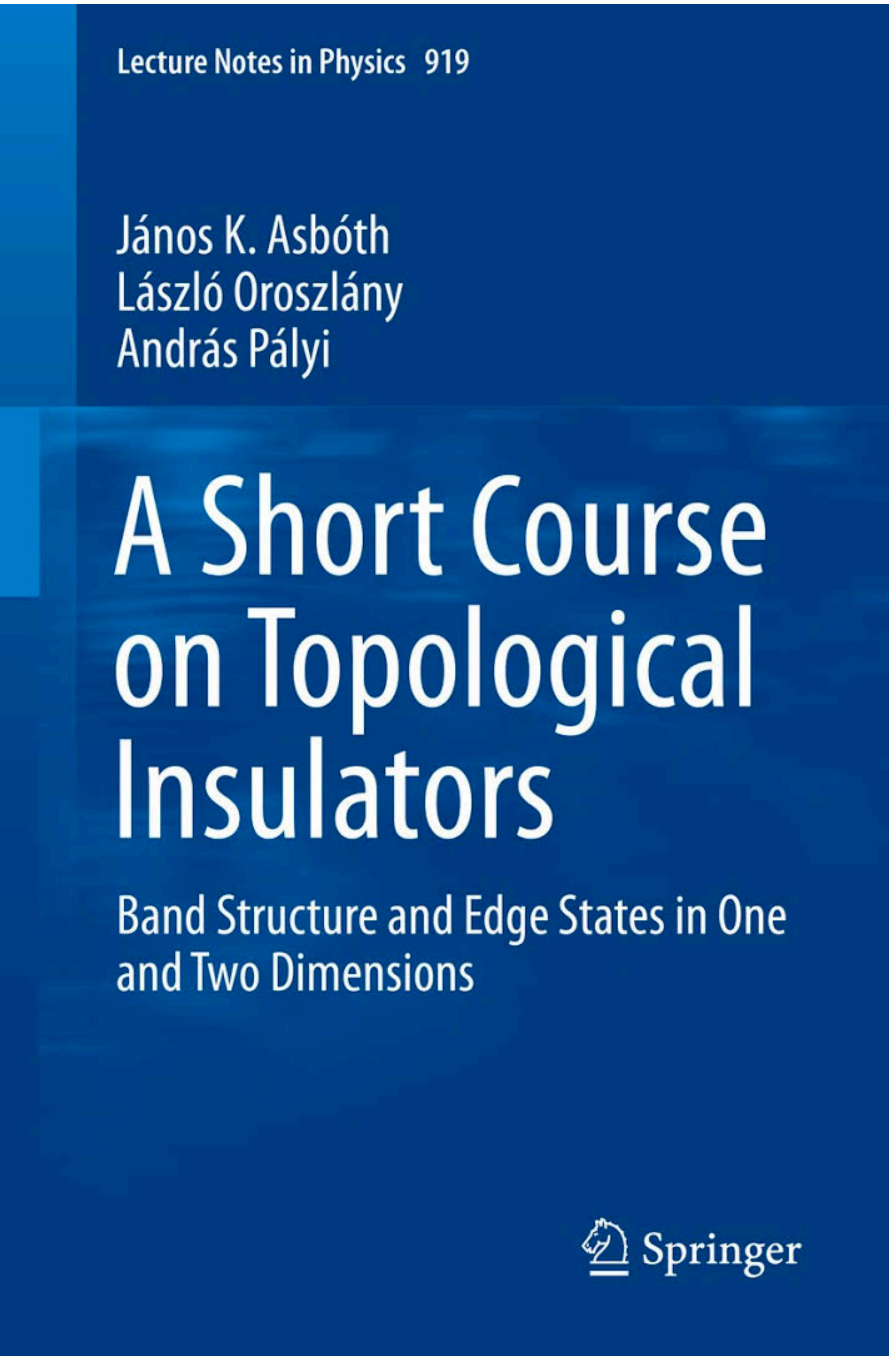
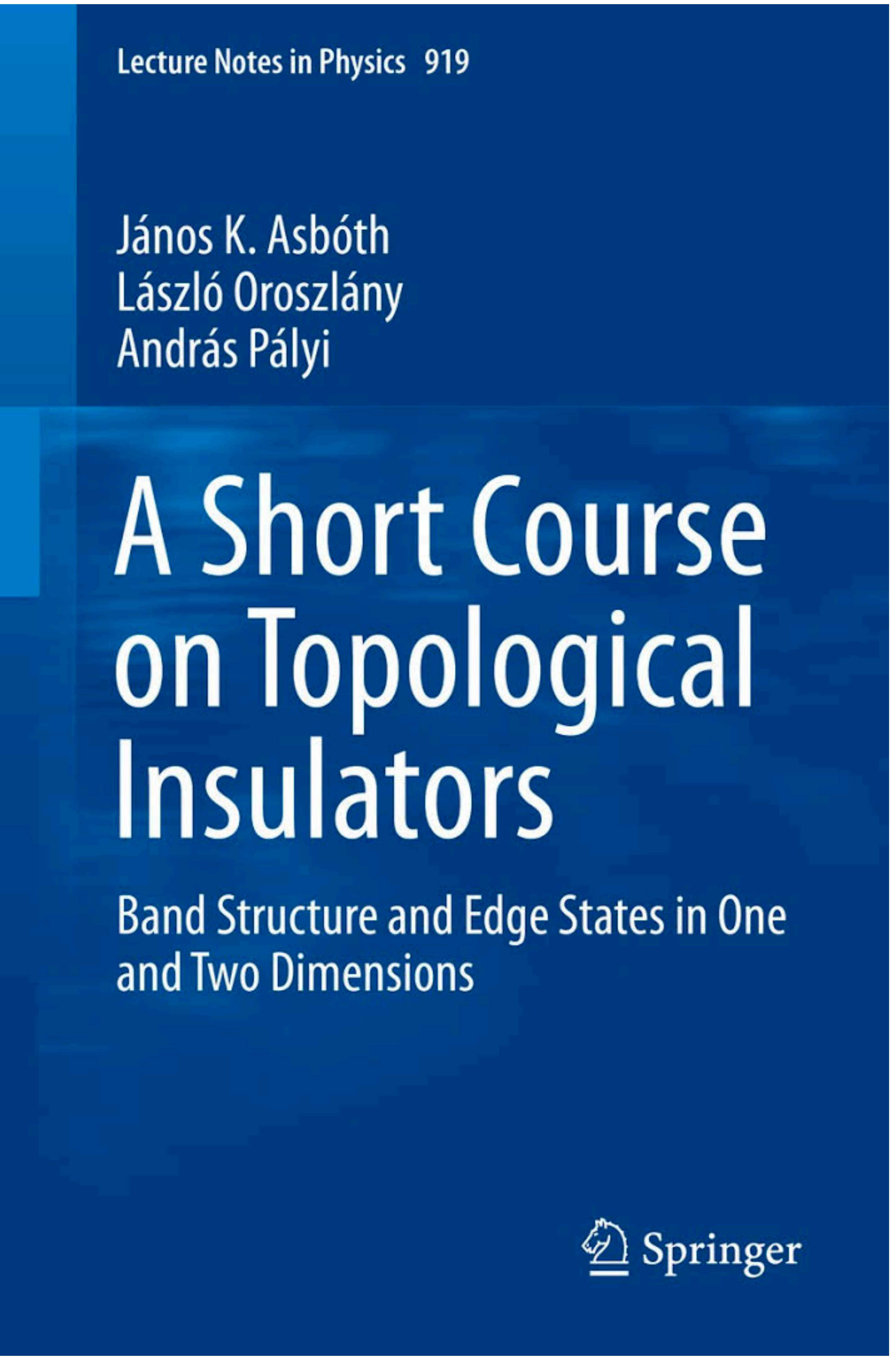
For next week, please read Chapter 5



