

For next week, please read Chapter 5

Lecture Notes in Physics 919

János K. Asbóth  
László Oroszlány  
András Pályi

# A Short Course on Topological Insulators

Band Structure and Edge States in One  
and Two Dimensions

 Springer

<b>5</b>	<b>Current operator and particle pumping</b> .....	<b>65</b>
5.1	Particle current at a cross section of the lattice .....	66
5.2	Time evolution governed by a quasi-adiabatic Hamiltonian .....	72
5.3	The pumped current is the Berry curvature .....	76

# Today

Lecture Notes in Physics 919

János K. Asbóth  
László Oroszlány  
András Pályi

## A Short Course on Topological Insulators

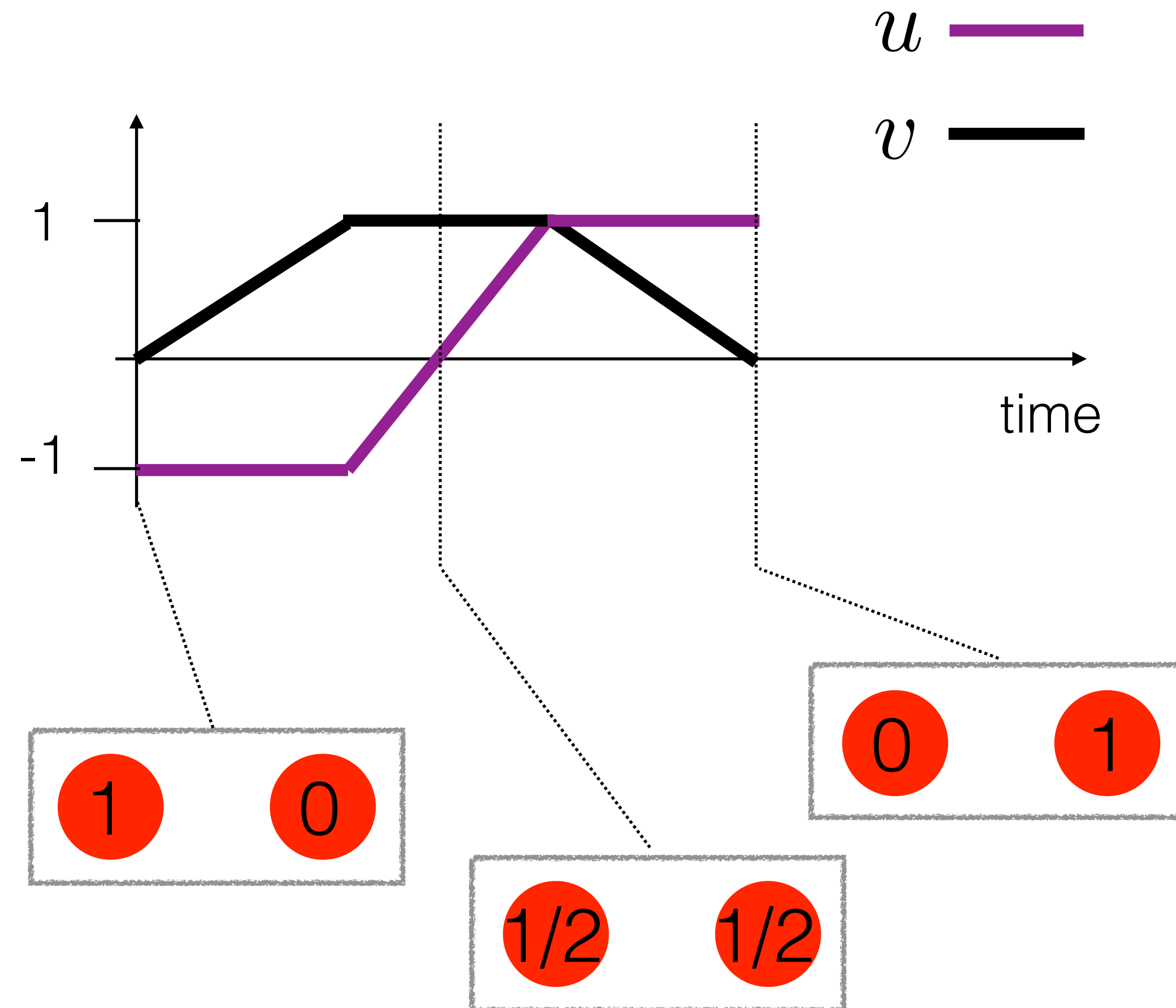
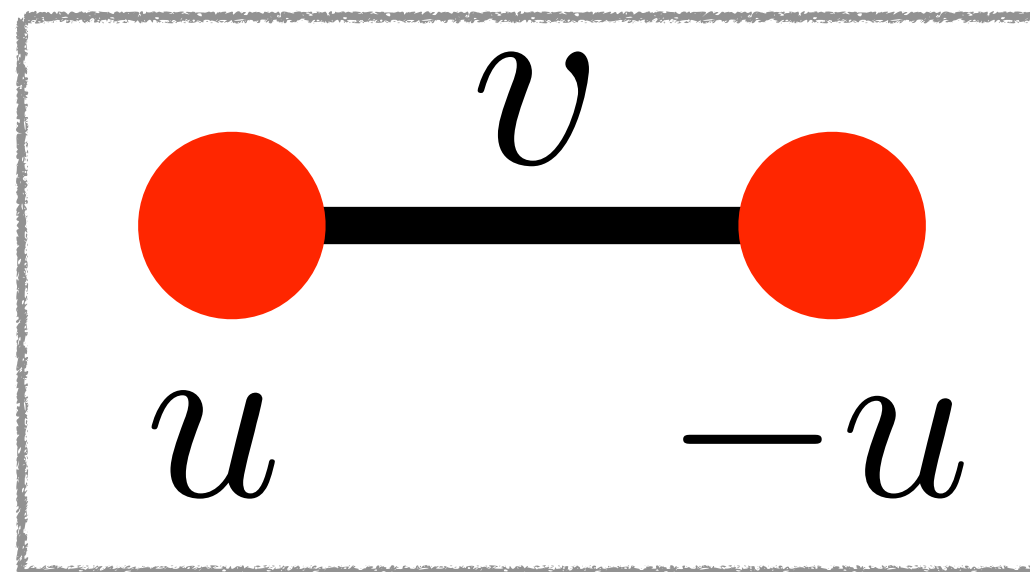
Band Structure and Edge States in One  
and Two Dimensions

 Springer

<b>3</b>	<b>Polarization and Berry phase</b> .....	<b>41</b>
3.1	Wannier states in the Rice-Mele model .....	42
3.2	Inversion symmetry and polarization .....	47
	Problems .....	49
<b>4</b>	<b>Adiabatic charge pumping, Rice-Mele model</b> .....	<b>51</b>
4.1	Charge pumping in a control freak way .....	51
4.2	Moving away from the control freak limit .....	55
4.3	Tracking the charges with Wannier states .....	59
	Problems .....	63

