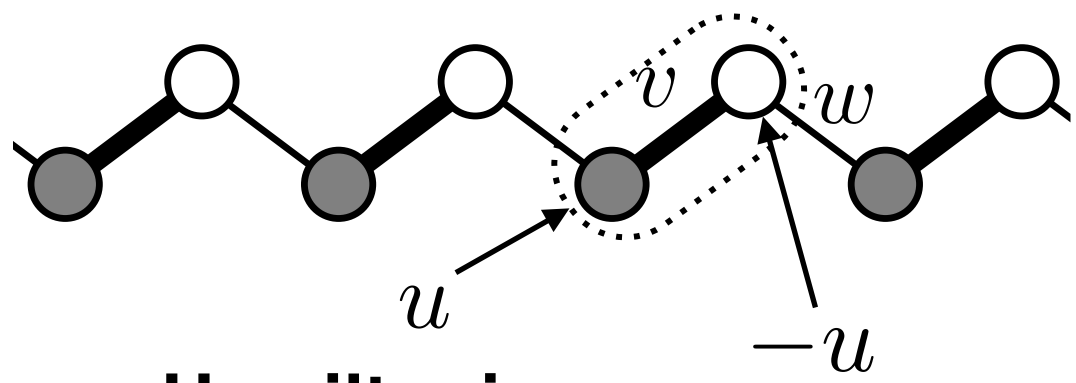
Rice & Mele, PRL 1982
Thouless PRB 1983

Adiabatic pumping in a chain

Example: time-dependent Rice-Mele model



$$H = \begin{pmatrix} u & v & 0 & 0 & 0 & 0 & 0 & 0 \\ v & -u & w & 0 & 0 & 0 & 0 & 0 \\ 0 & w & u & v & 0 & 0 & 0 & 0 \\ 0 & 0 & v & -u & w & 0 & 0 & 0 \\ 0 & 0 & 0 & w & u & v & 0 & 0 \\ 0 & 0 & 0 & 0 & v & -u & v & 0 \\ 0 & 0 & 0 & 0 & 0 & w & u & v \\ 0 & 0 & 0 & 0 & 0 & 0 & v & -u \end{pmatrix}$$

Real-space Hamiltonian:

$$\hat{H}(t) = v(t)\sum_{m=1}^{N}\left(|m,B\rangle\langle m,A| + h.c.\right) + w(t)\sum_{m=1}^{N-1}\left(|m+1,A\rangle\langle m,B| + h.c.\right)$$
$$+ u(t)\sum_{m=1}^{N}\left(|m,A\rangle\langle m,A| - |m,B\rangle\langle m,B|\right),$$

Momentum-space Hamiltonian:

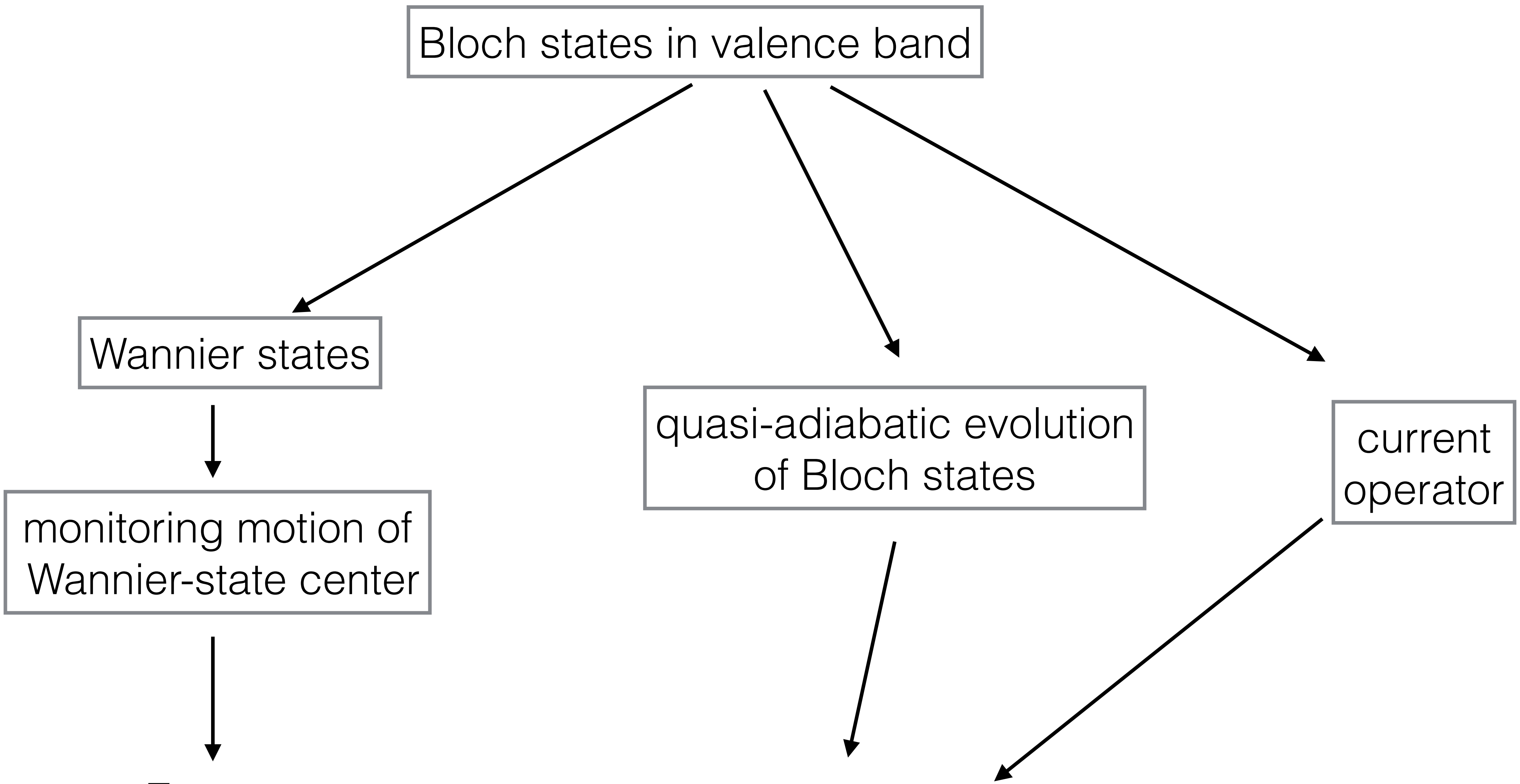
$$\hat{H}(k,t) = \mathbf{d}(k,t)\hat{\sigma} =$$
$$= (v(t) + w(t)\cos k)\hat{\sigma}_x$$
$$+ w(t)\sin k\hat{\sigma}_y + u(t)\hat{\sigma}_z,$$