Today


Lecture Notes in Physics 919
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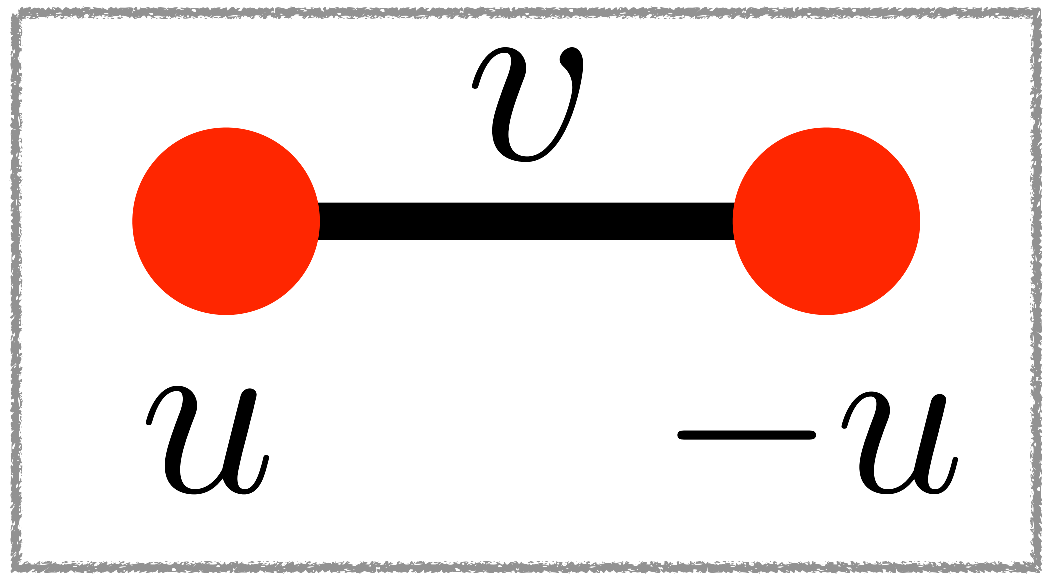
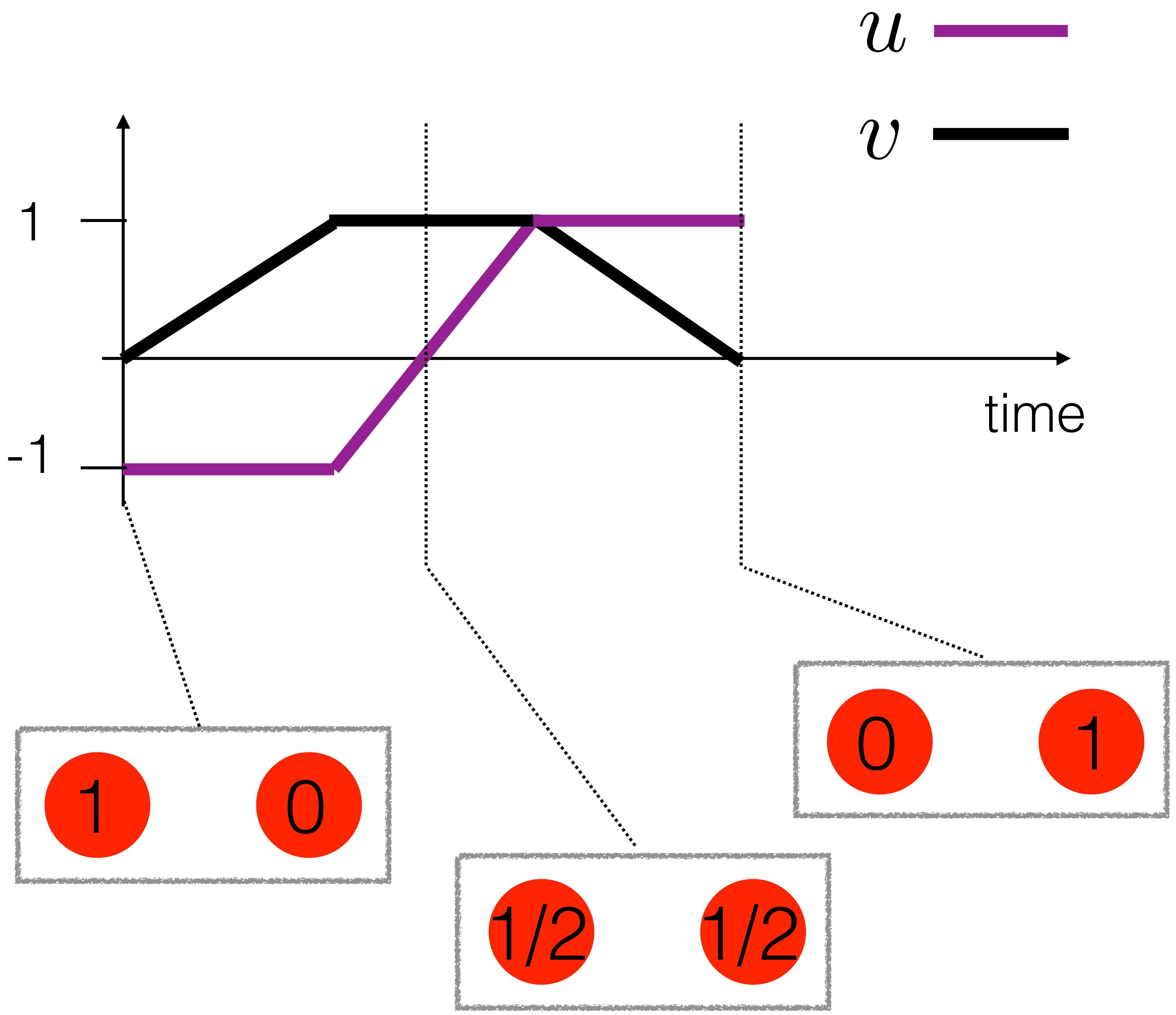
László Oroszlány
András Pályi
A Short Course on Topological Insulators
Band Structure and Edge States in One and Two Dimensions
Springer

Adiabatic pumping with two sites

- two sites
- single electron
- ground state



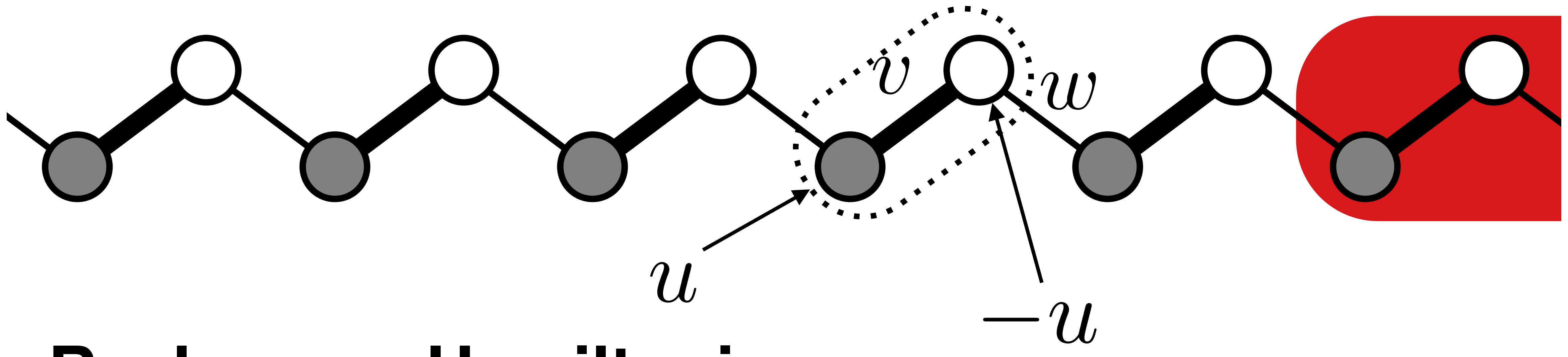


$$H(t) = \begin{pmatrix} u(t) & v(t) \\ v(t) & -u(t) \end{pmatrix}$$

This protocol pumps a single charge from one site to the other

Adiabatic pumping in a lattice

Example: time-dependent Rice-Mele model



$$H = \begin{pmatrix} u & v & 0 & 0 & 0 & 0 & 0 & 0 \\ v & -u & w & 0 & 0 & 0 & 0 & 0 \\ 0 & w & u & v & 0 & 0 & 0 & 0 \\ 0 & 0 & v & -u & w & 0 & 0 & 0 \\ 0 & 0 & 0 & w & u & v & 0 & 0 \\ 0 & 0 & 0 & 0 & v & -u & v & 0 \\ 0 & 0 & 0 & 0 & 0 & w & u & v \\ 0 & 0 & 0 & 0 & 0 & 0 & v & -u \end{pmatrix}$$