# Adiabatic pumping with two sites

- two sites
- single electron
- ground state

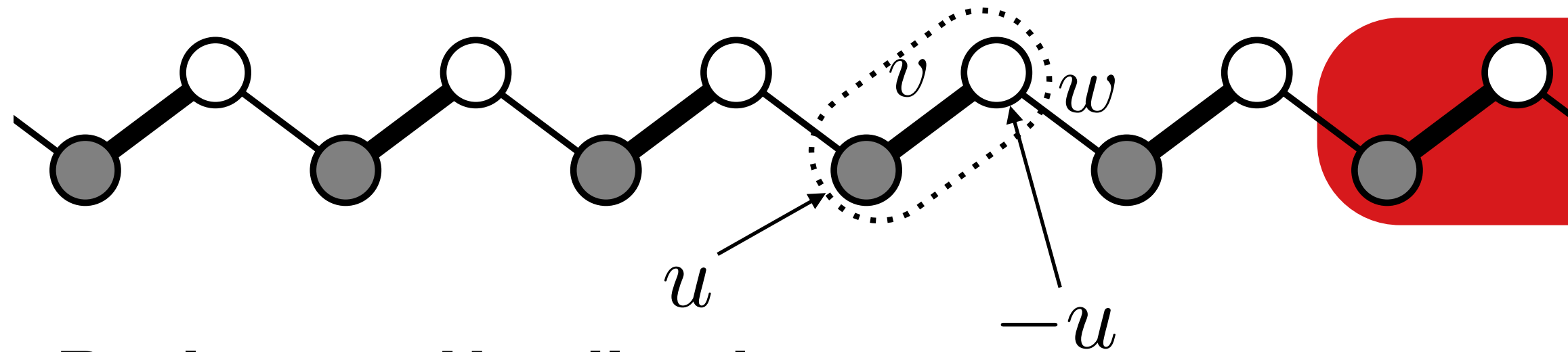


$$H(t) = \begin{pmatrix} u(t) & v(t) \\ v(t) & -u(t) \end{pmatrix}$$

This protocol pumps a single charge from one site to the other

# Adiabatic pumping in a lattice

**Example:** time-dependent Rice-Mele model



$$H = \begin{pmatrix} u & v & 0 & 0 & 0 & 0 & 0 & 0 \\ v & -u & w & 0 & 0 & 0 & 0 & 0 \\ 0 & w & u & v & 0 & 0 & 0 & 0 \\ 0 & 0 & v & -u & w & 0 & 0 & 0 \\ 0 & 0 & 0 & w & u & v & 0 & 0 \\ 0 & 0 & 0 & 0 & v & -u & v & 0 \\ 0 & 0 & 0 & 0 & 0 & w & u & v \\ 0 & 0 & 0 & 0 & 0 & 0 & v & -u \end{pmatrix}$$

**Real-space Hamiltonian:**

$$\hat{H}(t) = v(t) \sum_{m=1}^N (|m, B\rangle \langle m, A| + h.c.) + w(t) \sum_{m=1}^{N-1} (|m+1, A\rangle \langle m, B| + h.c.) \\ + u(t) \sum_{m=1}^N (|m, A\rangle \langle m, A| - |m, B\rangle \langle m, B|),$$

**Momentum-space Hamiltonian:**

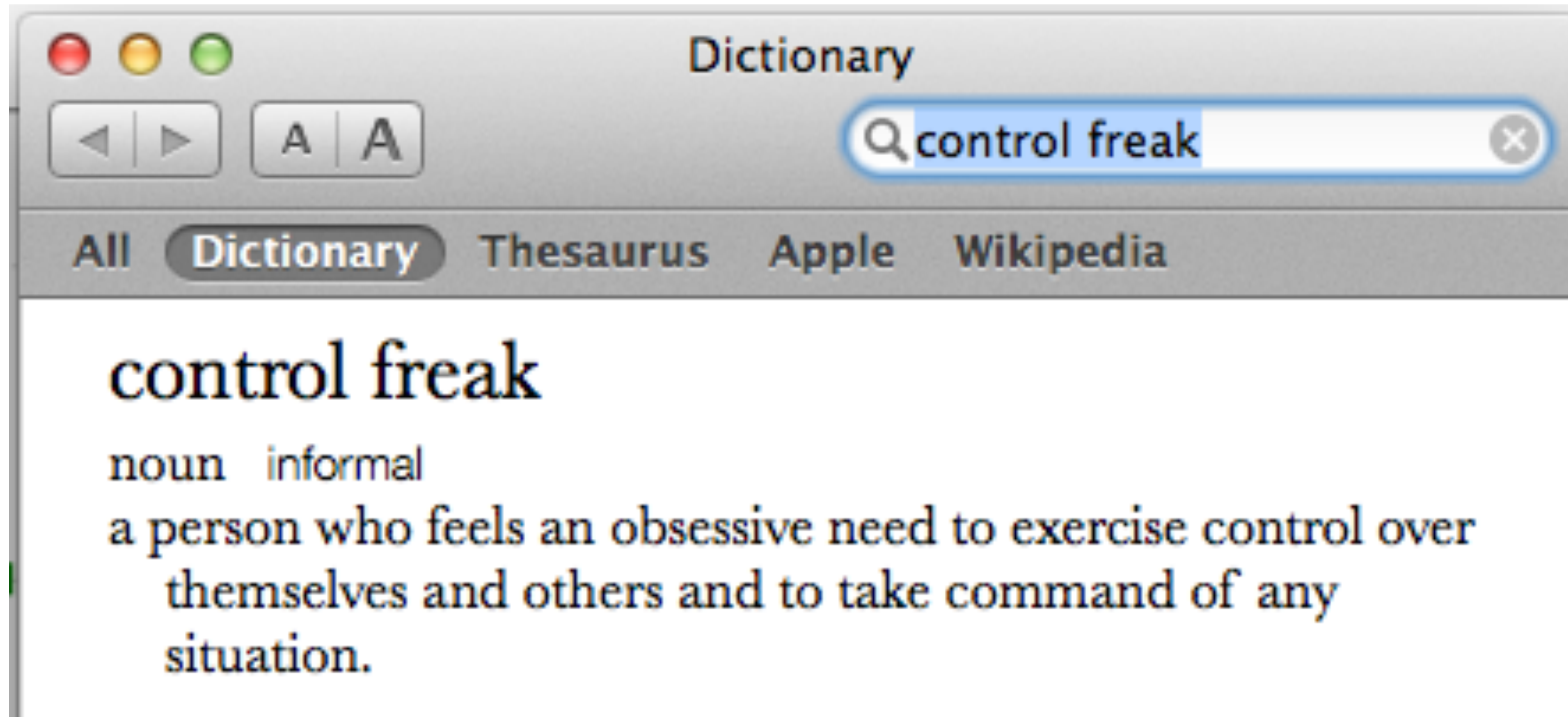
$$\hat{H}(k, t) = \mathbf{d}(k, t) \hat{\sigma} = \\ = (v(t) + w(t) \cos k) \hat{\sigma}_x \\ + w(t) \sin k \hat{\sigma}_y + u(t) \hat{\sigma}_z,$$