Adiabatic pumping:

1. Gap doesn't close: $|\boldsymbol{d}| > 0$.
2. Cyclic time depedence (period T)
3. $T \to \infty$

How much charge is pumped through a cross section?

Adiabatic pumping in a chain



$$\mathcal{Q} = -i\frac{1}{2\pi}\int_0^T dt \int_{\mathrm{BZ}} dk \left(\partial_k \langle u_1(k,t)|\,\partial_t u_1(k,t)\rangle - \partial_t \langle u_1(k,t)|\,\partial_k u_1(k,t)\rangle\right). \quad (5.3)$$

Pumped charge is the Chern number and hence and integer

Adiabatic pumping in a chain

Pumped charge (Q) per cycle:

$$\mathcal{Q} = \int_0^T dt \int_{\mathrm{BZ}} \frac{dk}{2\pi} j^{(1)}_{m+1/2}(k,t). \tag{5.45}$$

Momentum- and time-resolved current of the filled band:

$$j^{(1)}_{m+1/2}(k,t) = \frac{1}{N} \langle \tilde{u}_1(k,t) | \partial_k \hat{H}(k,t) | \tilde{u}_1(k,t) \rangle , \tag{5.44}$$

Quasi-adiabatic evolution of Bloch-states:

$$|\tilde{u}_1(t)\rangle = e^{-i\int_0^t dt' E_1(t')} \left[|u_1(t)\rangle + i \frac{\langle u_2(t)| \partial_t |u_1(t)\rangle}{E_t} |u_2(t)\rangle \right]. \tag{5.42}$$

Adiabatic pumping in a chain

Example: smoothly modulated Rice-Mele model

$$\hat{H}(k,t) = \mathbf{d}(k,t)\cdot\hat{\boldsymbol{\sigma}}, \qquad \mathbf{d}(k,t) = \begin{pmatrix} \bar{v} + \cos\Omega t + \cos k \\ \sin k \\ \sin\Omega t \end{pmatrix}$$