Real-space Hamiltonian:

$$\hat{H}(t) = v(t)\sum_{m=1}^{N}\left(|m,B\rangle\langle m,A| + h.c.\right) + w(t)\sum_{m=1}^{N-1}\left(|m+1,A\rangle\langle m,B| + h.c.\right)$$

$$+u(t)\sum_{m=1}^{N}\left(|m,A\rangle\langle m,A| - |m,B\rangle\langle m,B|\right),$$

Momentum-space Hamiltonian:

$$\hat{H}(k,t) = \mathbf{d}(k,t)\hat{\sigma} =$$

$$= (v(t)+w(t)\cos k)\hat{\sigma}_x$$

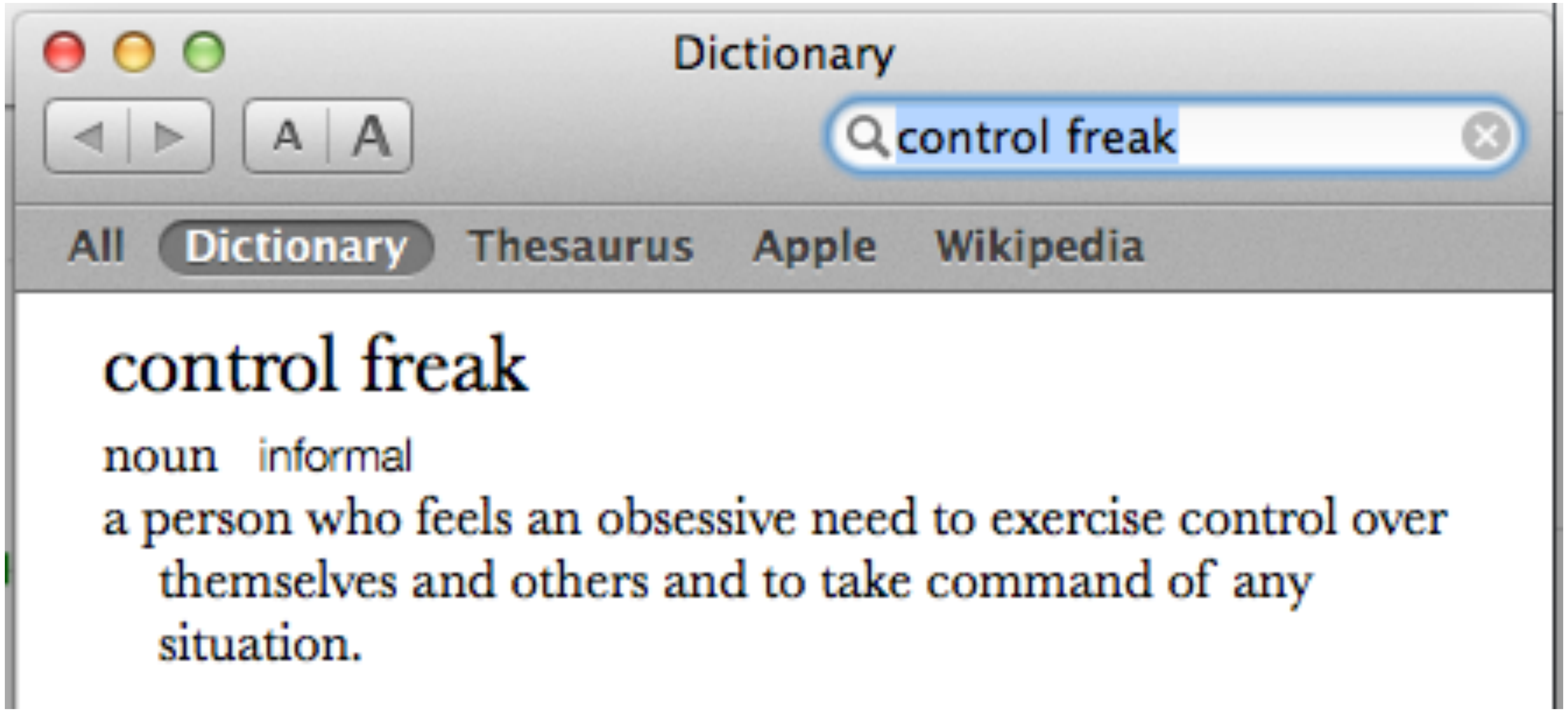
$$+w(t)\sin k\hat{\sigma}_y + u(t)\hat{\sigma}_z,$$

Adiabatic pumping:

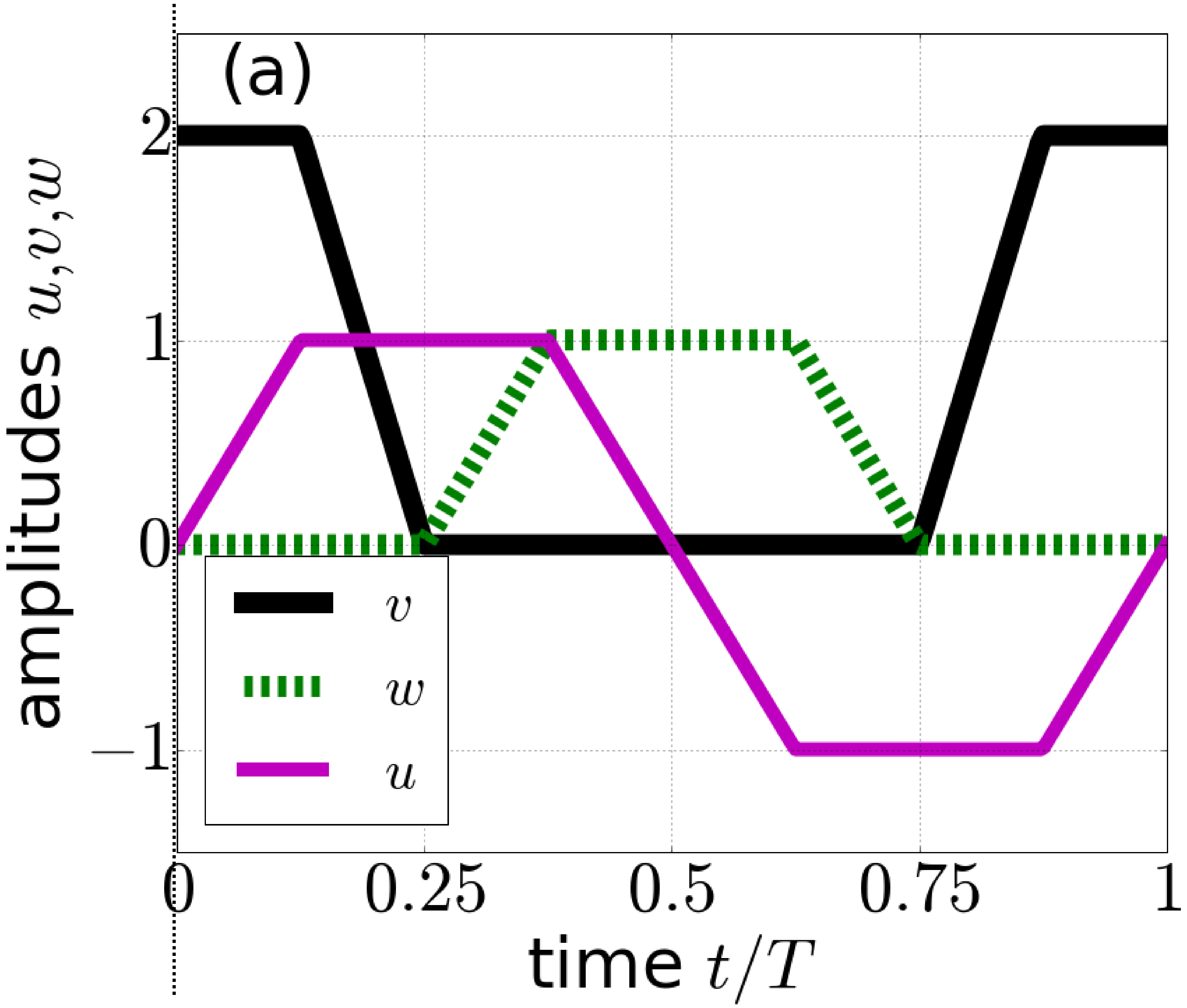
1. Gap doesn't close: $|\boldsymbol{d}| > 0$.
2. Cyclic time depedence (period T)
3. $T \to \infty$

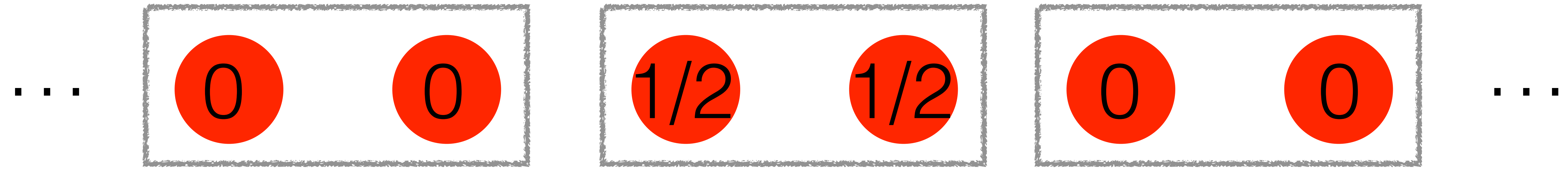
How much charge is pumped through a cross section?