**Adiabatic pumping:**

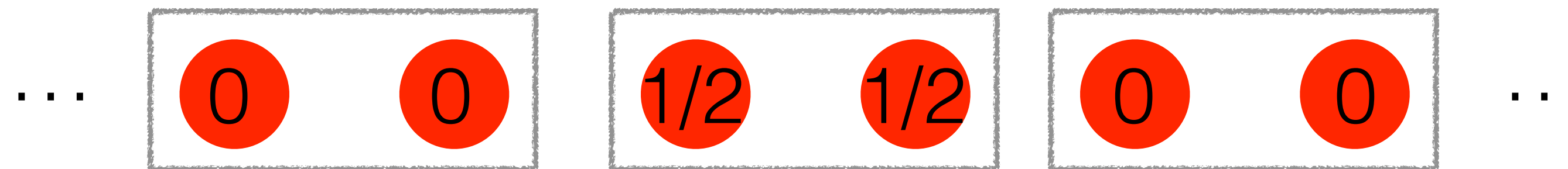
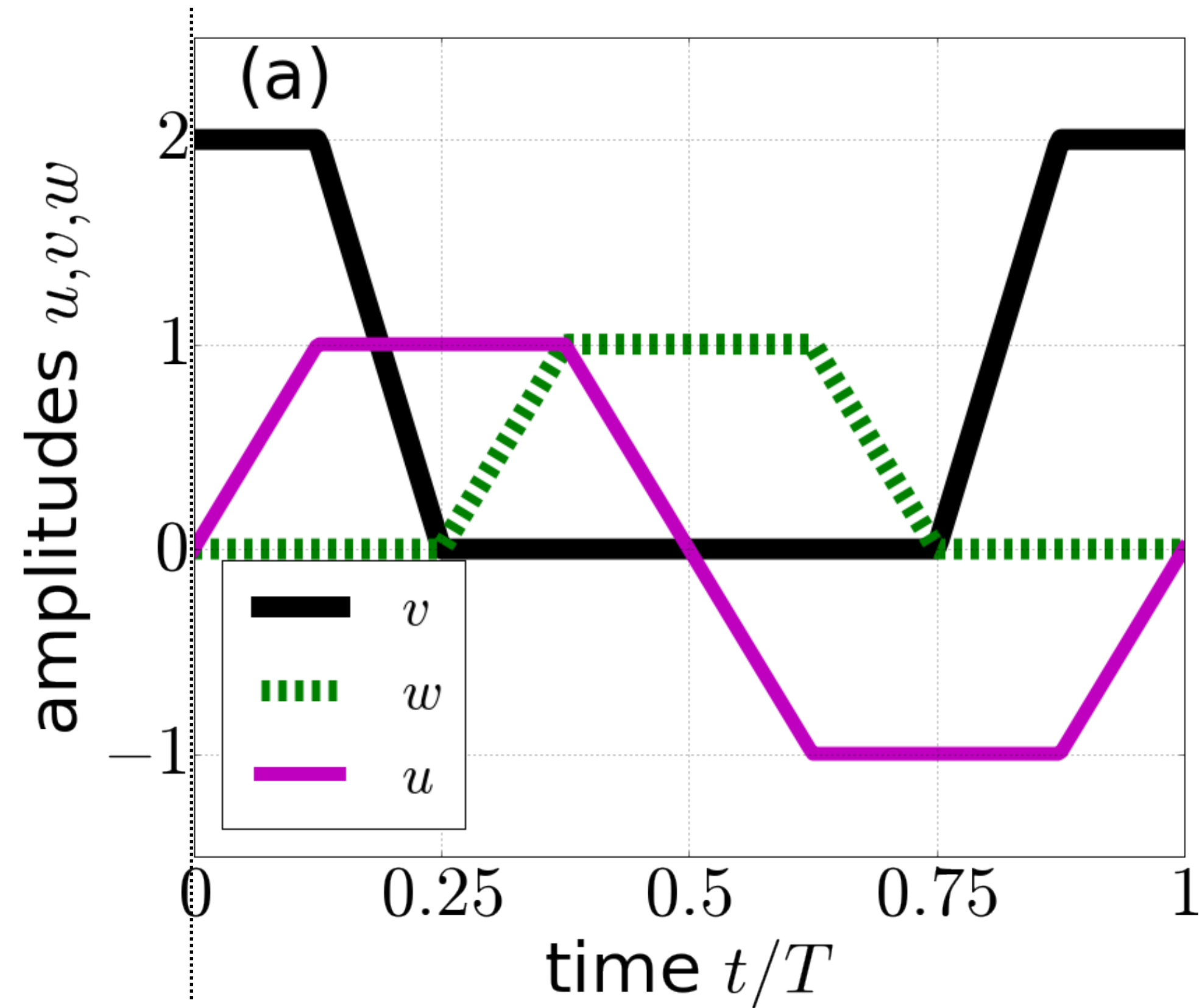
1. Gap doesn't close:  $|\mathbf{d}| > 0$ .
2. Cyclic time dependence (period  $T$ )
3.  $T \rightarrow \infty$

How much charge is pumped through a cross section?

