Experimental signatures

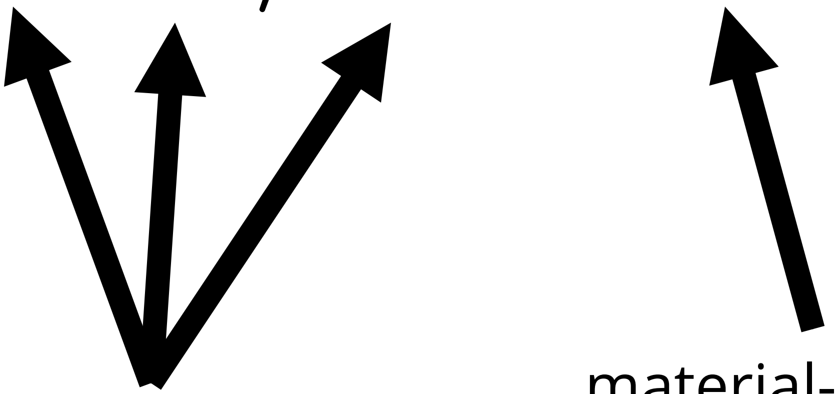
- Ohmic vs. non-ohmic conductors
- Landauer formalism
- Signatures of edgestates
- State of the art

Ohmic conductors

conductance/resistance

$$I/V = G \equiv R^{-1}$$

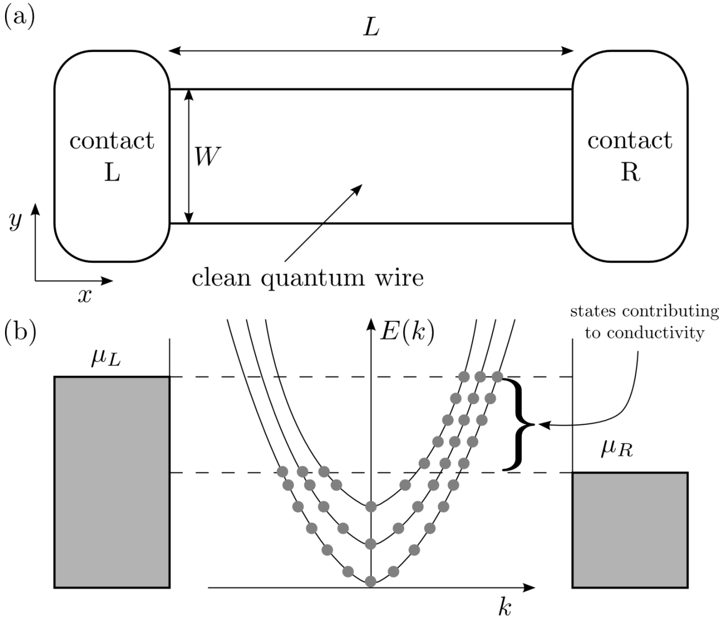
$$G_i L_i / A_i = \sigma$$



Sample specific

material-specific conductivity

Landauer formalism

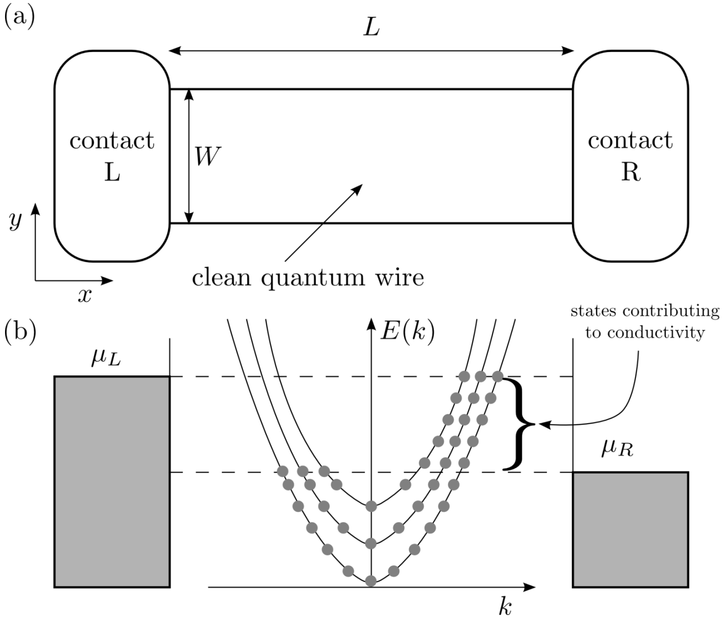


$$I = -|e| \sum_n \int_{-\pi/a}^{\pi/a} \frac{dk}{2\pi} \frac{1}{\hbar} \frac{dE_{lk}}{dk} \left[f(E_{lk} - \mu_L) - f(E_{lk} - \mu_R) \right]$$

$$I = -\frac{|e|}{h} M \int_{\mu_R}^{\mu_L} dE = -\frac{|e|}{h} (\mu_L - \mu_R) M = M \frac{e^2}{h} V$$

A clean wire can not be Ohmic!!

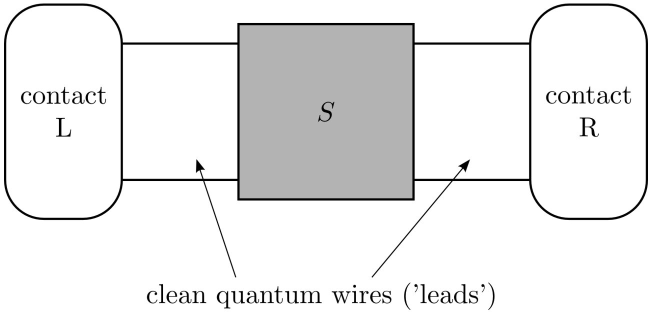
Landauer formalism



$$I = -|e|\sum_n \int_{-\pi/a}^{\pi/a} \frac{dk}{2\pi}\frac{1}{\hbar}\frac{dE_{lk}}{dk}\left[f(E_{lk}-\mu_L) - f(E_{lk}-\mu_R)\right]$$

$$I = -\frac{|e|}{h} M \int_{\mu_R}^{\mu_L} dE = -\frac{|e|}{h}(\mu_L-\mu_R)M = M\frac{e^2}{h}V$$

A clean wire can not be Ohmic!!



$$I = -\frac{|e|}{h}T\int_{\mu_R}^{\mu_L} dE\,[f_L(E)-f_R(E)] = \frac{e^2}{h}TV$$