A control-freak pumping protocol



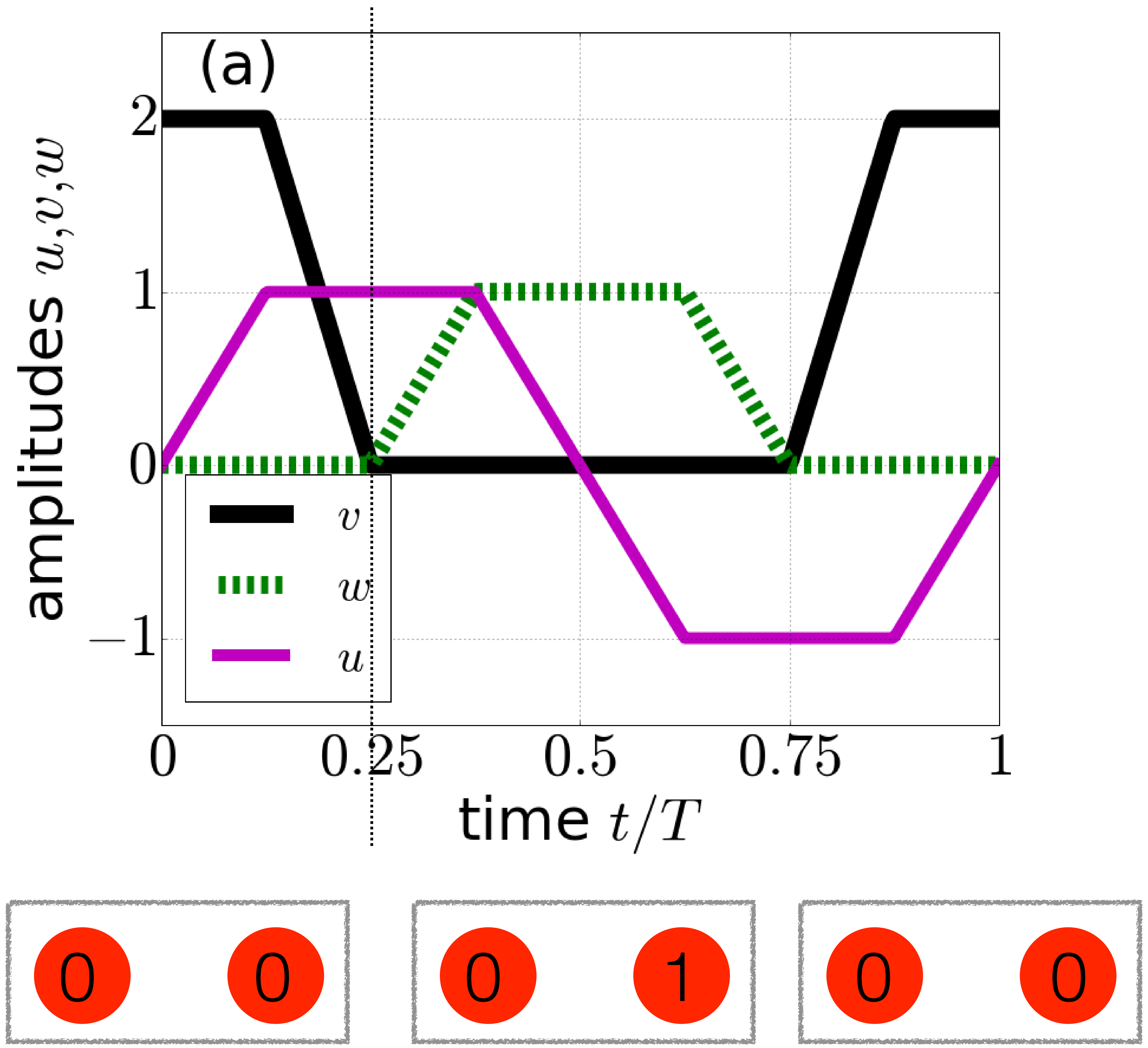
A control-freak pumping protocol





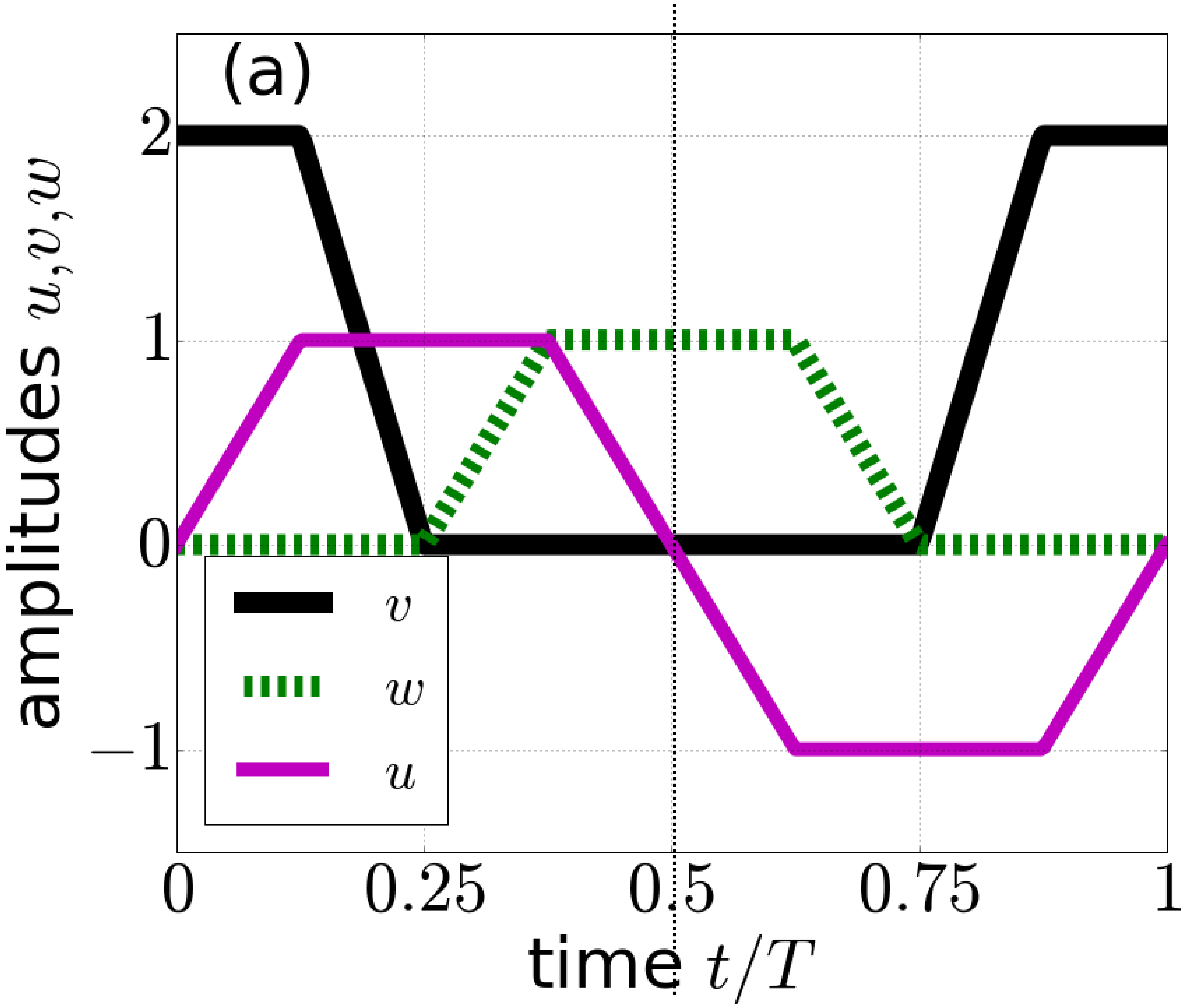
This scheme pumps 1 electron per cycle

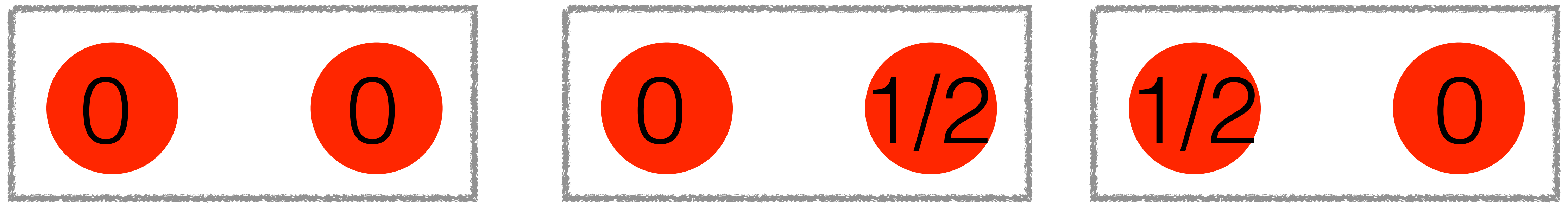
A control-freak pumping scheme



This scheme pumps 1 electron per cycle

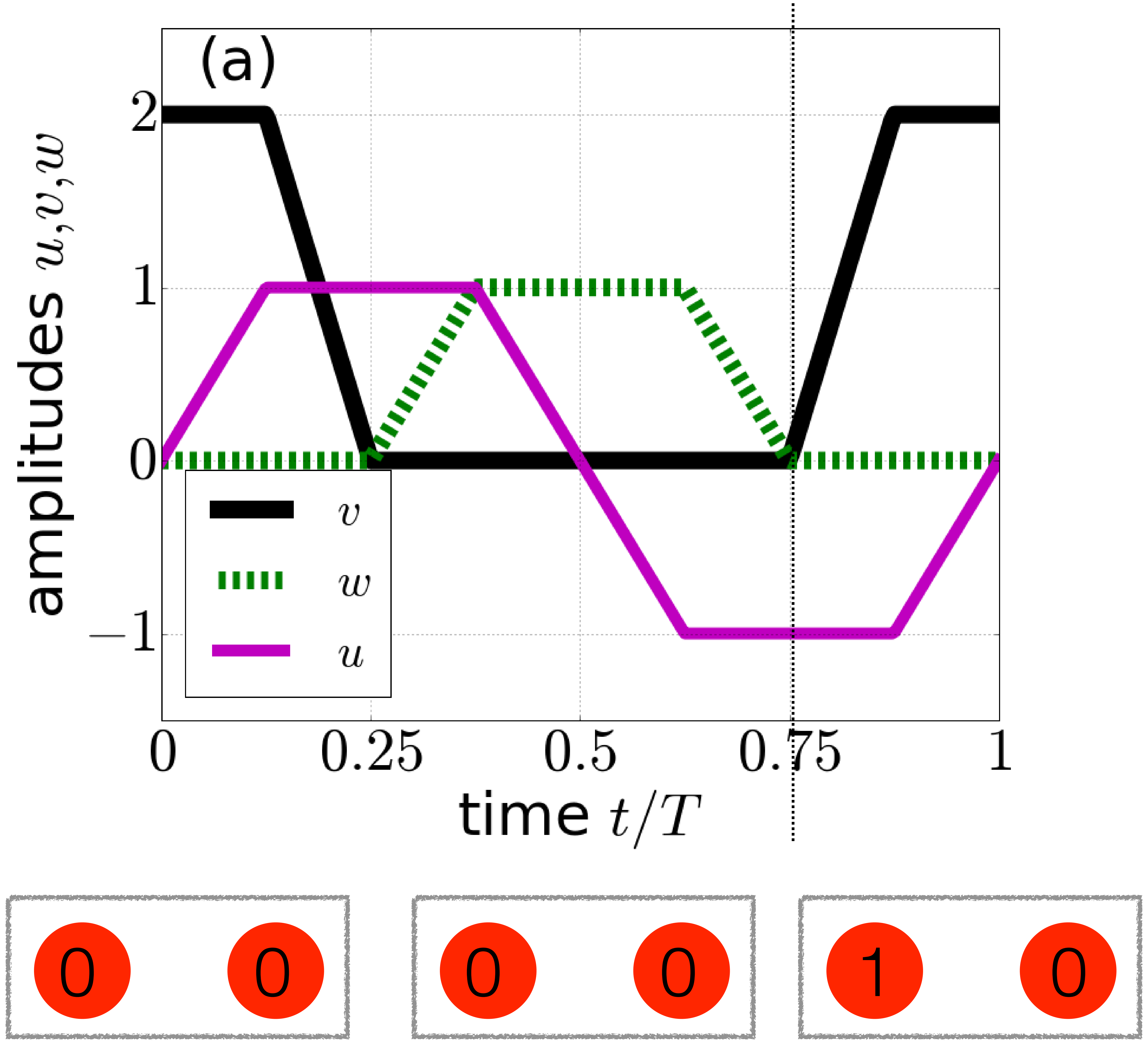
A control-freak pumping scheme