# A control-freak pumping protocol

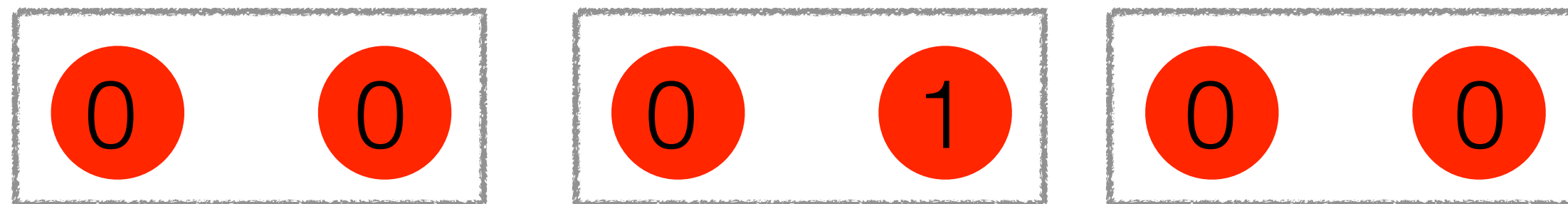
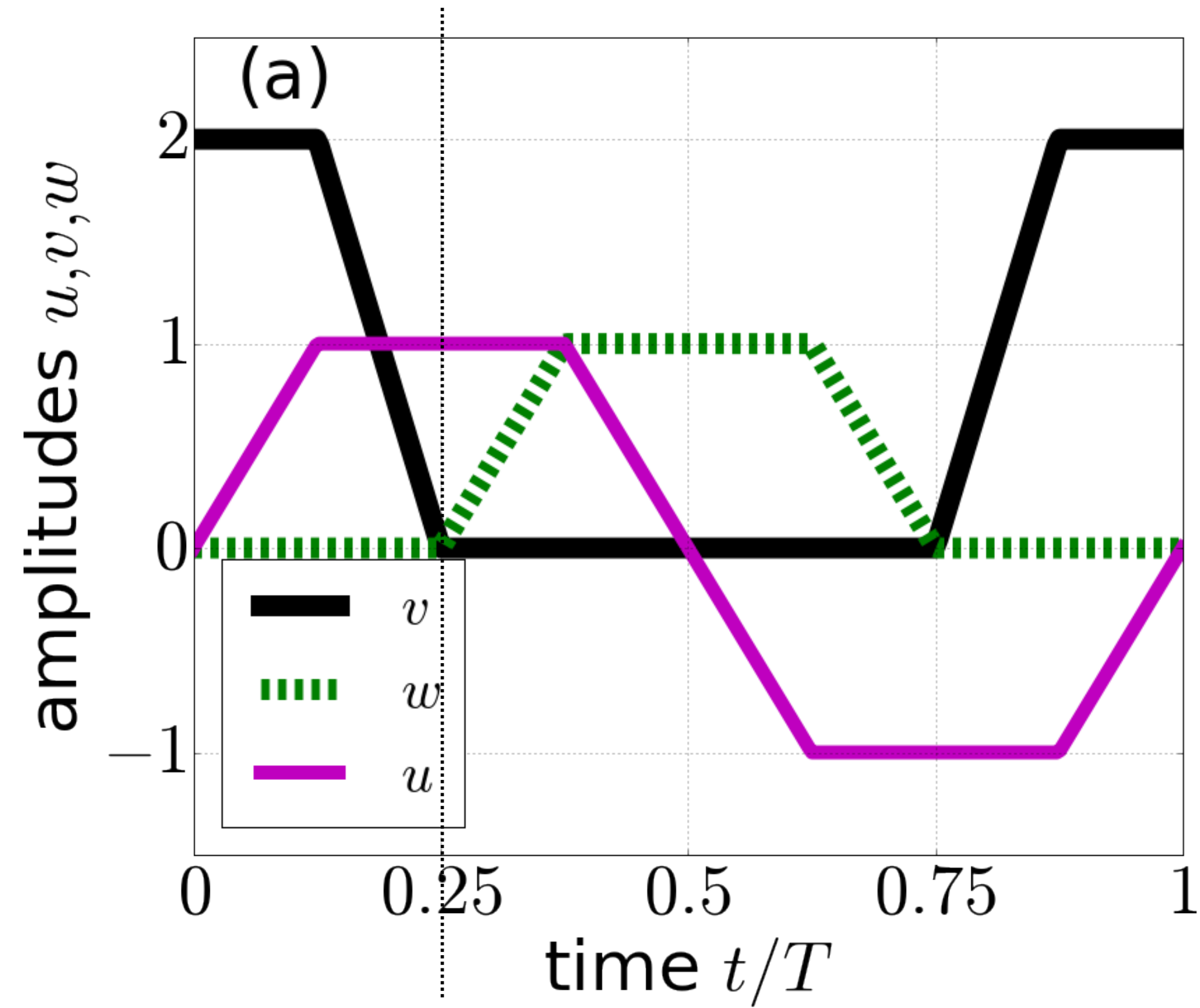


# A control-freak pumping protocol



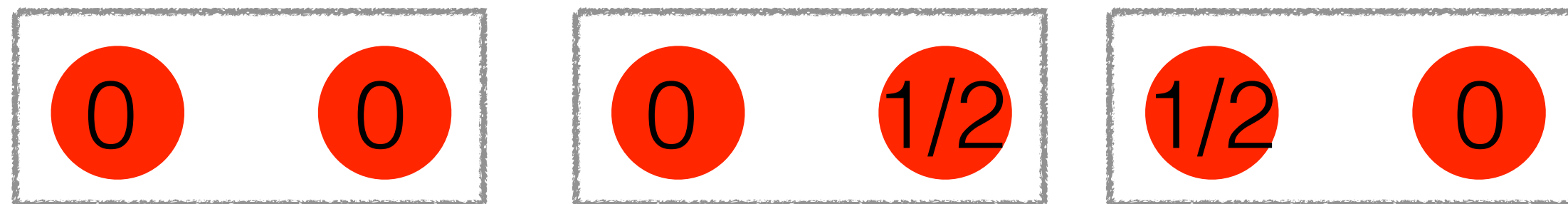
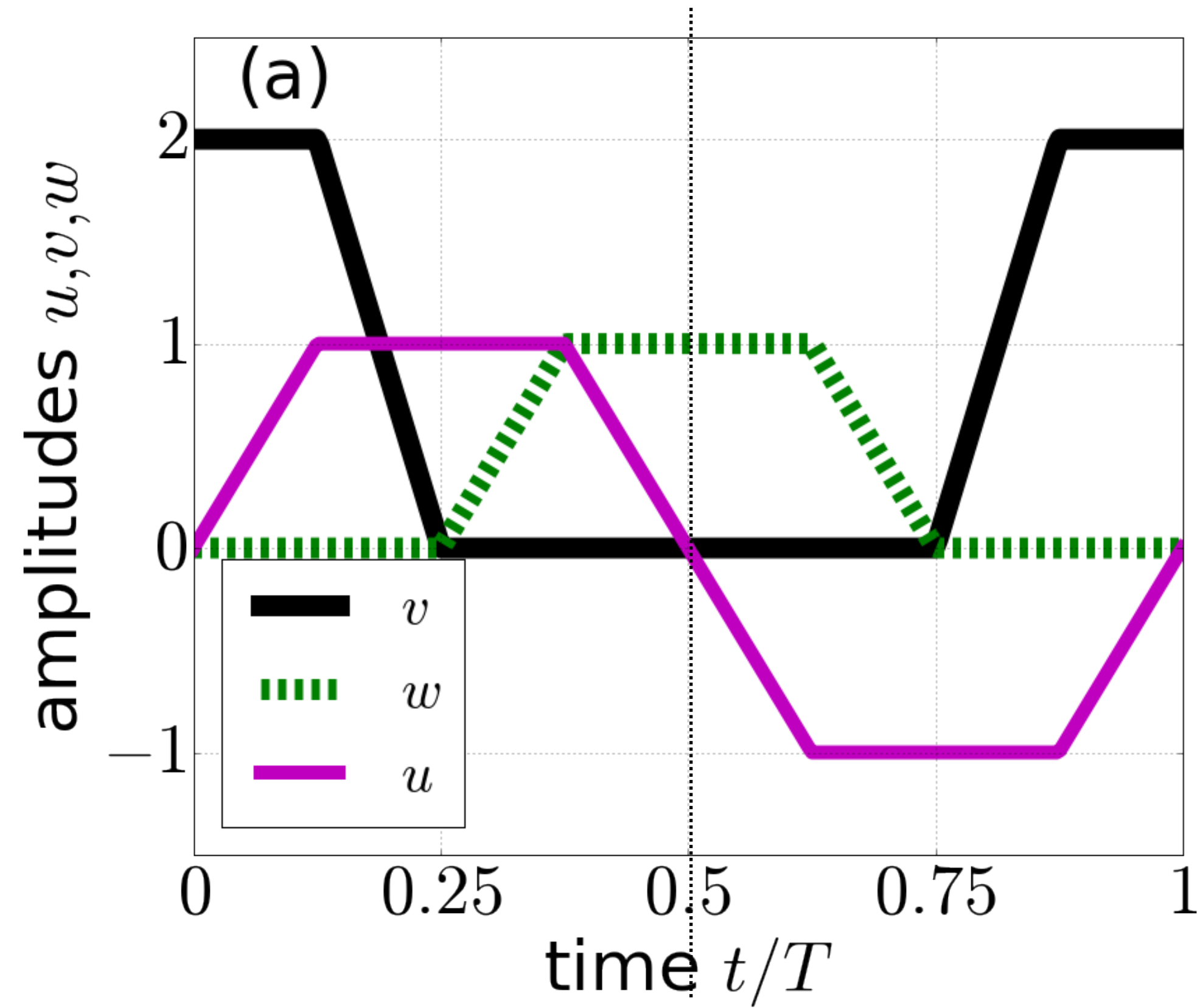
This scheme pumps 1 electron per cycle