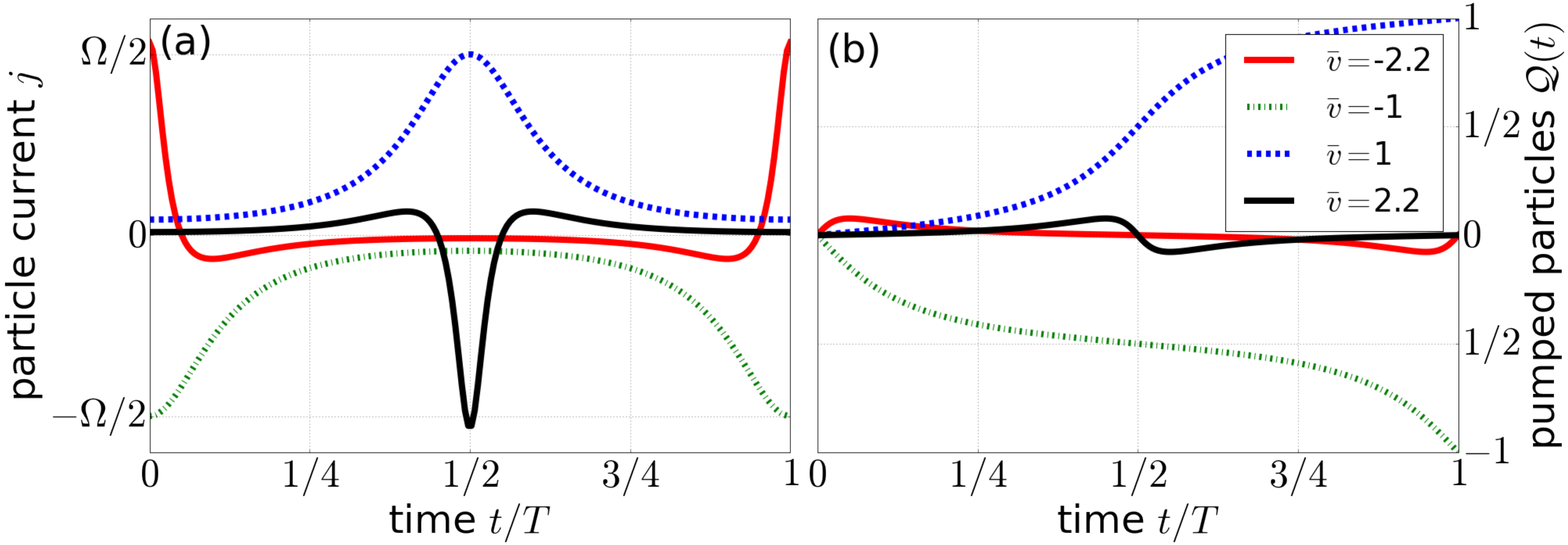


Fig. 5.2 Time dependence of the current and the number of pumped particles in an adiabatic cycle.

Expectation value

The expectation value of an observable $\hat{A}$ follows the equation ($\hbar = 1$):

$$\frac{d}{dt}\langle \hat{A} \rangle = -i \langle \left[\hat{A}, \hat{H}(t) \right] \rangle,$$

where $\langle \hat{A} \rangle$ stands for

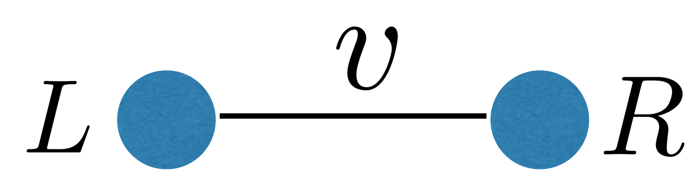
(a) the mean of the diagonal elements of $\hat{A}$.

(b) the mean of the eigenvalues of $\hat{A}$.

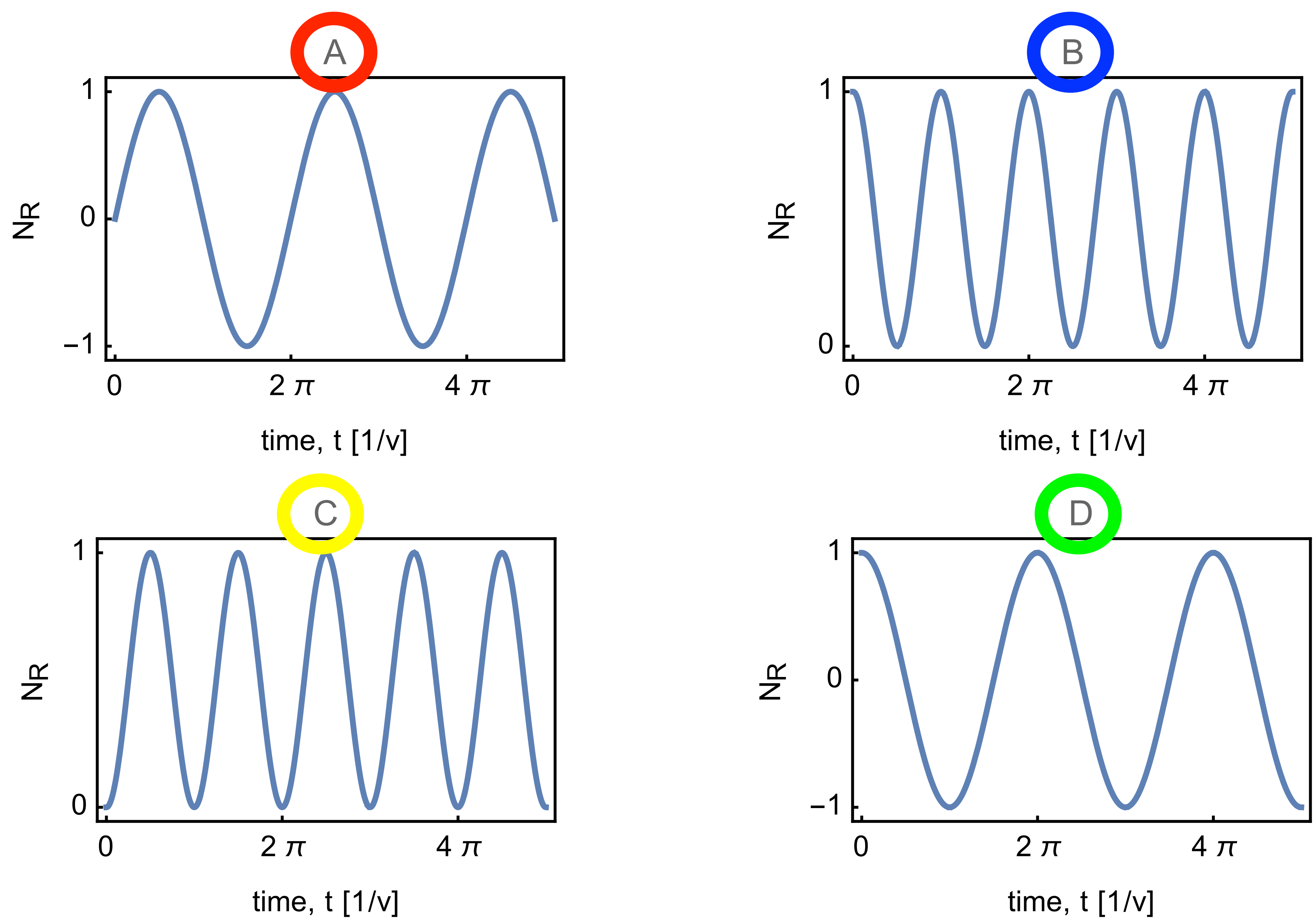
(c) the expectation value of $\hat{A}$ in a solution $\psi(t)$ of the time-dependent Schrödinger equation.