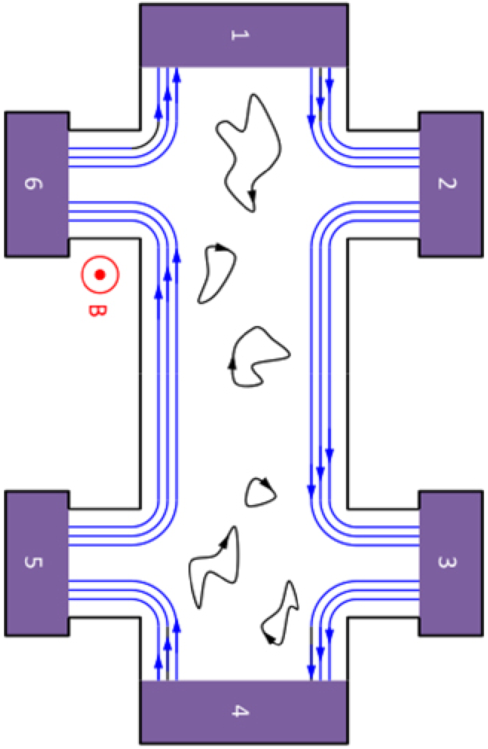
Signatures of Topological edge states in transport I: Chern Insulators

Disorder-free sample with a strip geometry

- Fermi energy lies in a band:

  the conductance is quantized and **insensitive to the length** of the sample, **grows with the width**
- Fermi energy lies in the gap:

  conductance is quantized, a **behaviour insensitive to both the length and the width** of the sample



Disordered sample with an irregular shape

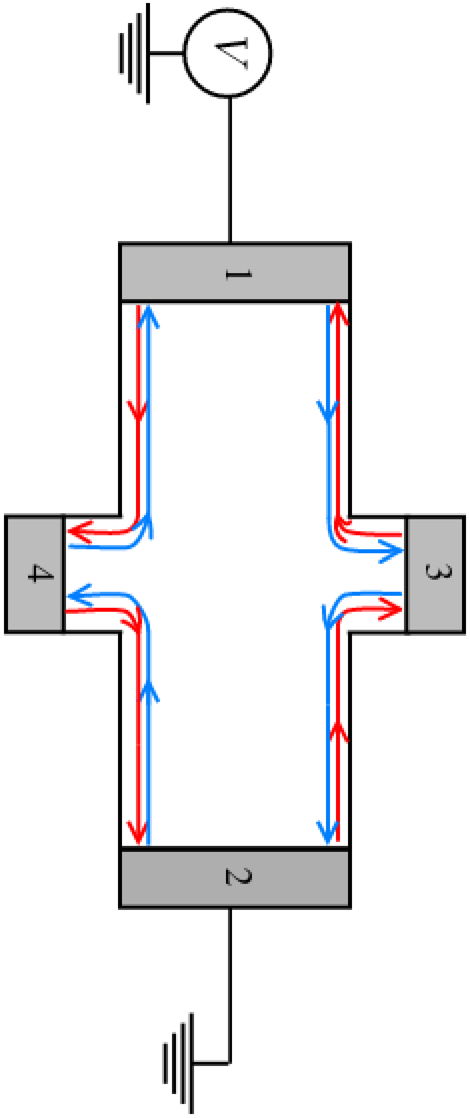
- Fermi energy lies in a band:

  Might be Ohmic. There are no protected edge states at the Fermi energy.

  Fermi energy lies in the gap:

  conductance is quantized, a **behaviour insensitive to both the length and the width** of the sample, **a hallmark of Chern Insulators.**

Signatures of Topological edge states in transport II: TRS Insulators



Disorder-free sample with a strip geometry

- Fermi energy lies in a band:

  the conductance is quantized and **insensitive to the length** of the sample,

  grows with the width
- Fermi energy lies in the gap:

  conductance is quantized to multiples of $\frac{2e^2}{h}$, since edge states come in pairs

  a **behaviour insensitive to both the length and the width** of the sample

Disordered sample with an irregular shape

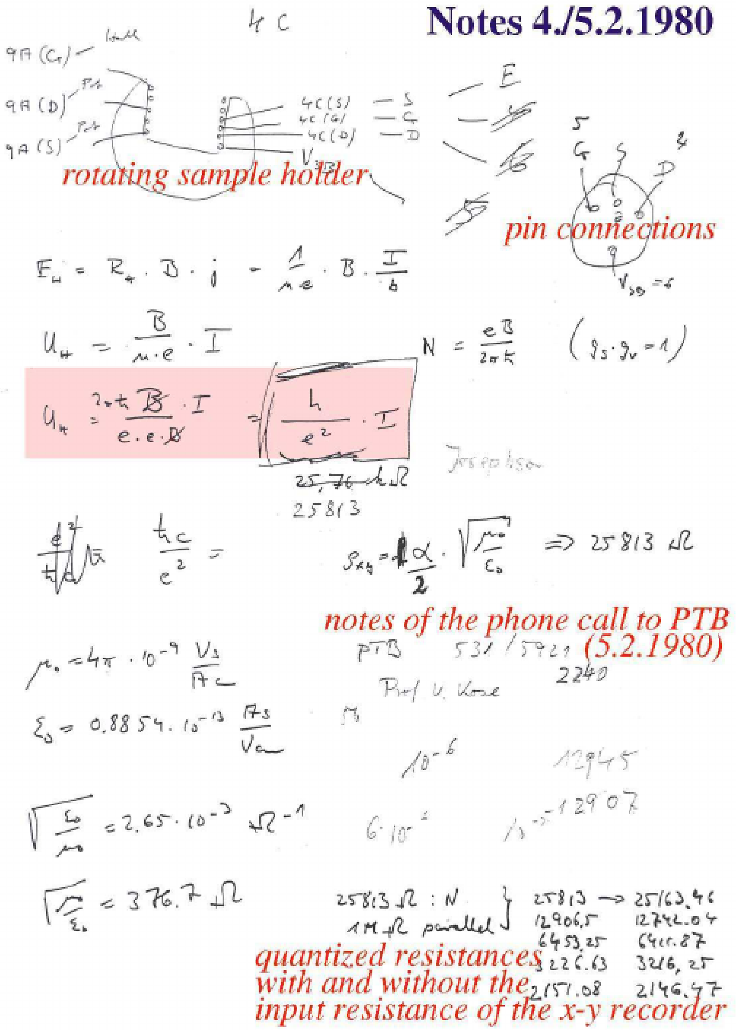
- Fermi energy lies in a band:

  Might be Ohmic.