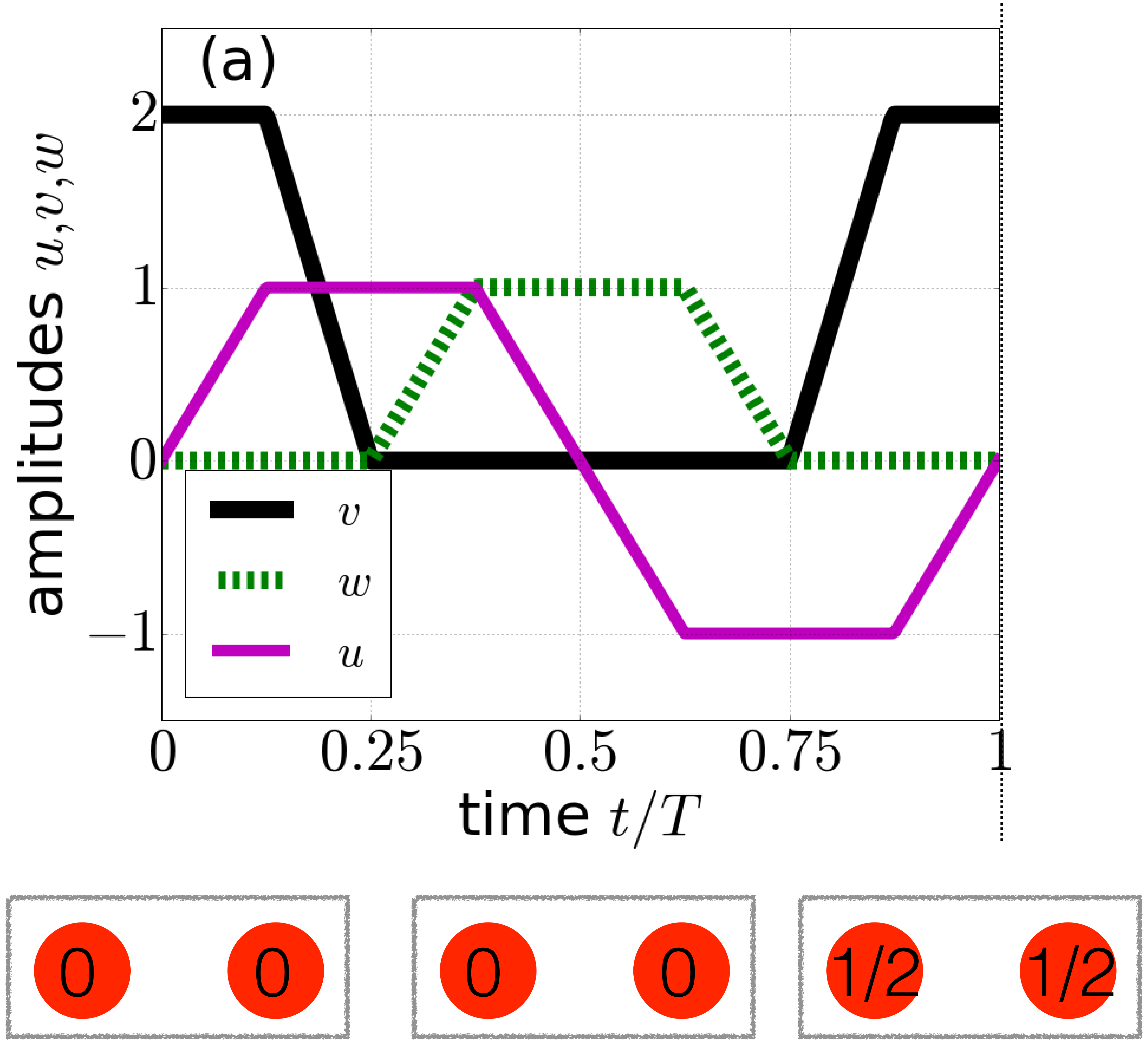
This scheme pumps 1 electron per cycle

A control-freak pumping scheme



This scheme pumps 1 electron per cycle

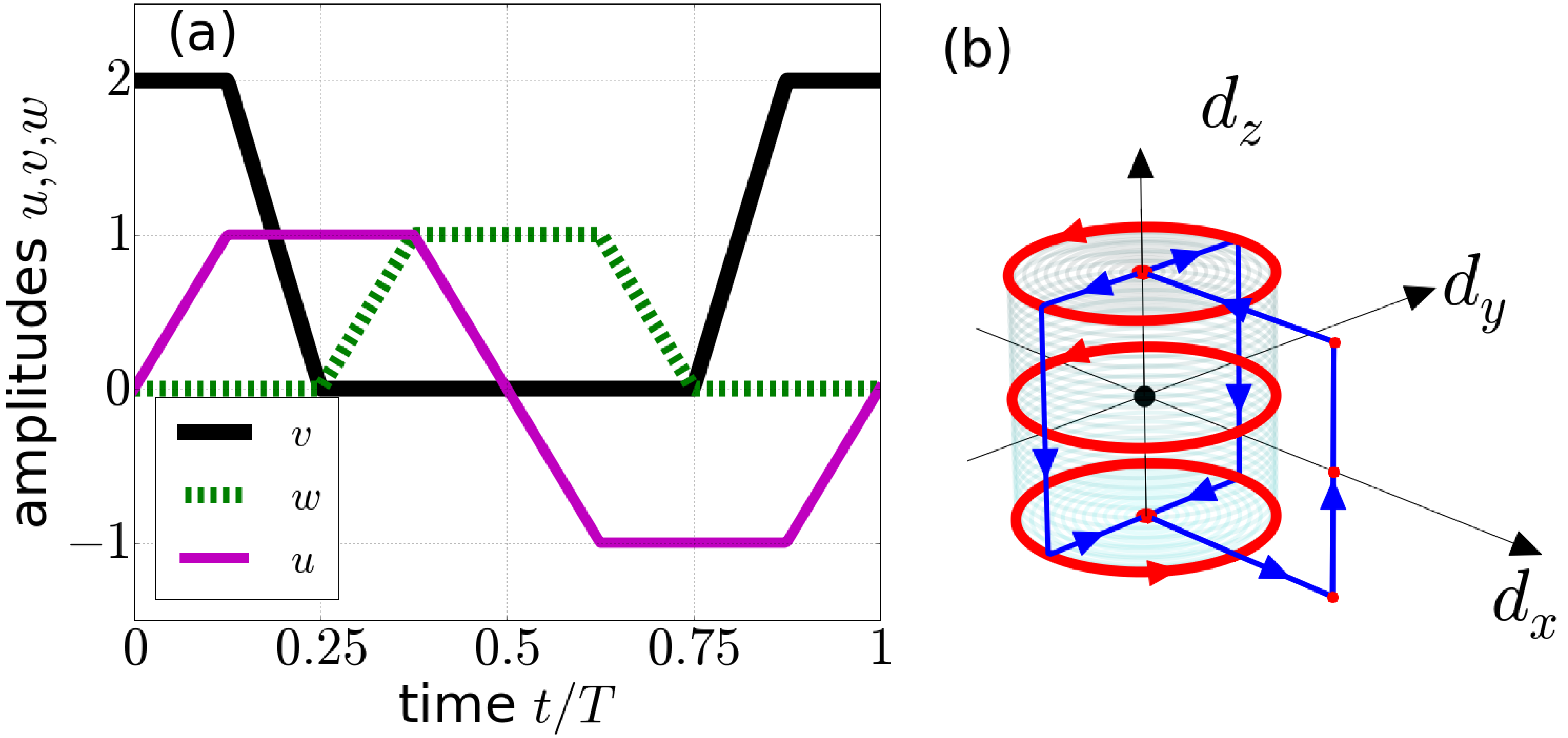
A control-freak pumping scheme



This scheme pumps 1 electron per cycle

The d-vector draws a torus

$$\hat{H}(k,t) = \mathbf{d}(k,t)\hat{\boldsymbol{\sigma}} = (v(t) + w(t)\cos