# A control-freak pumping scheme



This scheme pumps 1 electron per cycle

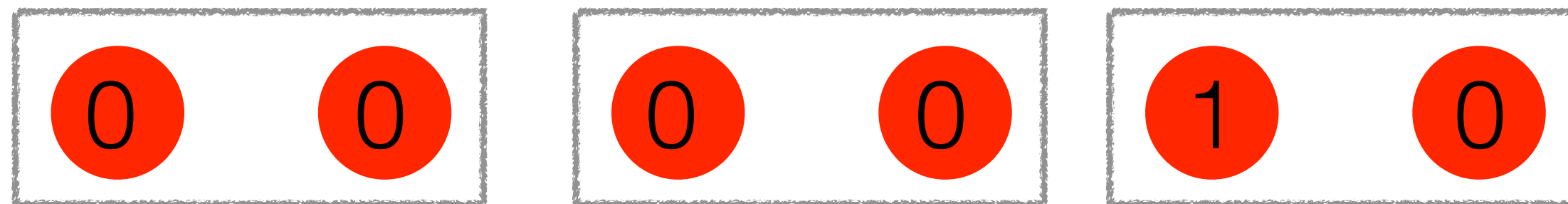
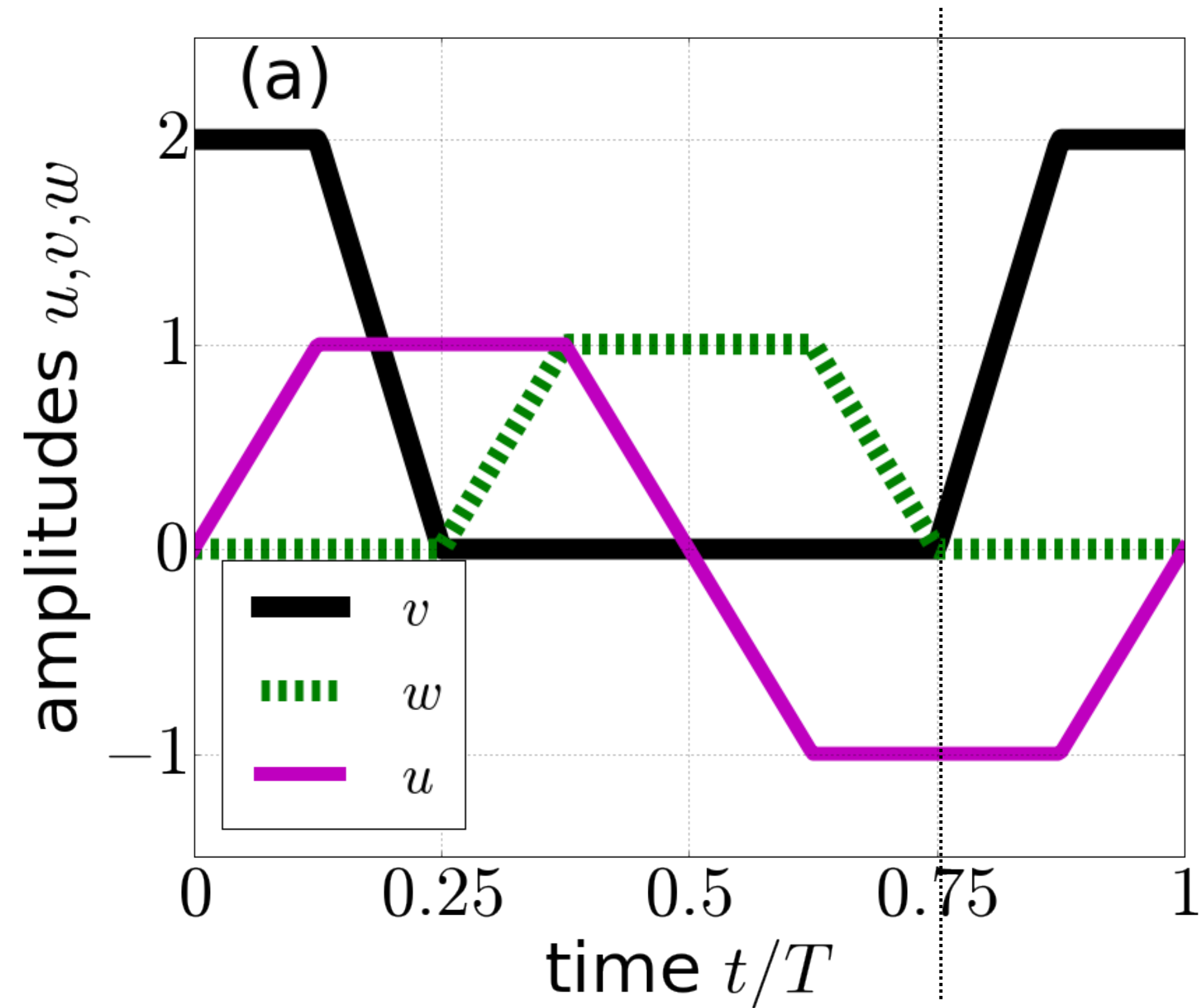
# A control-freak pumping scheme