  Fermi energy lies in the gap **and TRS is preserved**:

  conductance is quantized to $\frac{2e^2}{h}$, a **behaviour insensitive to both the length and the width** of the sample, **a hallmark of TRS Insulators.**

Quantum Hall Effect



Notes 4./5.2.1980

rotating sample holder

pin connections

$E_H = R_H \cdot B \cdot j = \frac{1}{ne} \cdot B \cdot \frac{I}{b}$

$U_H = \frac{B}{n \cdot e} \cdot I$

$N = \frac{eB}{2\pi\hbar}$ $(g_s \cdot g_v = 1)$

$U_H = \frac{2\pi\hbar \cancel{B}}{e \cdot e \cdot \cancel{B}} \cdot I = \boxed{\frac{h}{e^2} \cdot I}$

~~25,76 kΩ~~

25813

Josephson

$\frac{\hbar c}{e^2} =$

$\rho_{xy} = \frac{\alpha}{2} \cdot \sqrt{\frac{\mu_0}{\varepsilon_0}} \Rightarrow 25813\ \Omega$

notes of the phone call to PTB (5.2.1980)

PTB 531 / 5921
2240
Prof. v. Kose

$\mu_0 = 4\pi \cdot 10^{-9} \frac{Vs}{Acm}$

$\varepsilon_0 = 0{,}8854 \cdot 10^{-13} \frac{As}{Vcm}$

10^{-6} 12945

$\sqrt{\frac{\varepsilon_0}{\mu_0}} = 2{,}65 \cdot 10^{-3}\ \Omega^{-1}$

$6 \cdot 10^{-6}$ 12907

$\sqrt{\frac{\mu_0}{\varepsilon_0}} = 376.7\ \Omega$

25813 Ω : N
1MΩ parallel

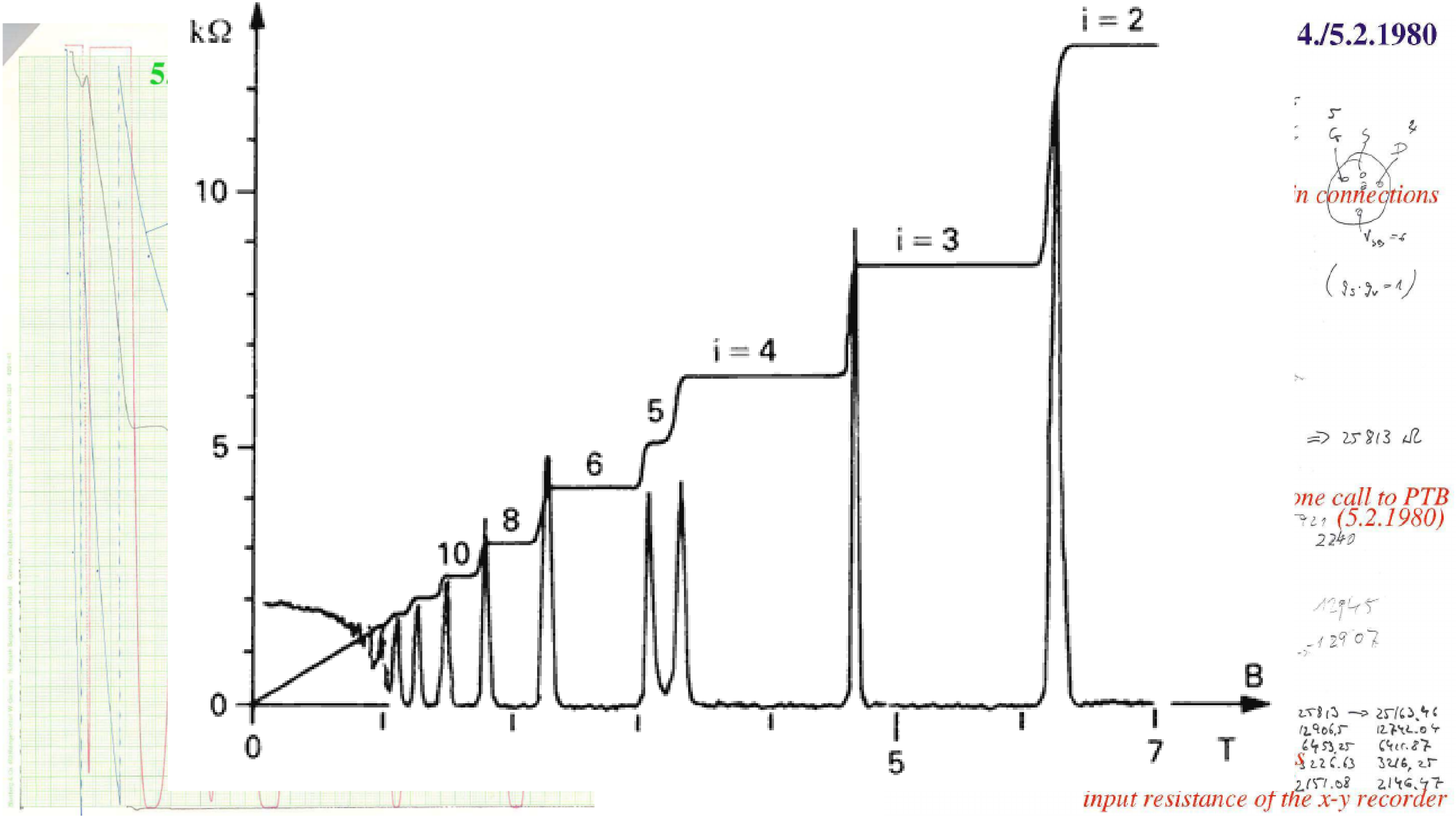
25813 →	25163,46
12906,5	12742.04
6453,25	6411.87
3226.63	3216,25
2151.08	2146,47

quantized resistances with and without the input resistance of the x-y recorder

Klitzing, K. v., Dorda, G., Pepper, M. Phys. Rev. Lett., **45**, 494 (1980)

Klitzing, K. v. Seminaire Poincare **2**, 1 (2004)

Quantum Hall Effect



Klitzing, K. v., Dorda, G., Pepper, M. Phys. Rev. Lett., **45**, 494 (1980)