(d) the expectation value of $\hat{A}$ in the instantaneous eigenstate of $H(t)$.

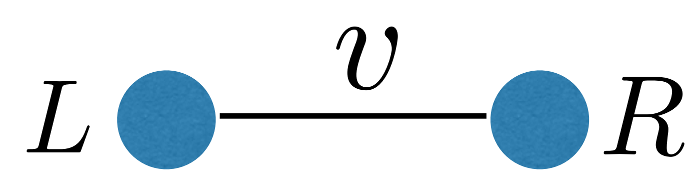
Particle number in a two-site model



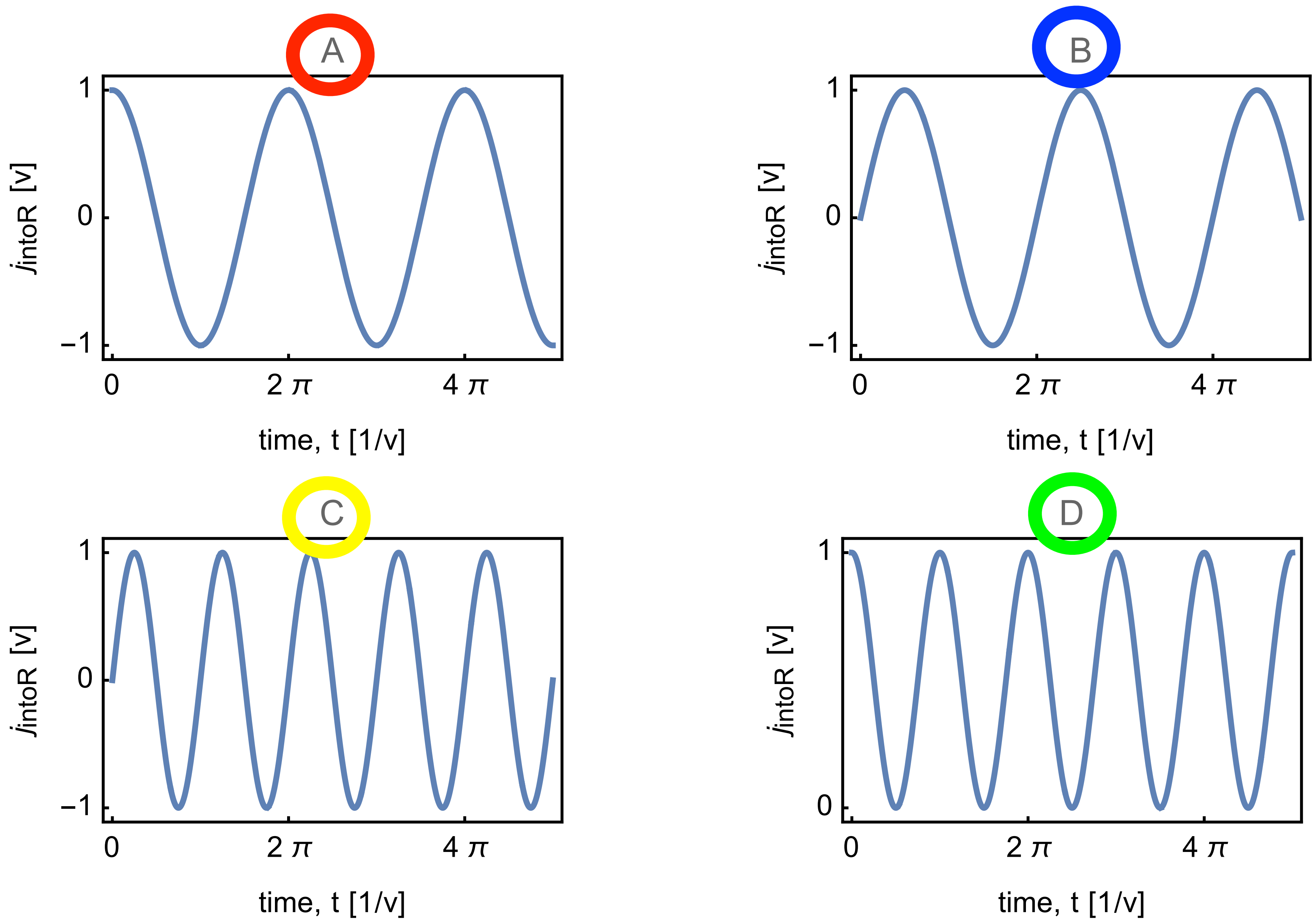
Consider the two-site system described by the Hamiltonian $H = v\sigma_x$. The initial state at $t = 0$ is localized on the left site, $\psi_i(t = 0) = (1, 0)$. How does the particle number $N_R(t)$ on the right site evolve in time?