This scheme pumps 1 electron per cycle

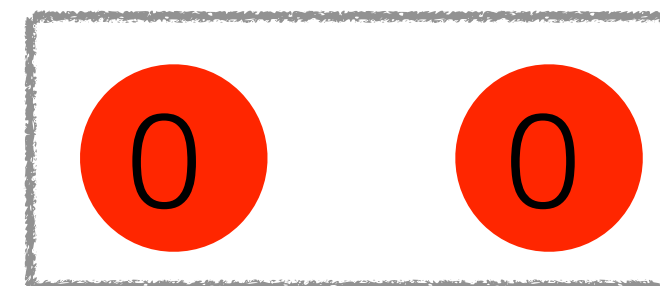
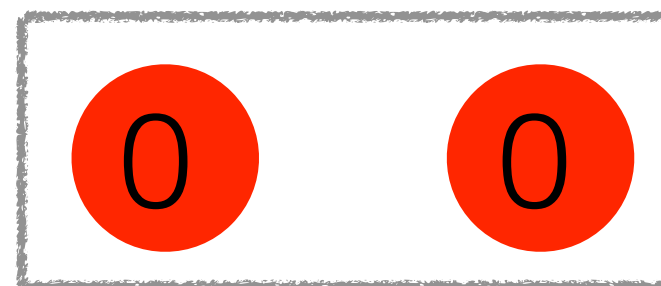
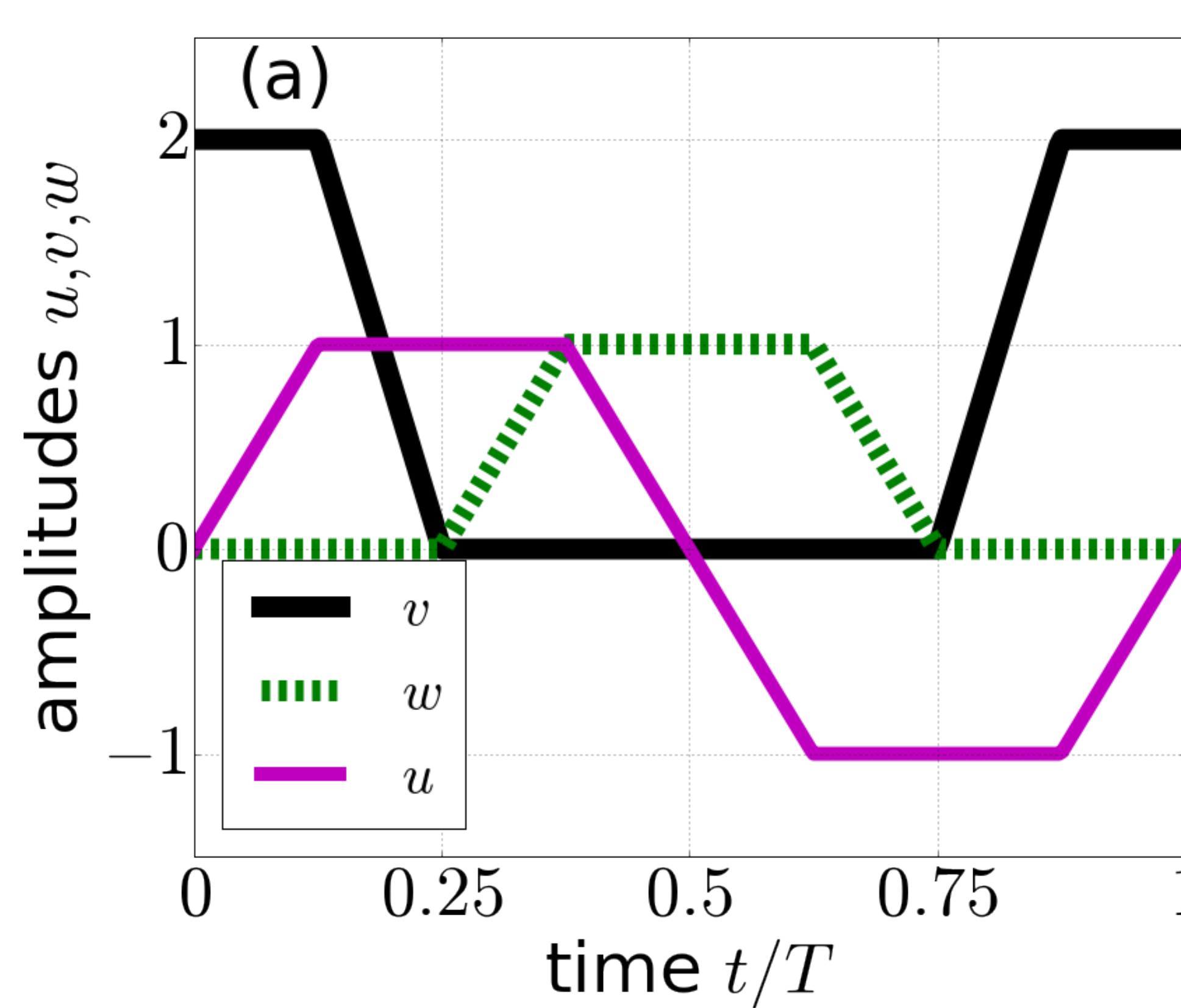


# A control-freak pumping scheme



This scheme pumps 1 electron per cycle

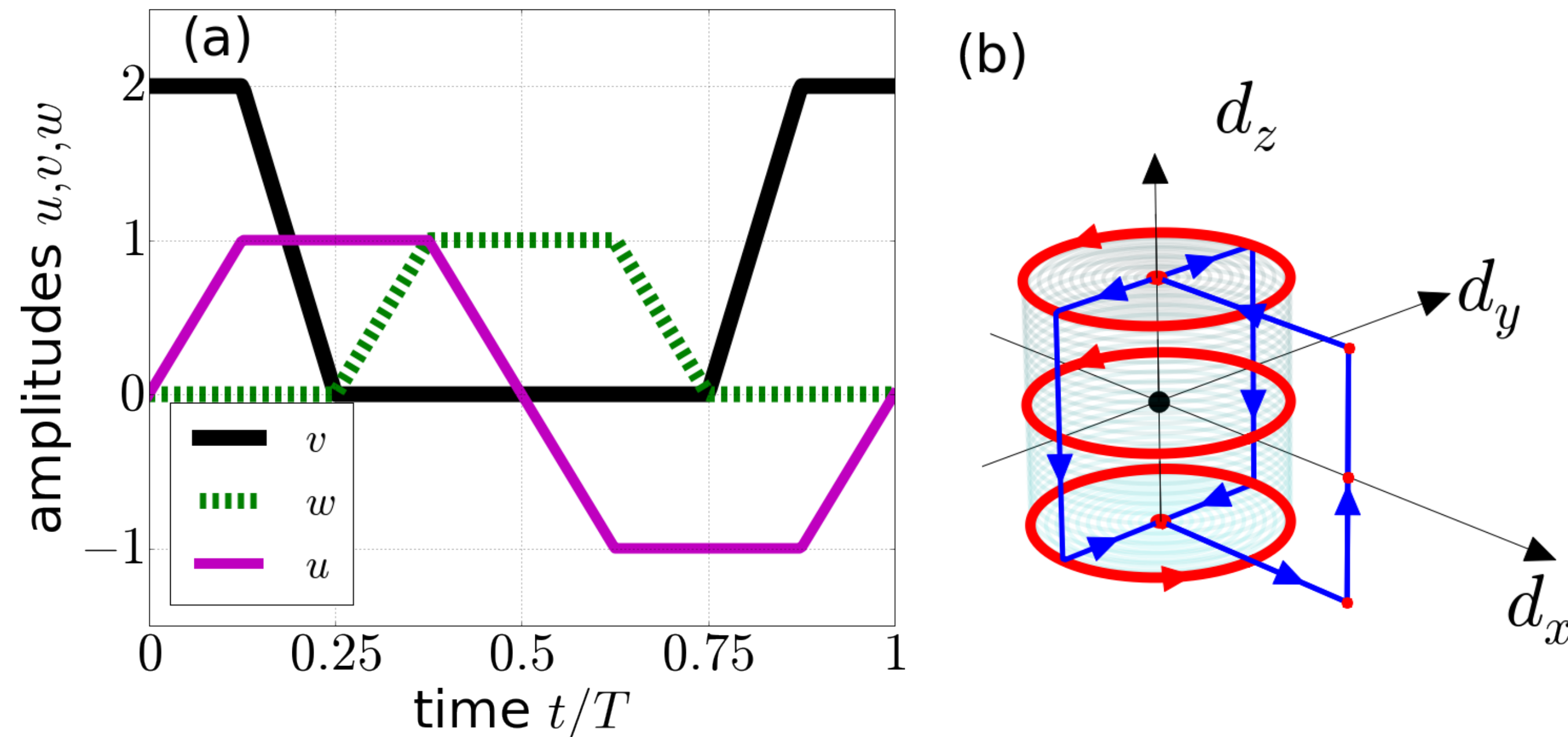
# A control-freak pumping scheme



This scheme pumps 1 electron per cycle

# The d-vector draws a torus

$$\hat{H}(k, t) = \mathbf{d}(k, t) \hat{\boldsymbol{\sigma}} = (v(t) + w(t) \cos k) \hat{\boldsymbol{\sigma}}_x + w(t) \sin k \hat{\boldsymbol{\sigma}}_y + u(t) \hat{\boldsymbol{\sigma}}_z,$$



$$\mathbf{d}(k, t) = \begin{pmatrix} v(t) + w \cos(k) \\ w(t) \sin(k) \\ u(t) \end{pmatrix}$$