k)\hat{\sigma}_x + w(t)\sin k\hat{\sigma}_y + u(t)\hat{\sigma}_z,$$



$$\boldsymbol{d}(k,t) = \begin{pmatrix} v(t) + w\cos(k) \\ w(t)\sin(k) \\ u(t) \end{pmatrix}$$

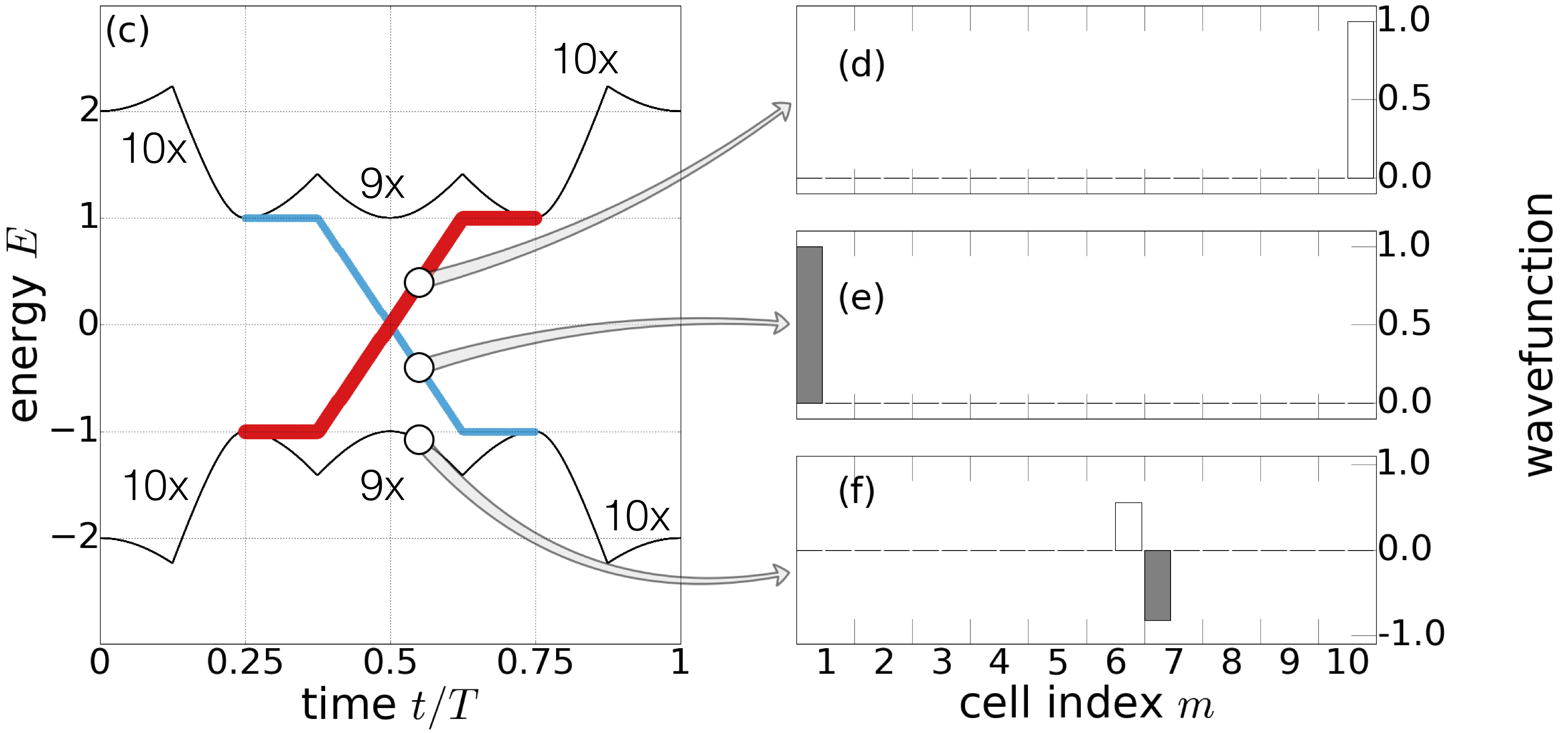
Topological invariant (Chern number)

1. Sit at $\boldsymbol{d} = 0$.
2. Go to infinity along a straight line.
3. Count the number of intersections with the torus.