Current in a two-site model



Consider the two-site system described by the Hamiltonian $H = v\sigma_x$. The initial state at $t = 0$ is localized on the left site, $\psi_i(t = 0) = (1, 0)$. How does the current into the right site, j_{intoR}, evolve in time?



Current in a two-site model II.

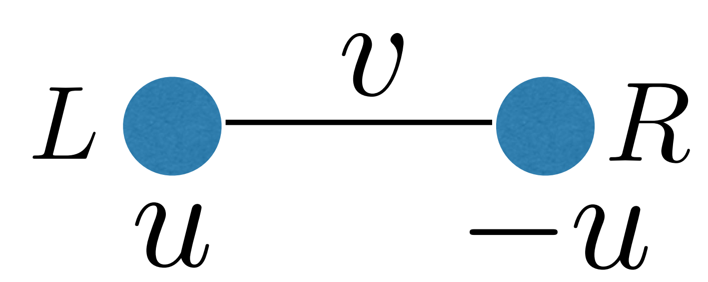
Consider the time-dependent two-site Hamiltonian $H = u(t)\sigma_z + v(t)\sigma_x$. Which of the operators below represents the influx of particles into site R?

a) $-v(t)\sigma_x$

b) $-v(t)\sigma_y$

c) $-iv(t)\sigma_y$

d) $-u(t)\sigma_y$



Particle influx into a segment of a molecule

Consider the 5-atom molecule shown on the right.
The spatial structure of the nonzero hopping amplitudes is indicated by the graph.
Otherwise, hopping amplitudes and on-site energies are arbitrary.

Denote the current operator describing the influx of electrons into the orange segment as $\hat{j}_S$.
The matrix representation of $\hat{j}_S$ in the real-space basis (shown in the figure) is a 5x5 matrix.

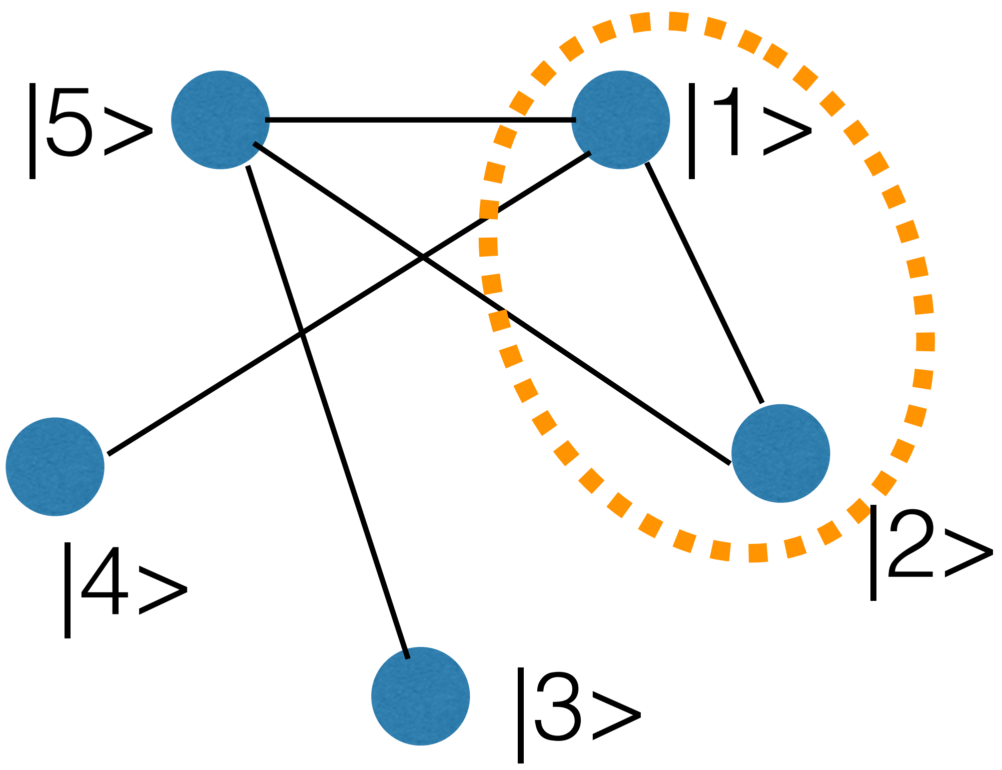
How many nonzero elements does it have?

a) 3

b) 6

c) 8

d) 10



Adiabatic limit of a quasi-adiabatic pumping cycle

Consider the adiabatic limit of a quasi-adiabatic pumping cycle in a 1D crystal.
Which statement is true?