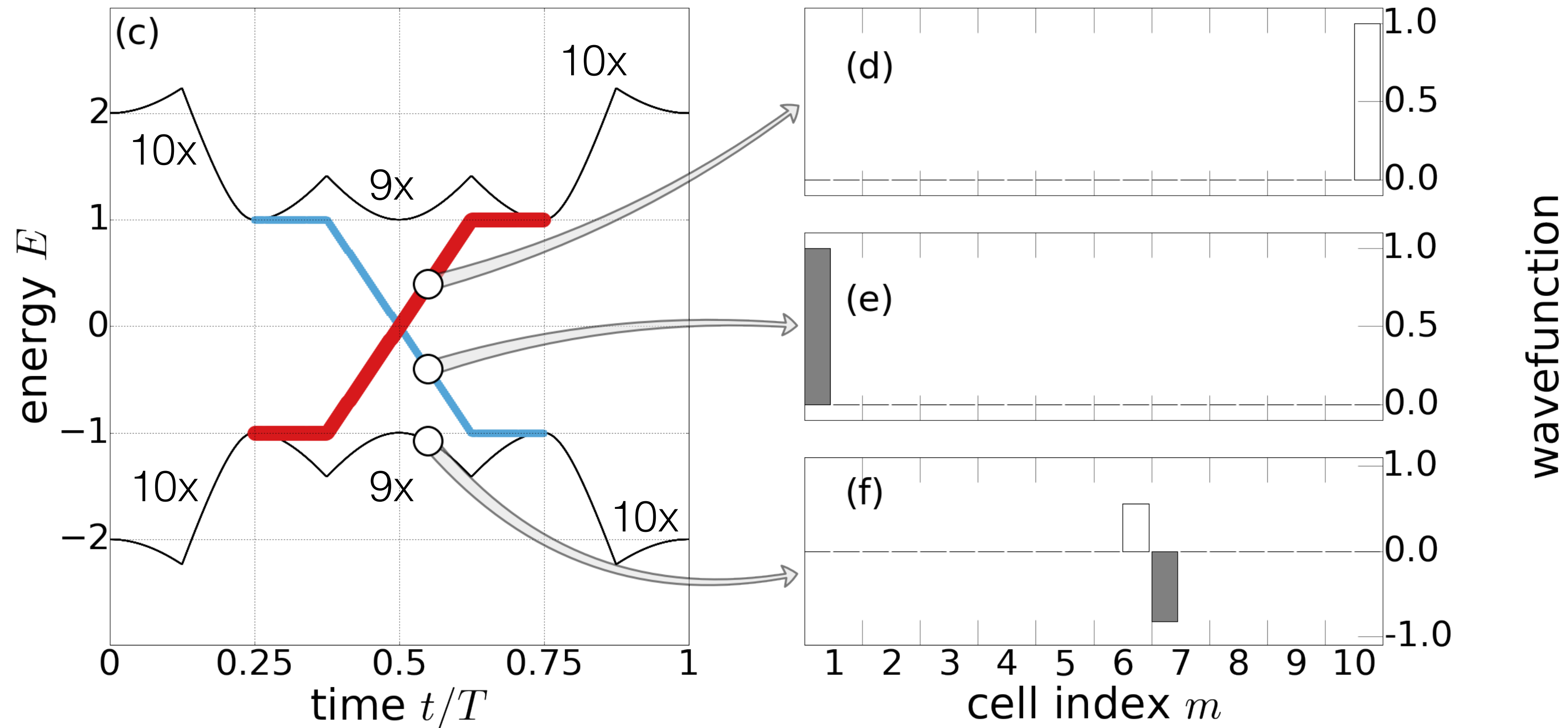
Topological invariant (Chern number)

1. Sit at  $\mathbf{d} = 0$ .
2. Go to infinity along a straight line.
3. Count the number of intersections with the torus.

Topological invariant of this torus is 1.