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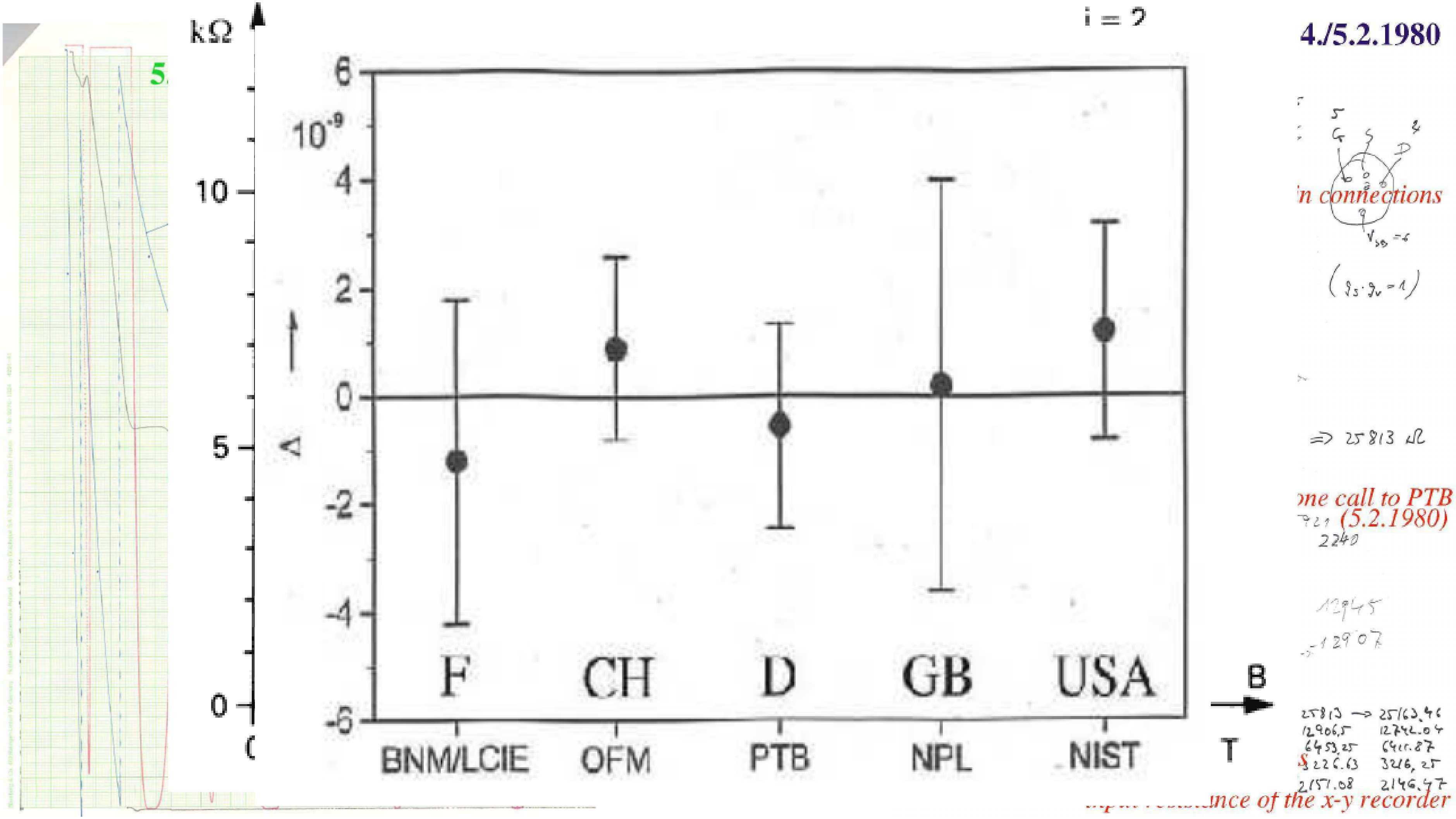
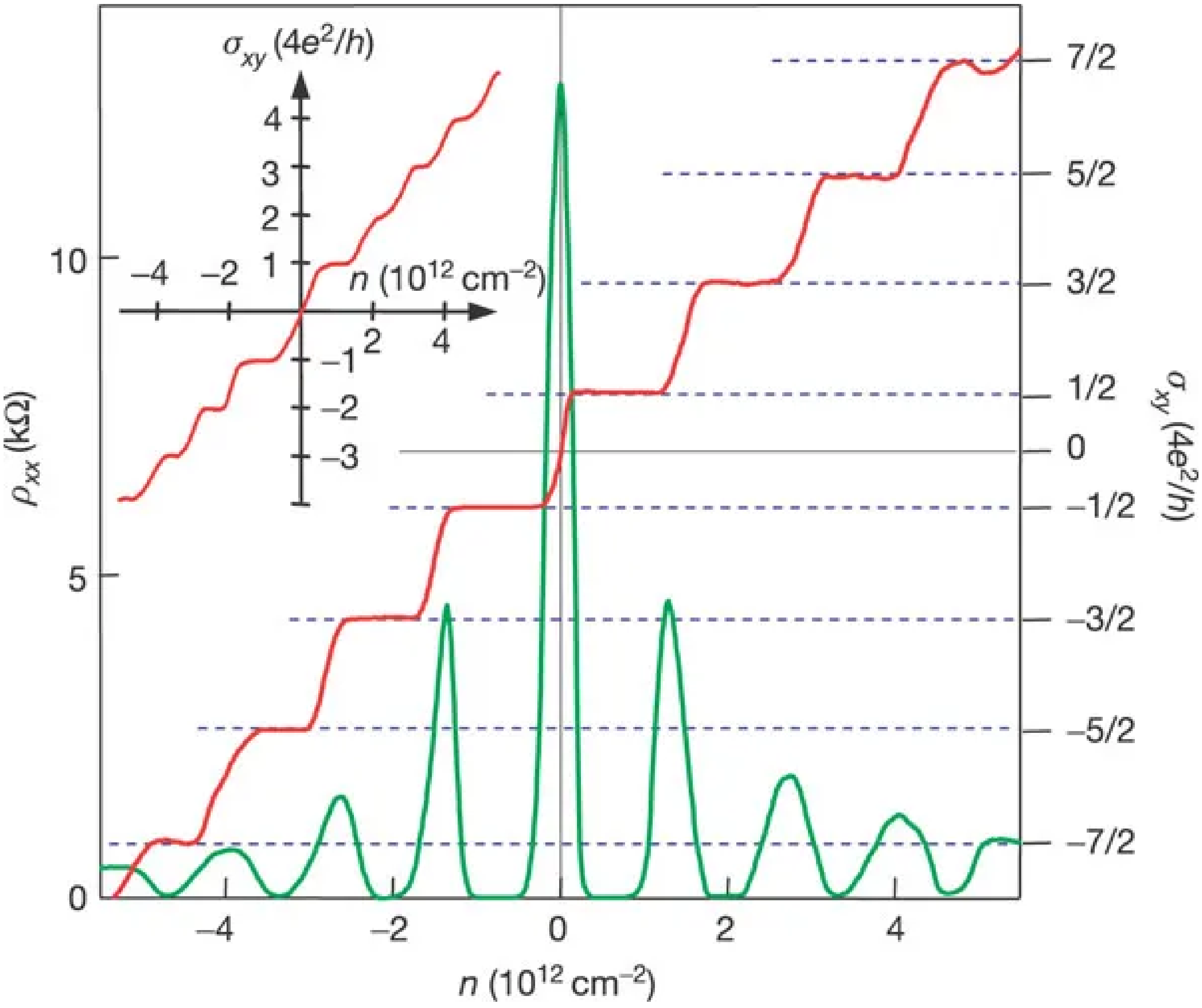
Quantum Hall Effect