In the adiabatic limit,

a) the momentum- and time-resolved current through a cross section approaches zero.

b) the time-resolved current through a cross section approaches zero.

c) the number of particles pumped through a cross section during the whole cycle approaches zero.

d) More than one of the above statements is true.

Current from a filled band?

Take the filled lower-energy band of a static,
insulating one-dimensional, two-band lattice model.
Assume periodic boundary condition, allow for complex-valued hopping amplitudes,
but consider the thermodynamic limit, $N \to \infty$.

Then,

a) the current carried by each occupied Bloch state is zero.

b) the net current carried by the electrons of the filled band is zero.

c) the net current carried by the electrons of the filled band is always nonzero.

d) the net current carried by the electrons of the filled band can be nonzero.

Parallel-transport time parametrization

Consider a spin aligned with a B-field along z.
Adiabatically rotate the B-field 360 degrees in the x-z plane, such that it returns to its original alignment at the end of the cycle:

$$H(t) = \boldsymbol{B}(t) \cdot \boldsymbol{\sigma}, \text{ where } \boldsymbol{B}(t) = B(\sin(2\pi t/T), 0, \cos(2\pi t/T)).$$

Let us describe the instantaneous ground state of this Hamiltonian with the parallel-transport time parametrization that starts with $\psi(t=0) = (0, 1)$.

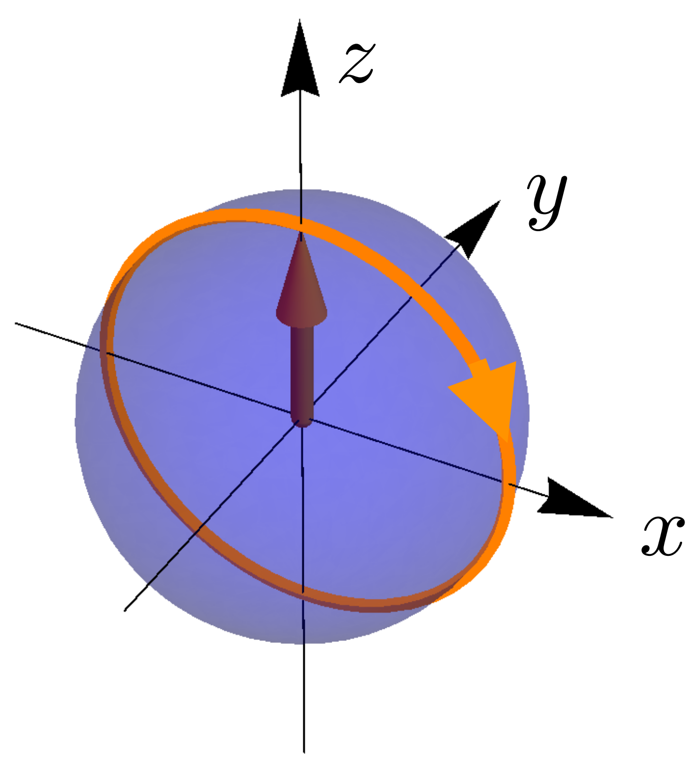
What is the value of this parametrization in the final point $t = T$?