Topological invariant of this torus is 1.

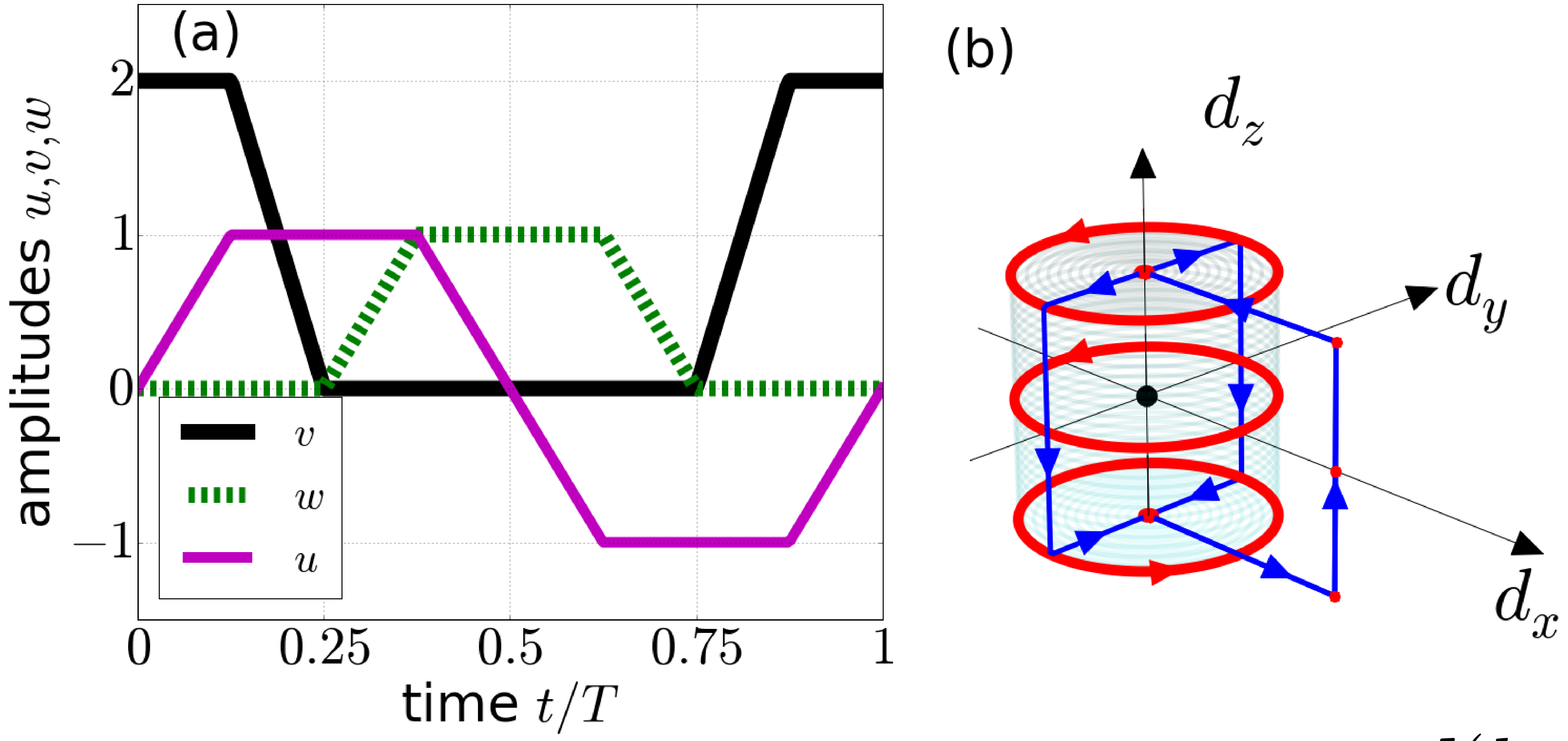
Control-freak pumping in a finite wire

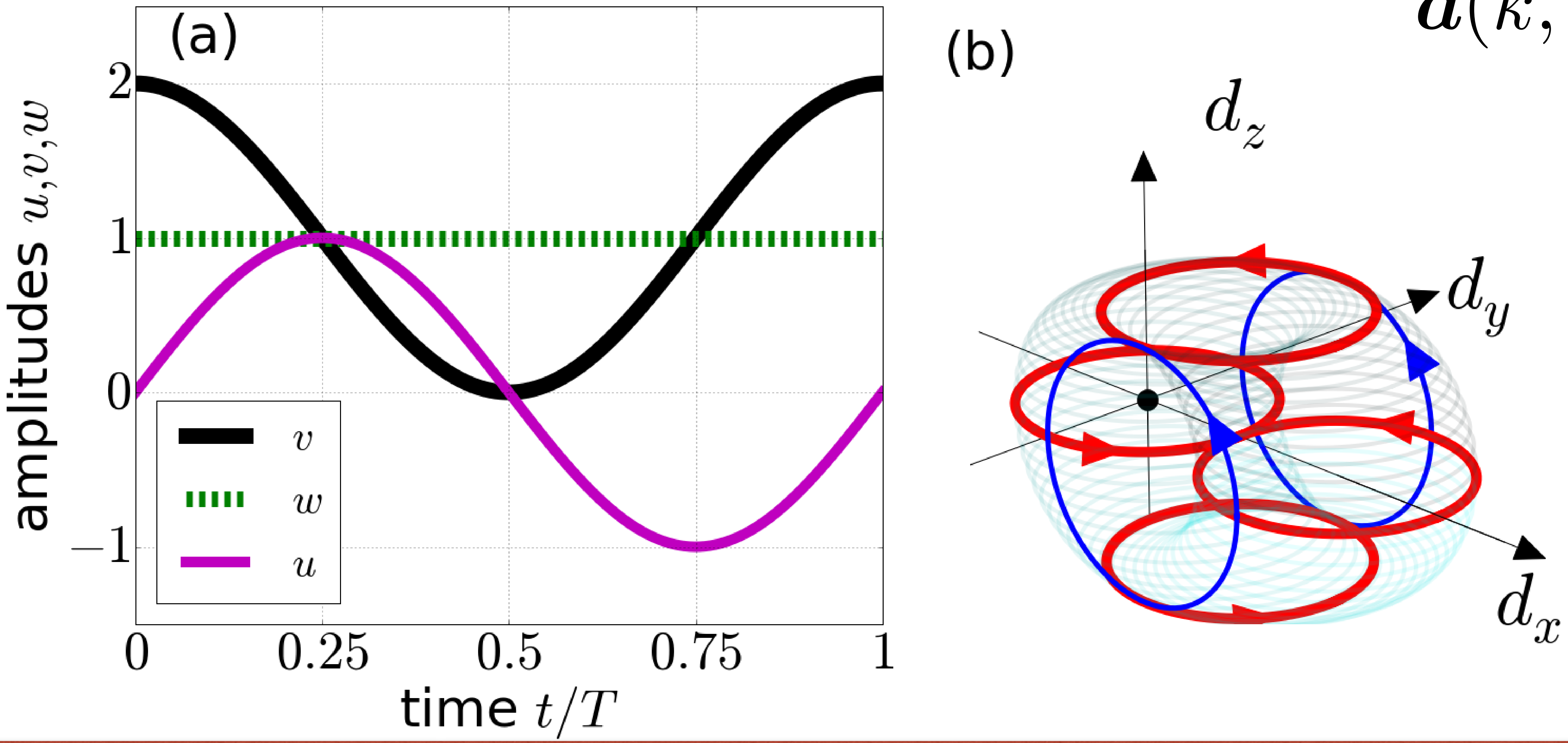
Same pumping cycle as before, with $N = 10$ unit cells.