Klitzing, K. v., Dorda, G., Pepper, M. Phys. Rev. Lett., **45**, 494 (1980)

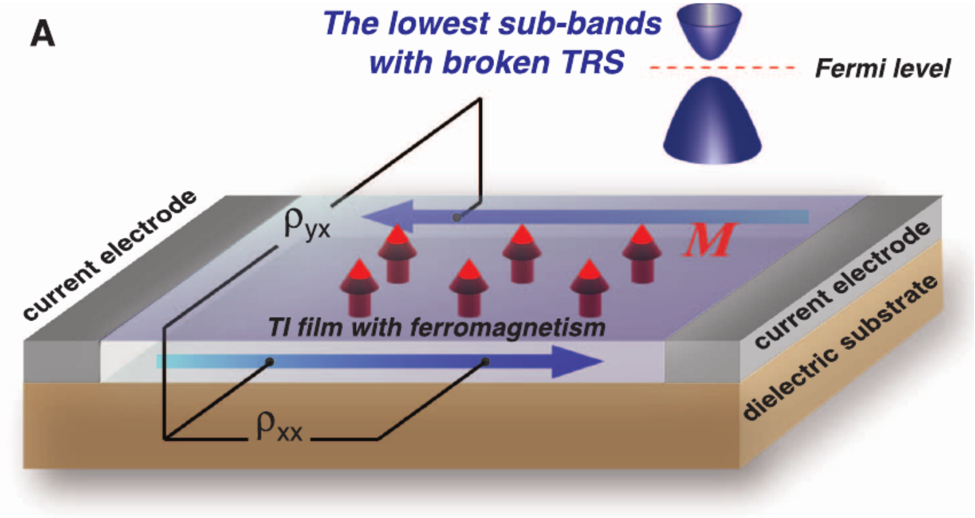
Klitzing, K. v. Seminaire Poincare **2**, 1 (2004)