a) $\psi(T) = (0, 1)$

b) $\psi(T) = -(0, 1)$

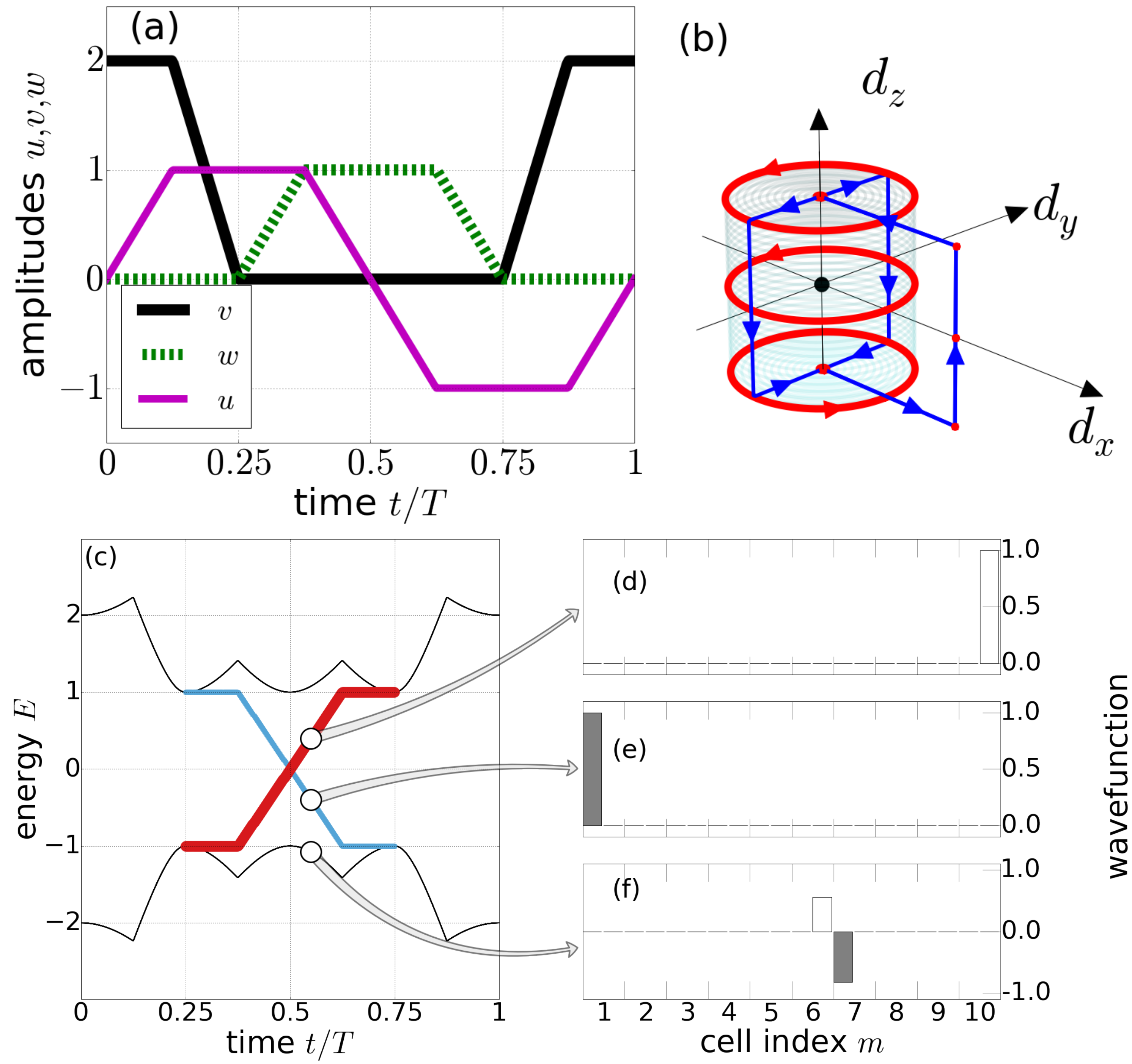
c) $\psi(T) = e^{iBT}(0, 1)$

d) $\psi(T) = -e^{iBT}(0, 1)$



Adiabatic pumping in finite chain

Fig: control freak cycle from the book, *N* = 10



Initial state: ground state with 10 electrons.

How many cycles should we pump to arrive to the ground state again?

(a) 1

(b) 2

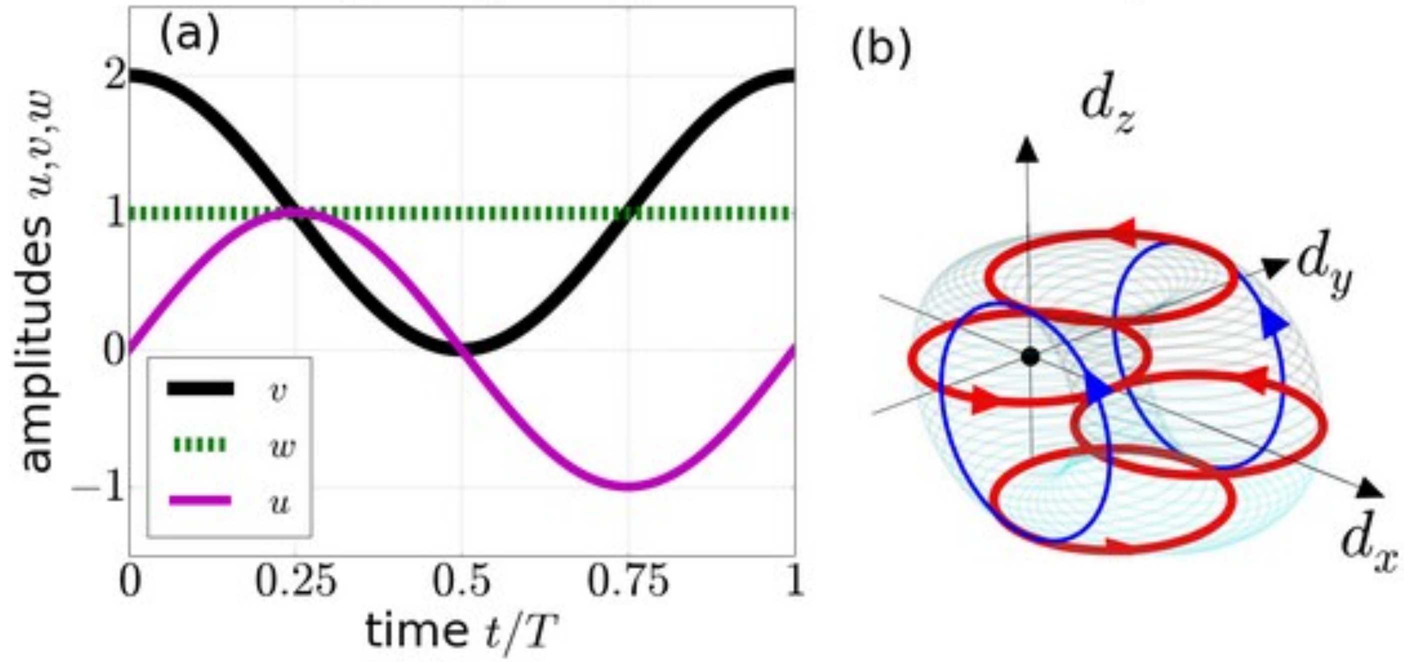
(c) 10

(d) 20

Adiabatic pumping in a finite chain I.