1 electron is taken from VB to CB per cycle

Moving away from the control-freak limit

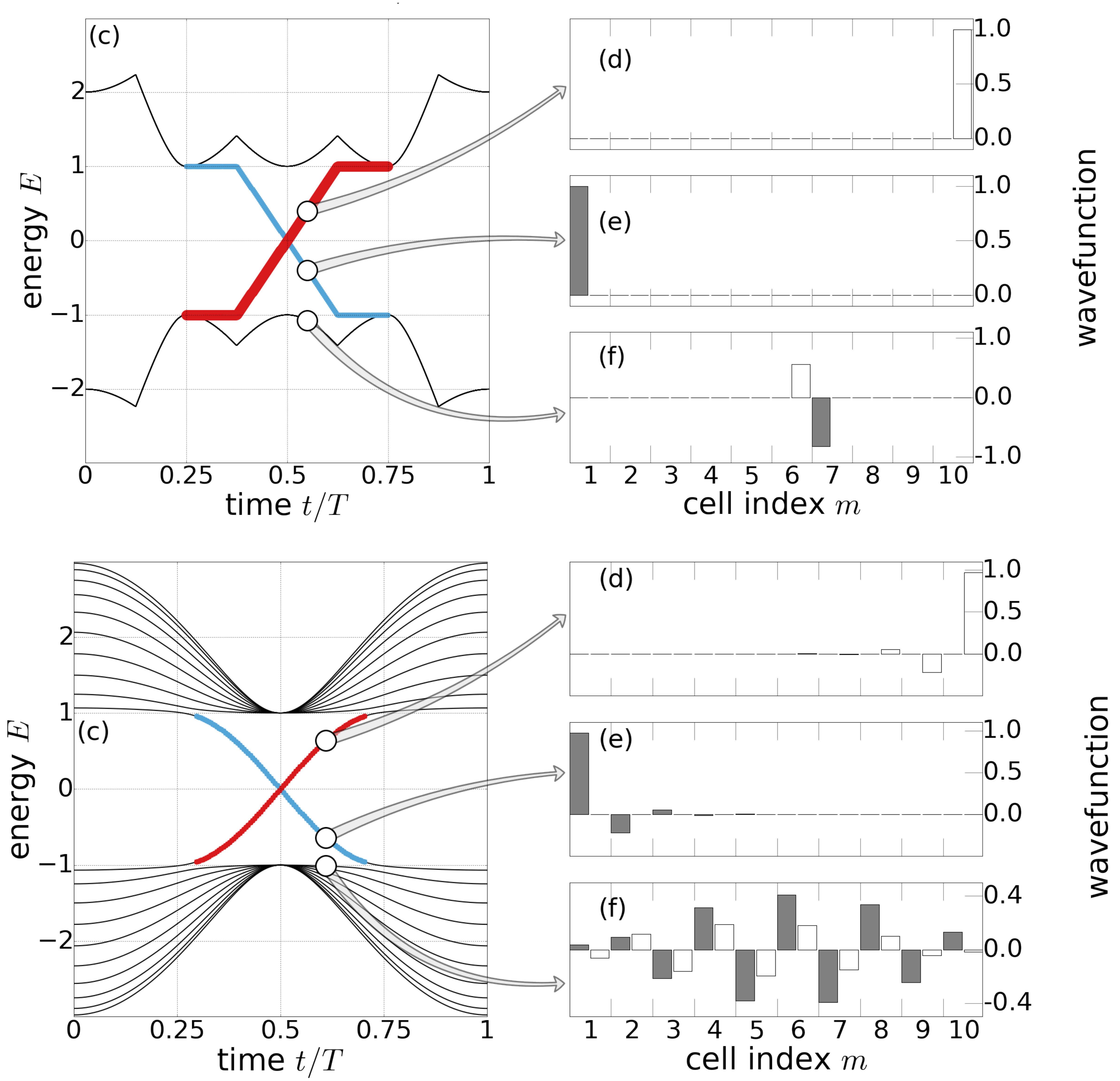




$$\boldsymbol{d}(k,t) = \begin{pmatrix} v(t) + w\cos(k) \\ w(t)\sin(k) \\ u(t) \end{pmatrix}$$

Continuous deformation can't change the topological invariant

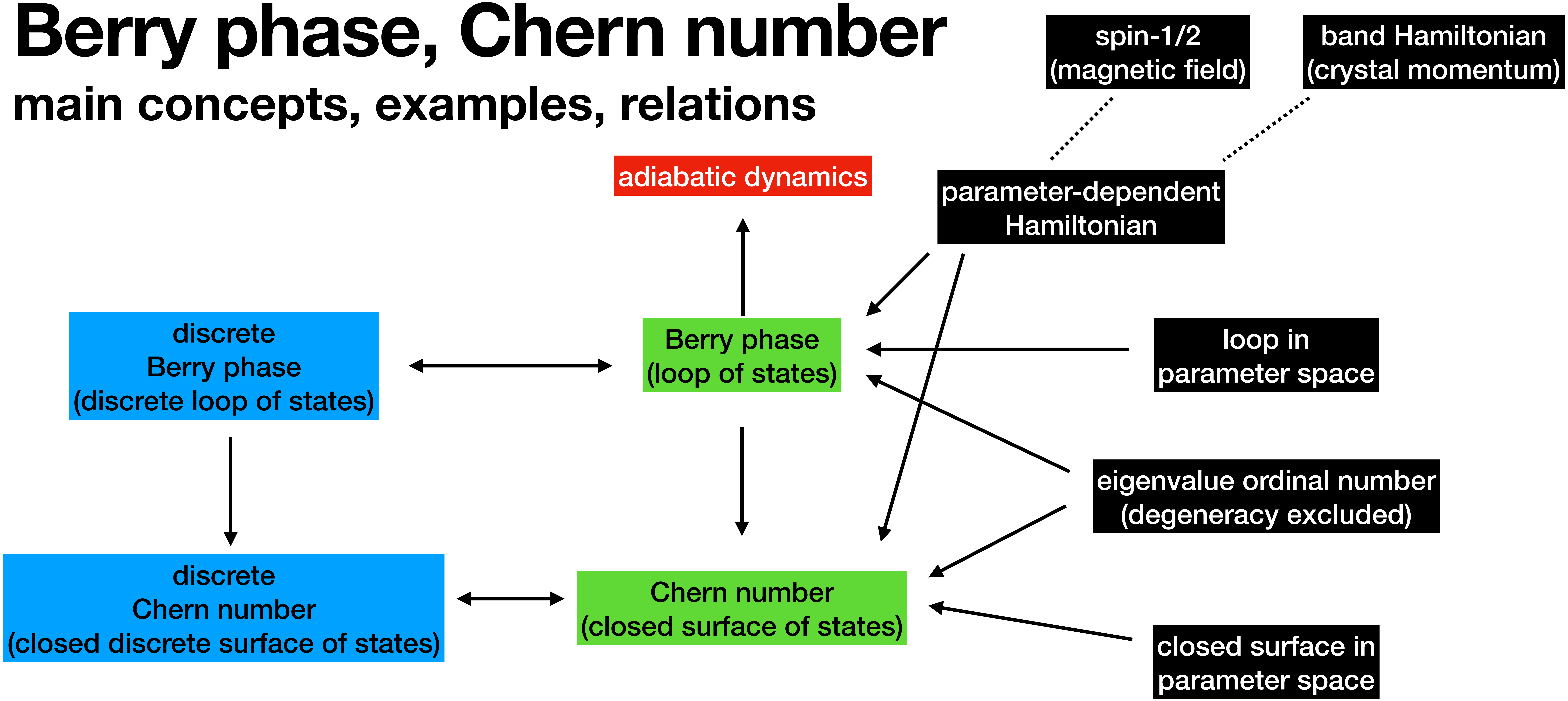
Moving away from the control-freak limit



Continuous deformation can't change the # of edge states

Berry phase, Chern number

main concepts, examples, relations



remark: terminology: *Berry phase factor* lives on the complex unit circle, *Berry phase* lives in]-pi,pi] or [0,2pi[

Polarization and Berry phase

start with concepts from last week, think of 1D and 2-band tight binding models