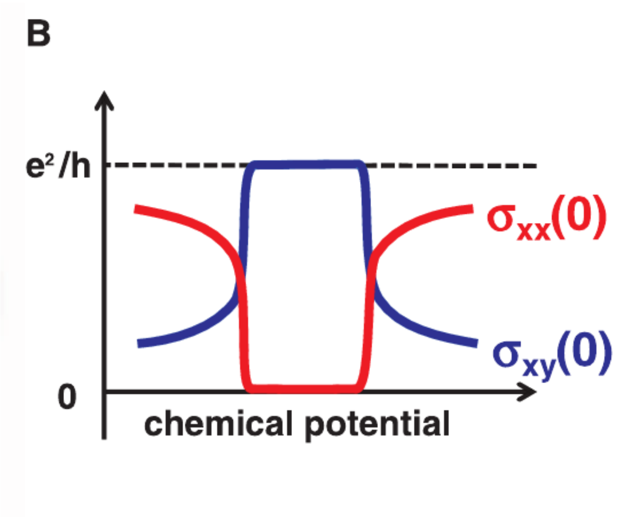
Quantum Hall Effect in graphene

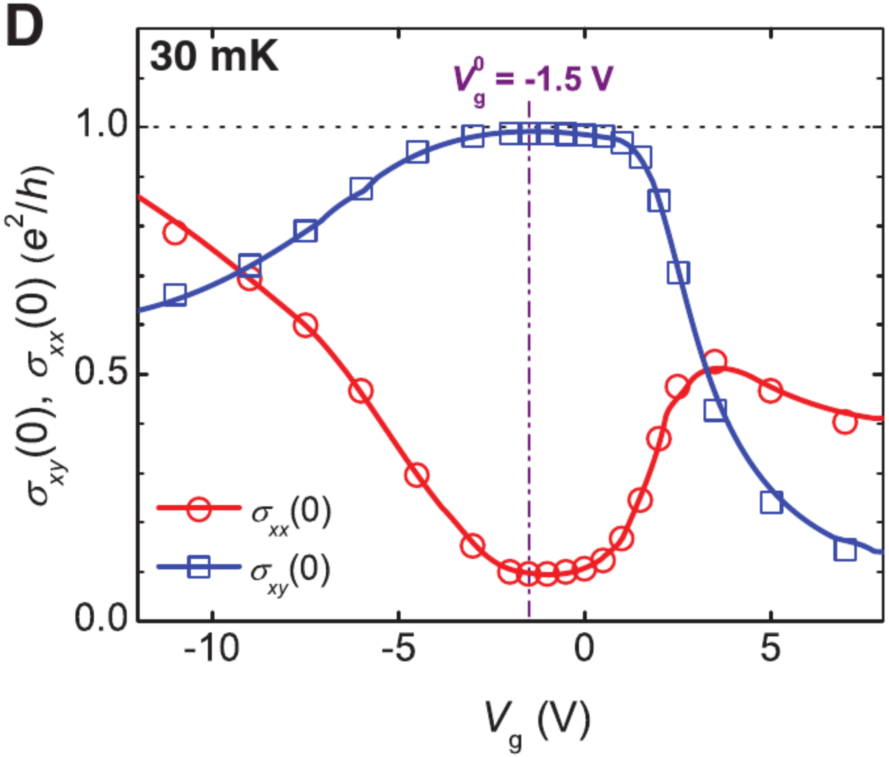


K. S. Novoselov *et al.* Nature **438**, 197 (2005)

Quantum Anomalous Hall Effect in 3D TI

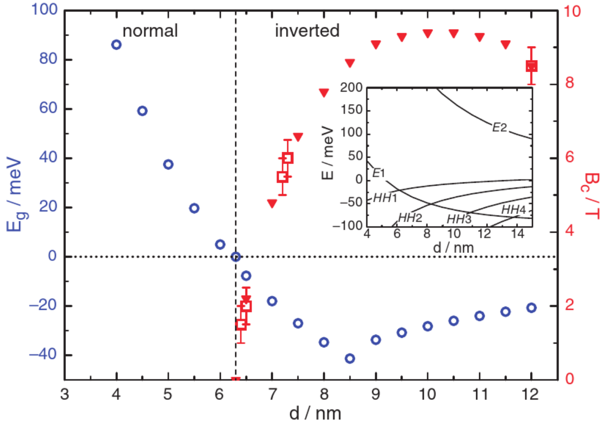


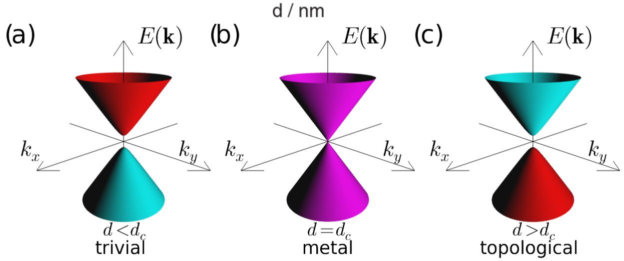


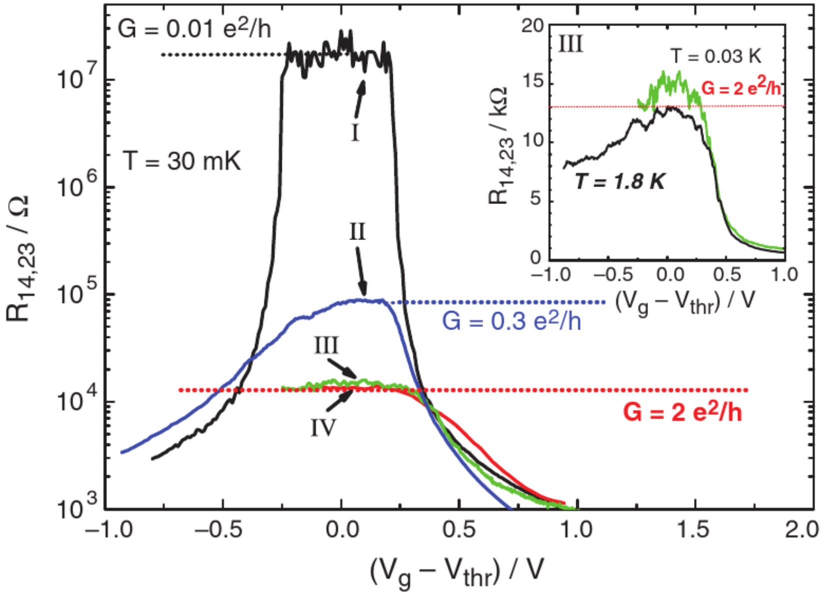


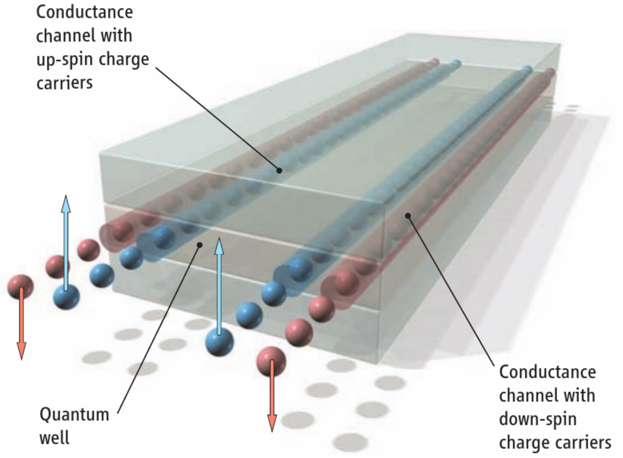
Cui-Zu Chang *et al.* Science **340**, 167(2013)

Quantum Spin Hall Effect in HgTe/HgCdTe



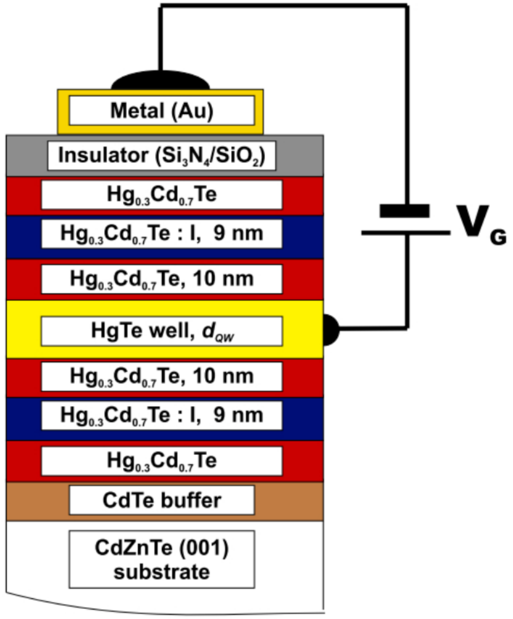





Conductance channel with up-spin charge carriers

Quantum well

Conductance channel with down-spin charge carriers


Metal (Au)

Insulator (Si_3N_4/SiO_2)