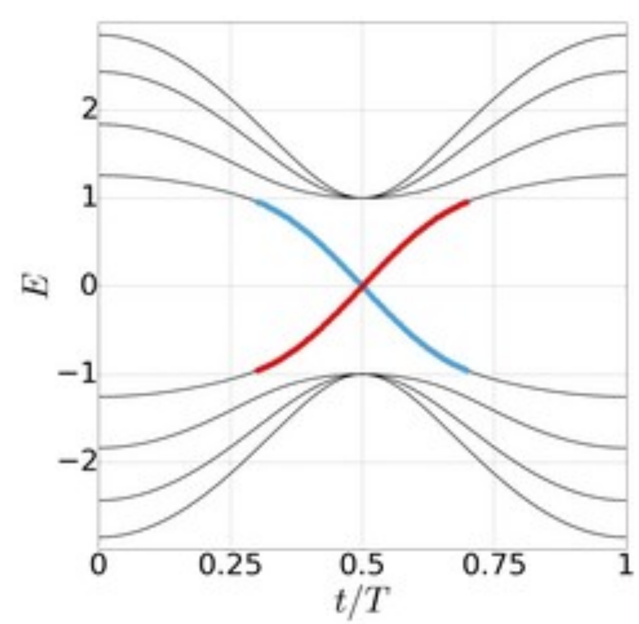
The figures represent the $\bar{v} = 1$ case of the pump sequence defined by

$$
\begin{aligned}
u(t) &= \sin(2\pi t/T), \\
v(t) &= \bar{v} + \cos(2\pi t/T), \\
w(t) &= 1,
\end{aligned}
$$



in the finite-sized Rice-Mele model with N=4 unit cells.

Assume that initially the electronic system is in its ground state: the four electrons occupy the negative-energy states. At least how many cycles should be completed to arrive to this ground state again?



a) 1
b) 2
c) 4
d) 8

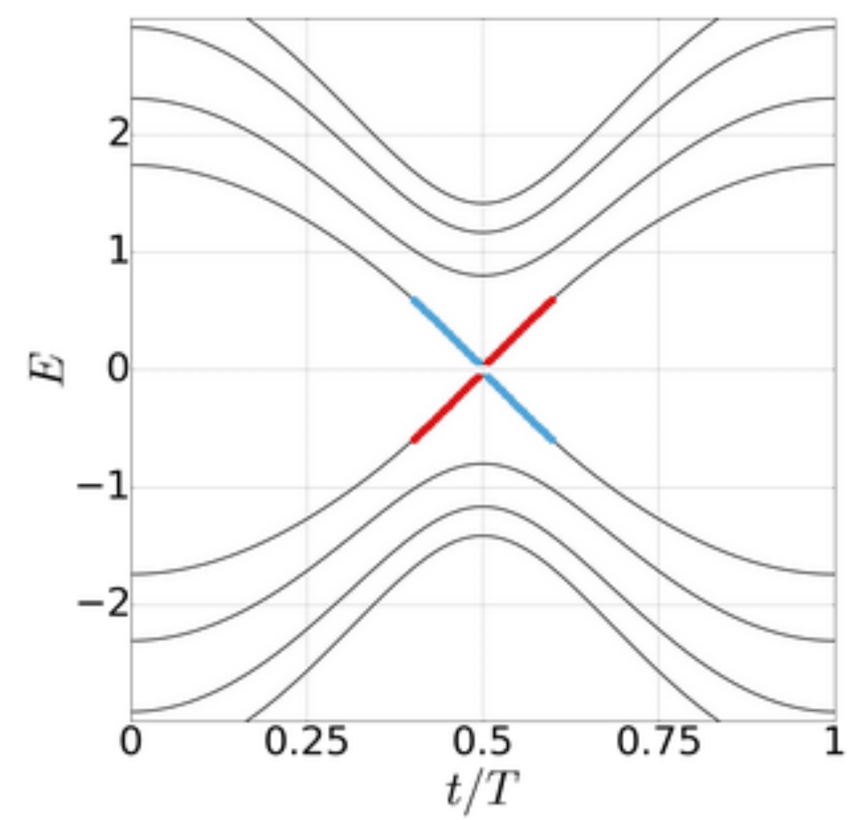
Adiabatic pumping in a finite chain II.

The figure represent the $\bar{v} = 1.5$ case of the pump sequence defined by

$$
\begin{aligned}
u(t) &= \sin(2\pi t/T), \\
v(t) &= \bar{v} + \cos(2\pi t/T), \\
w(t) &= 1,
\end{aligned}
$$

in the finite-sized Rice-Mele model with N=4 unit cells.

Assume that initially the electronic system is in its ground state: the four electrons occupy the negative-energy states. At least how many cycles should be completed to arrive to this ground state again?



a) 1
b) 2
c) 4
d) 8