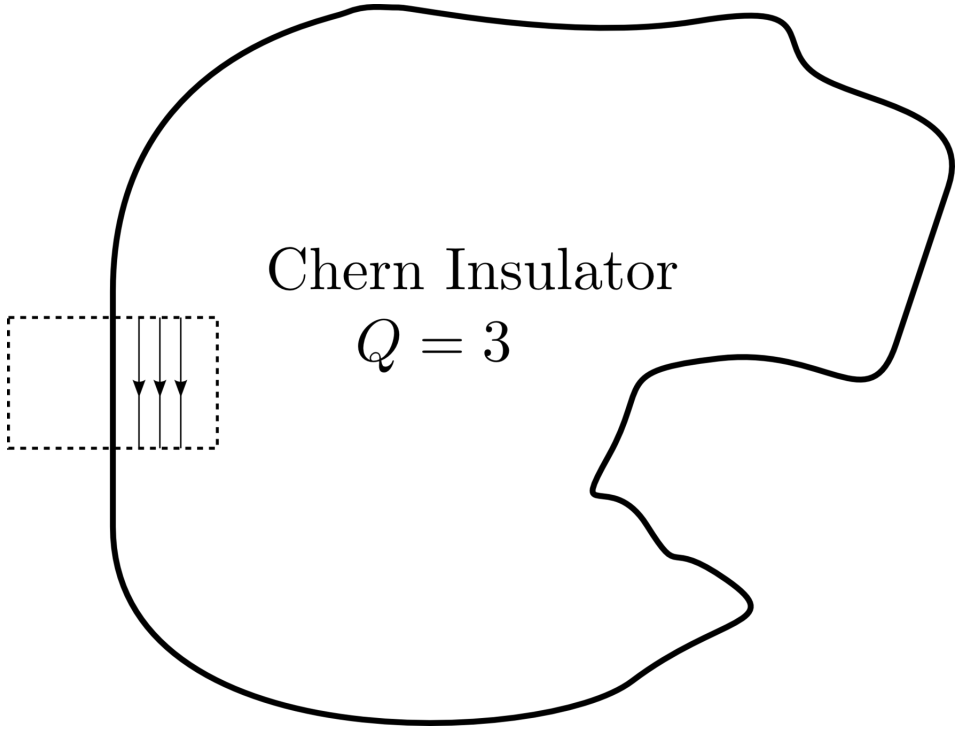


C=0 | C=-1 | C=1 | C=0

Net number of clockwise-propagating edge state modes in the gap is protected against continuous deformations



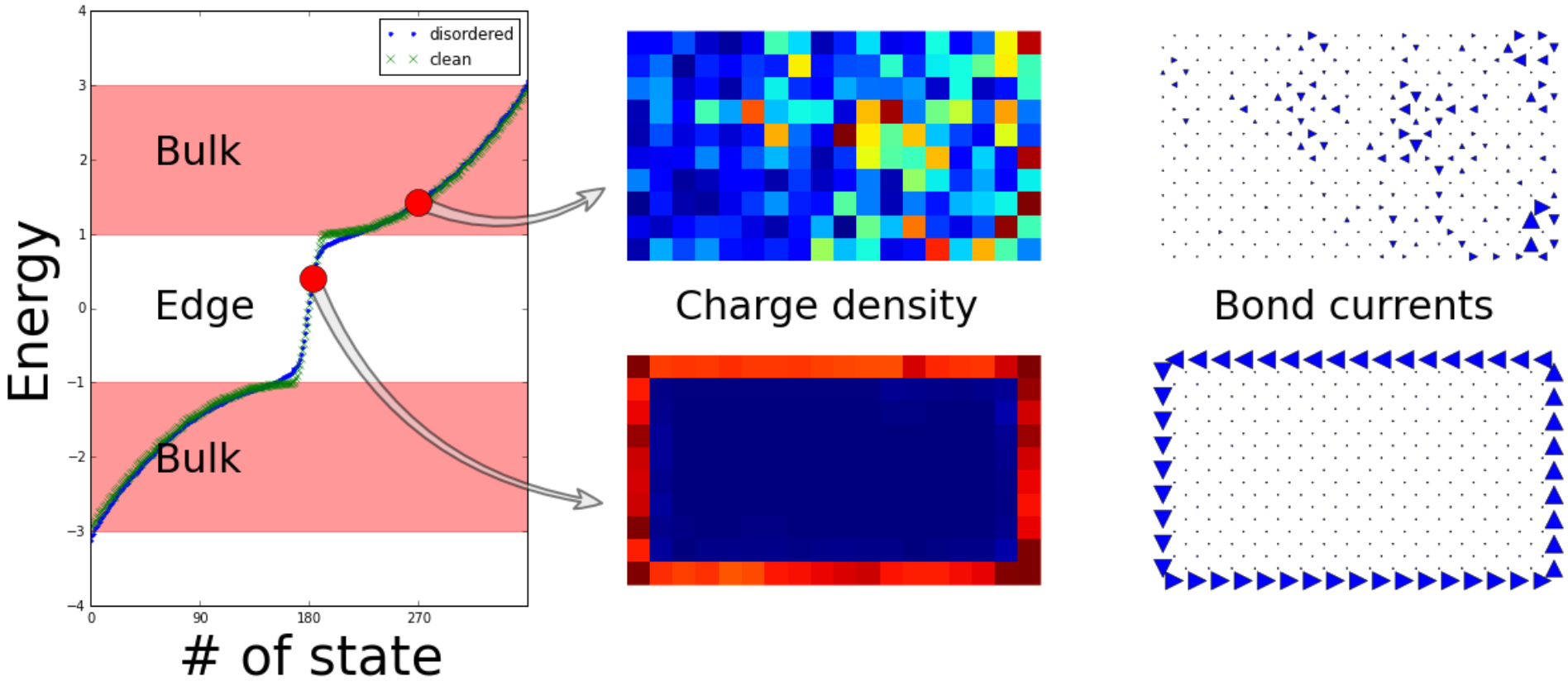
Net edge states at some section of edge → edge states all around (unitarity → particles cannot accumulate)



Topologically protected = robust against:
- Arbitrary disorder on edges
- Some disorder in bulk

(interesting variation on Anderson localization)

Net edge states at some section of edge → edge states all around (unitarity → particles cannot accumulate)



Summary: Chern Insulators have robust edge states predicted by bulk Chern

- Required:Thouless pumping (ensure edge states, Chern #)
- New theory tool: Promoting time t → quasimomentum k
- Main results: Edge states in two-dimensional systems

  Bulk Chern number predicts edge states

  Topological protection due to no backscattering

  Robust against disorder (large edge, small bulk)
- Toy model: Qi-Wu-Zhang

  Tune Chern number by onsite magnetic field u (-2, 0, 2)

$$\hat{H}_{\mathrm{QWZ}}(k_x, k_y) = \sin(k_x)\hat{\sigma}_x + \sin(k_y)\hat{\sigma}_y + \left(\bar{v} + \cos(k_x) + \cos(k_y)\right)\hat{\sigma}_z$$