$Hg_{0.3}Cd_{0.7}Te$

$Hg_{0.3}Cd_{0.7}Te$: I, 9 nm

$Hg_{0.3}Cd_{0.7}Te$, 10 nm

HgTe well, d_{QW}

$Hg_{0.3}Cd_{0.7}Te$, 10 nm

$Hg_{0.3}Cd_{0.7}Te$: I, 9 nm

$Hg_{0.3}Cd_{0.7}Te$

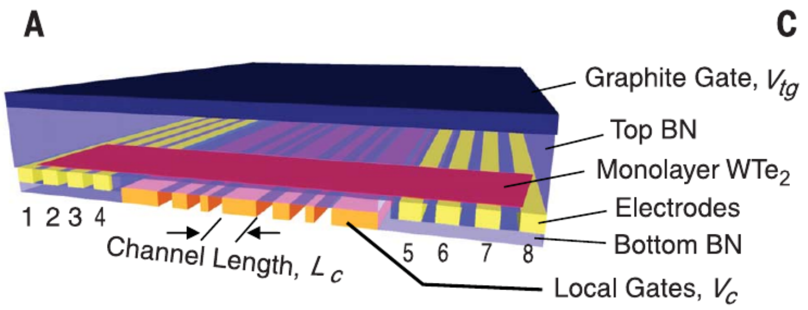
CdTe buffer

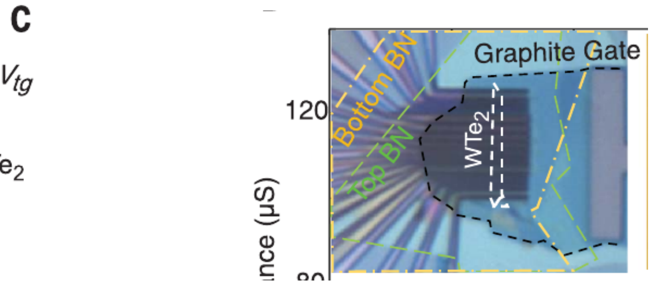
CdZnTe (001) substrate

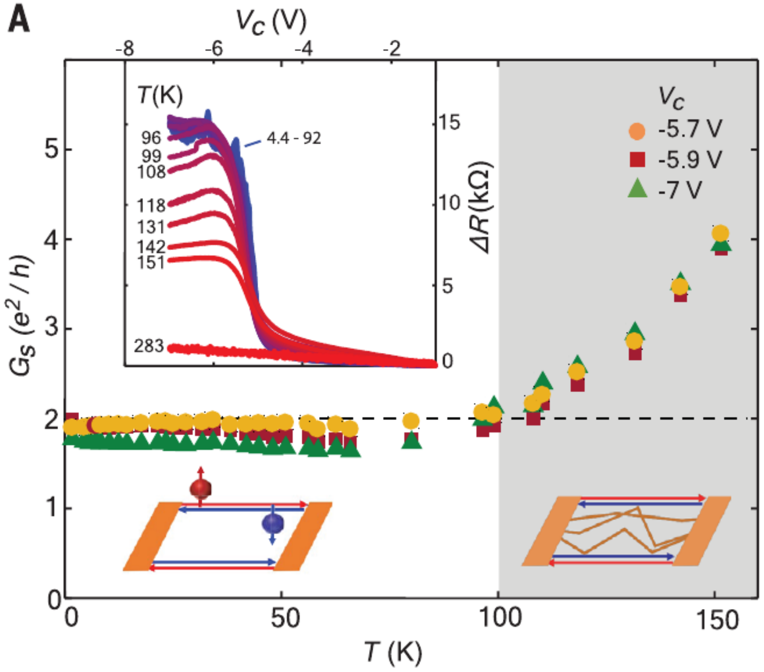
V_G

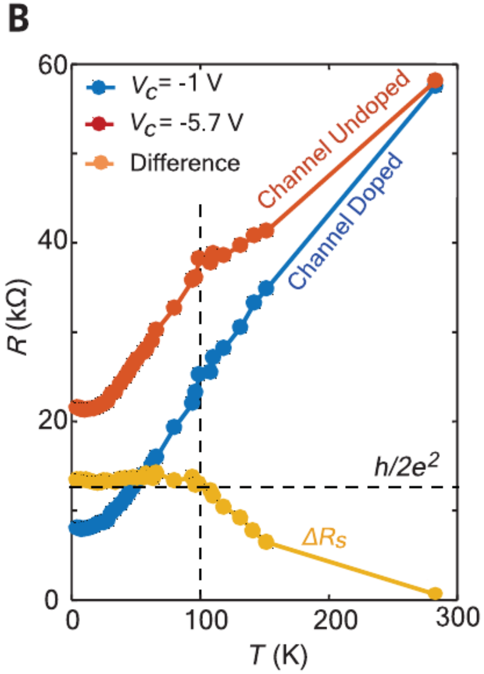
König *et al.* Science **318,** 766 (2007)

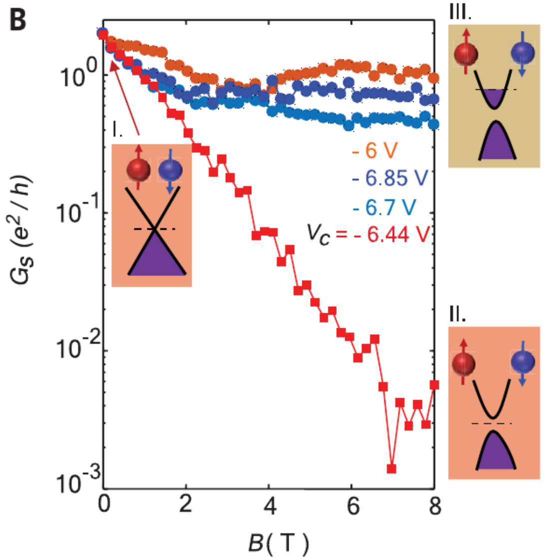
Quantum Spin Hall Effect in WTe_2





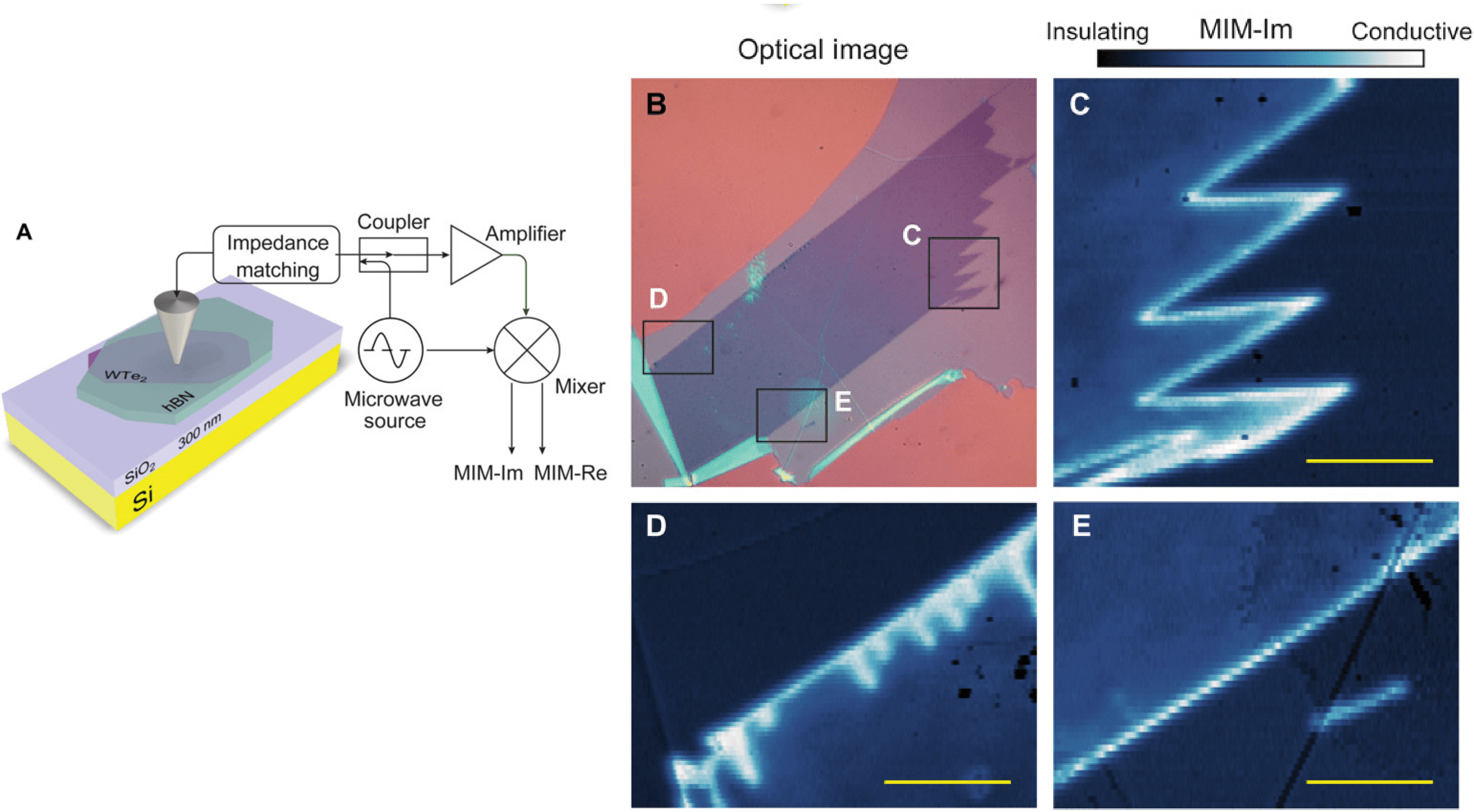






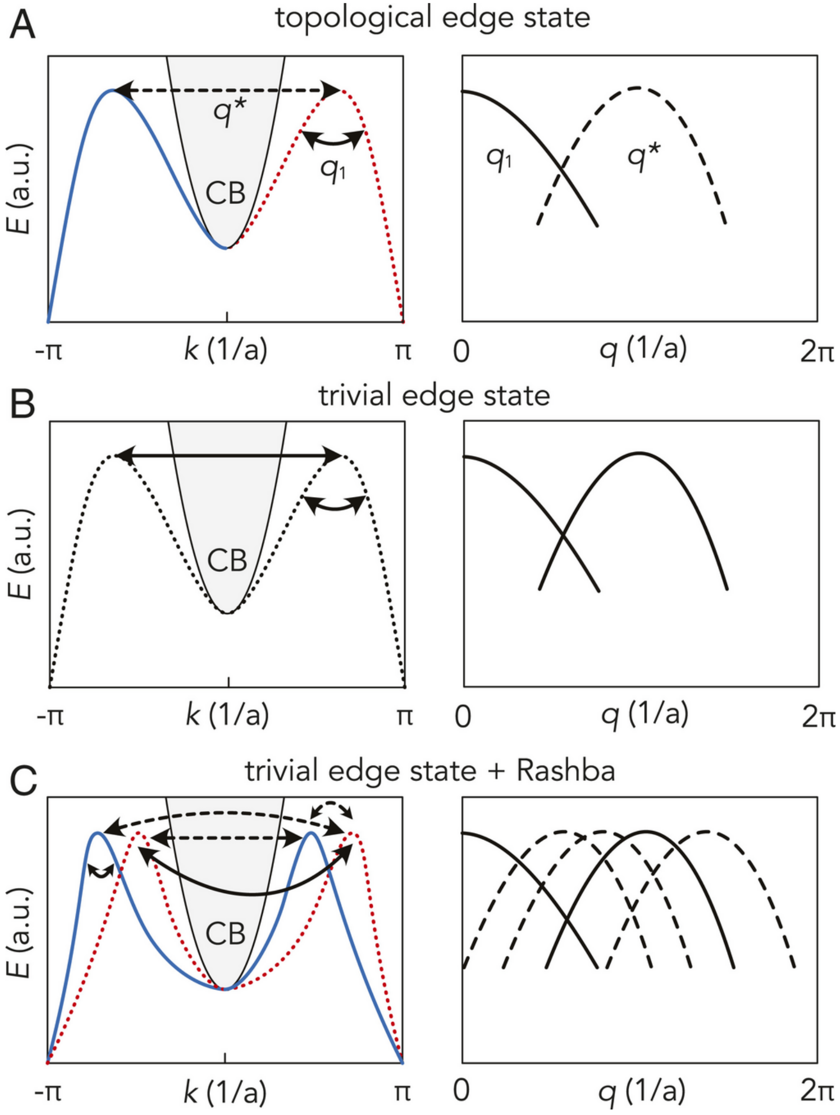
Wu et al., Science **359**, 76 (2018)

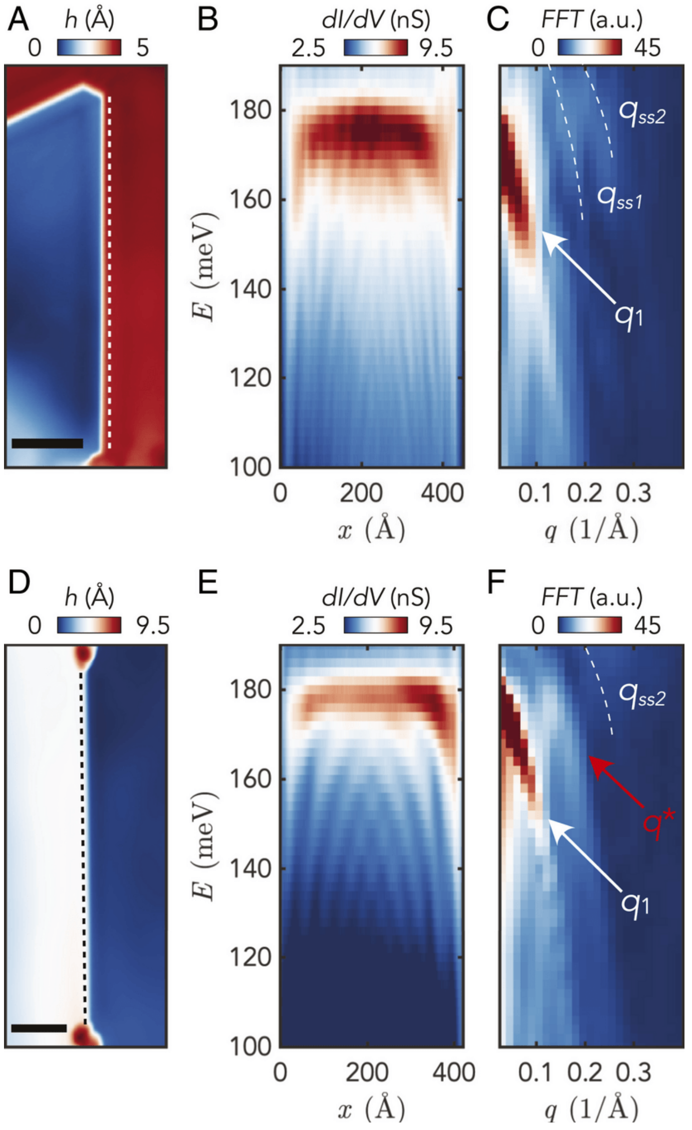
Quantum Spin Hall Effect in WTe_2



Yanmeng et al. Science Advances **5**, eaat8799 (2019)

Quasiparticle Interference @ QSH edge of Bi$_2$





Jäck, Berthold, *et al.* PNAS **117** 16214 (2020)