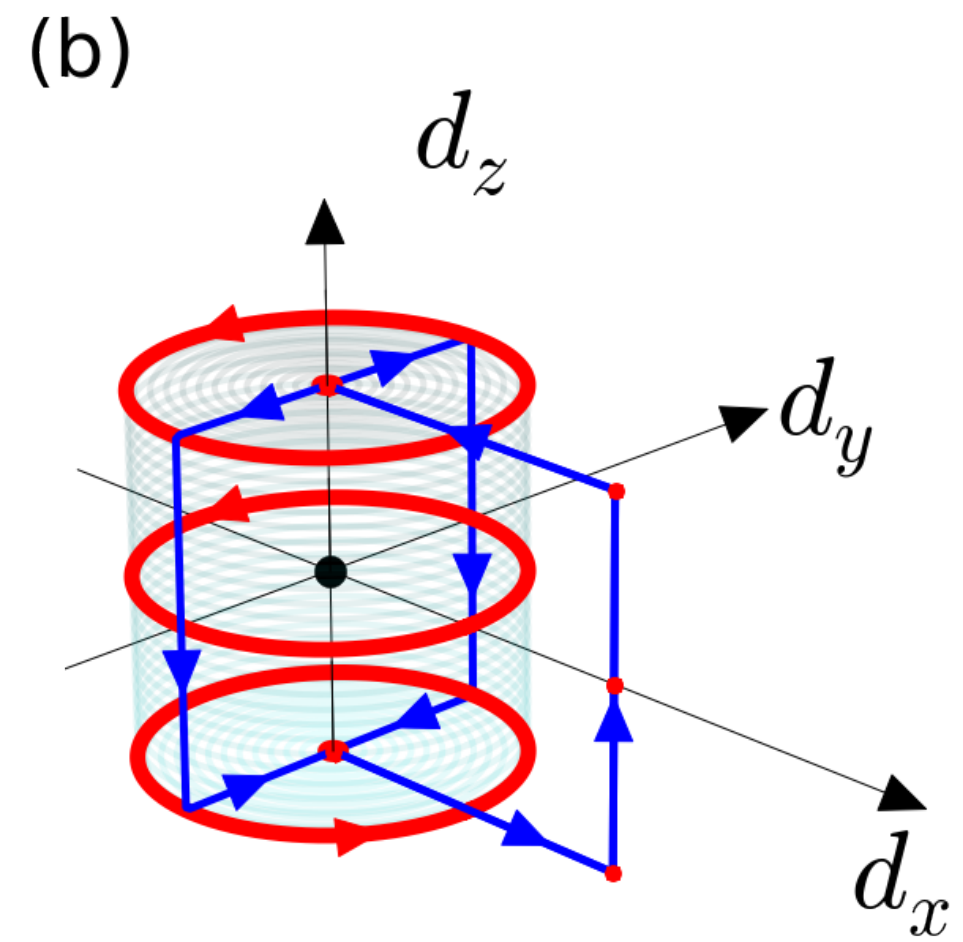
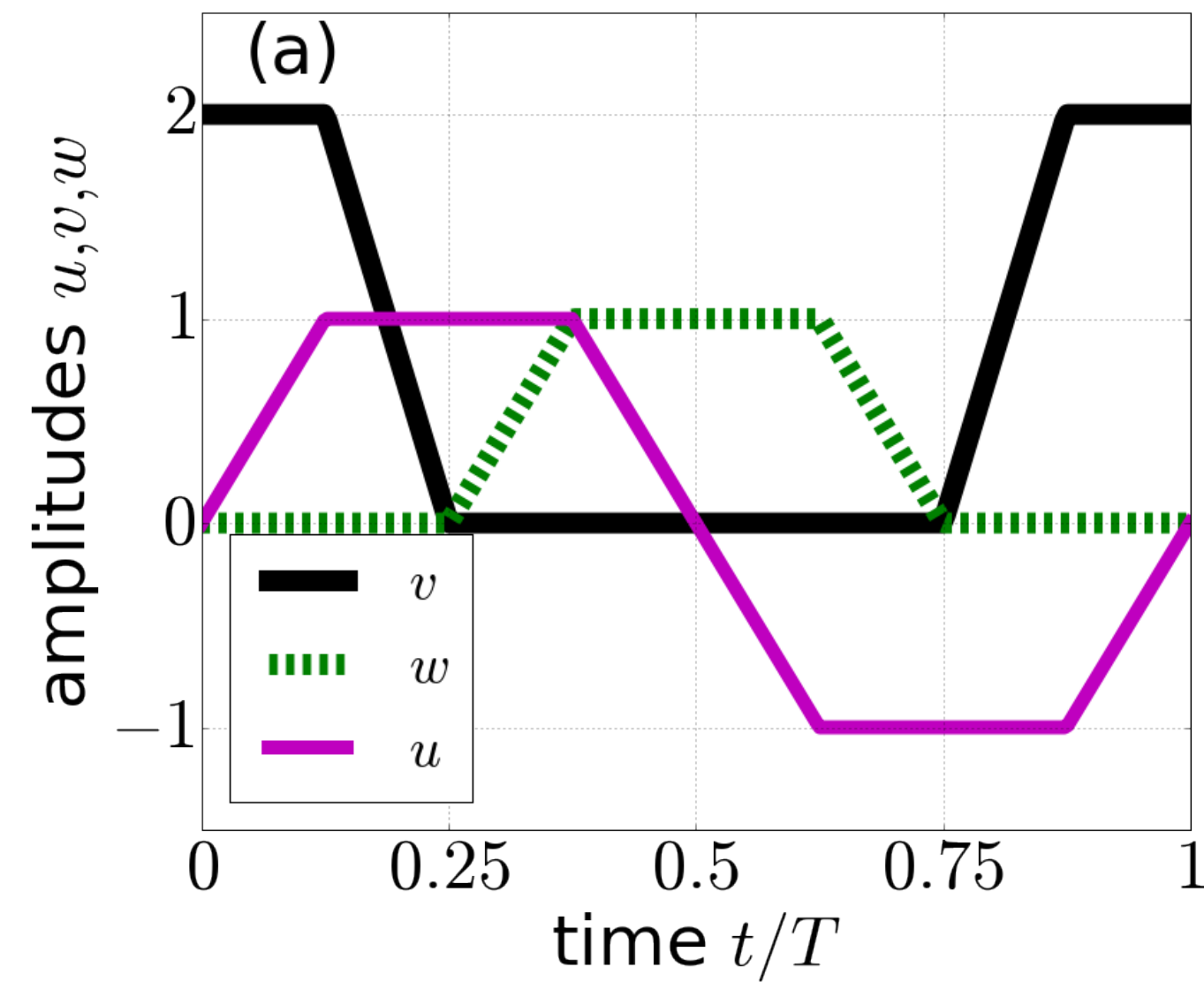
# Control-freak pumping in a finite wire

Same pumping cycle as before, with  $N = 10$  unit cells.

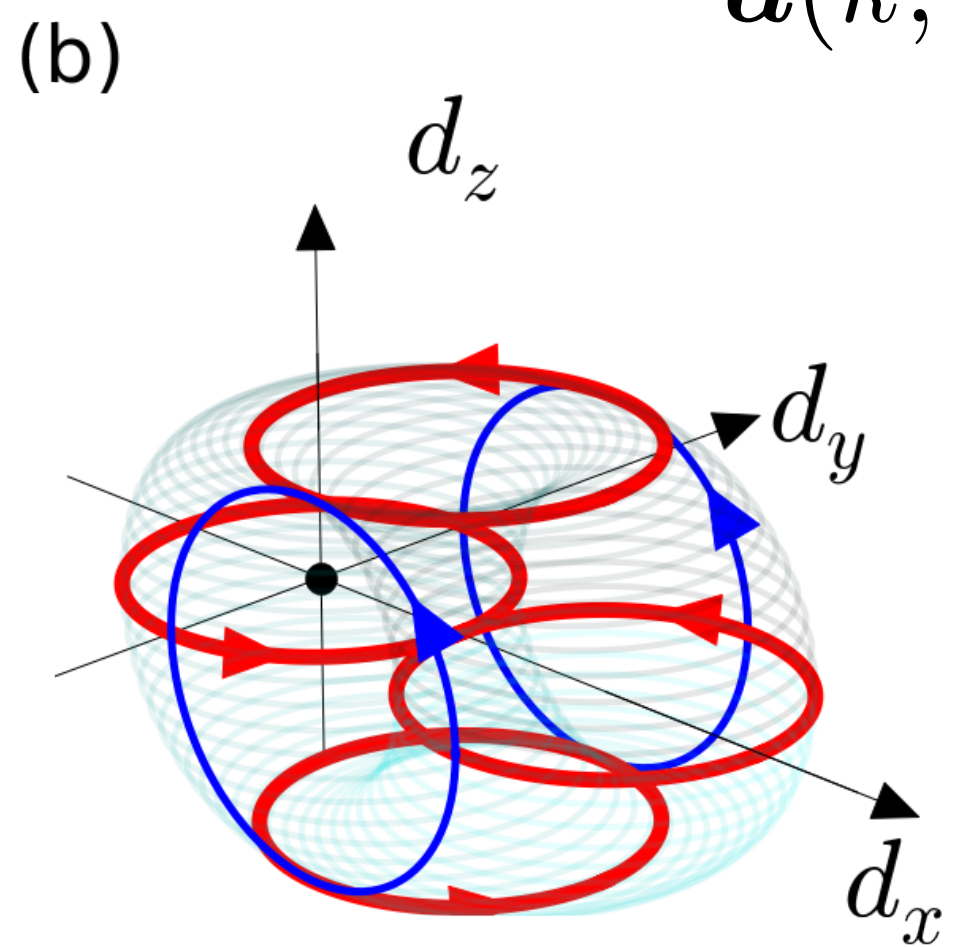
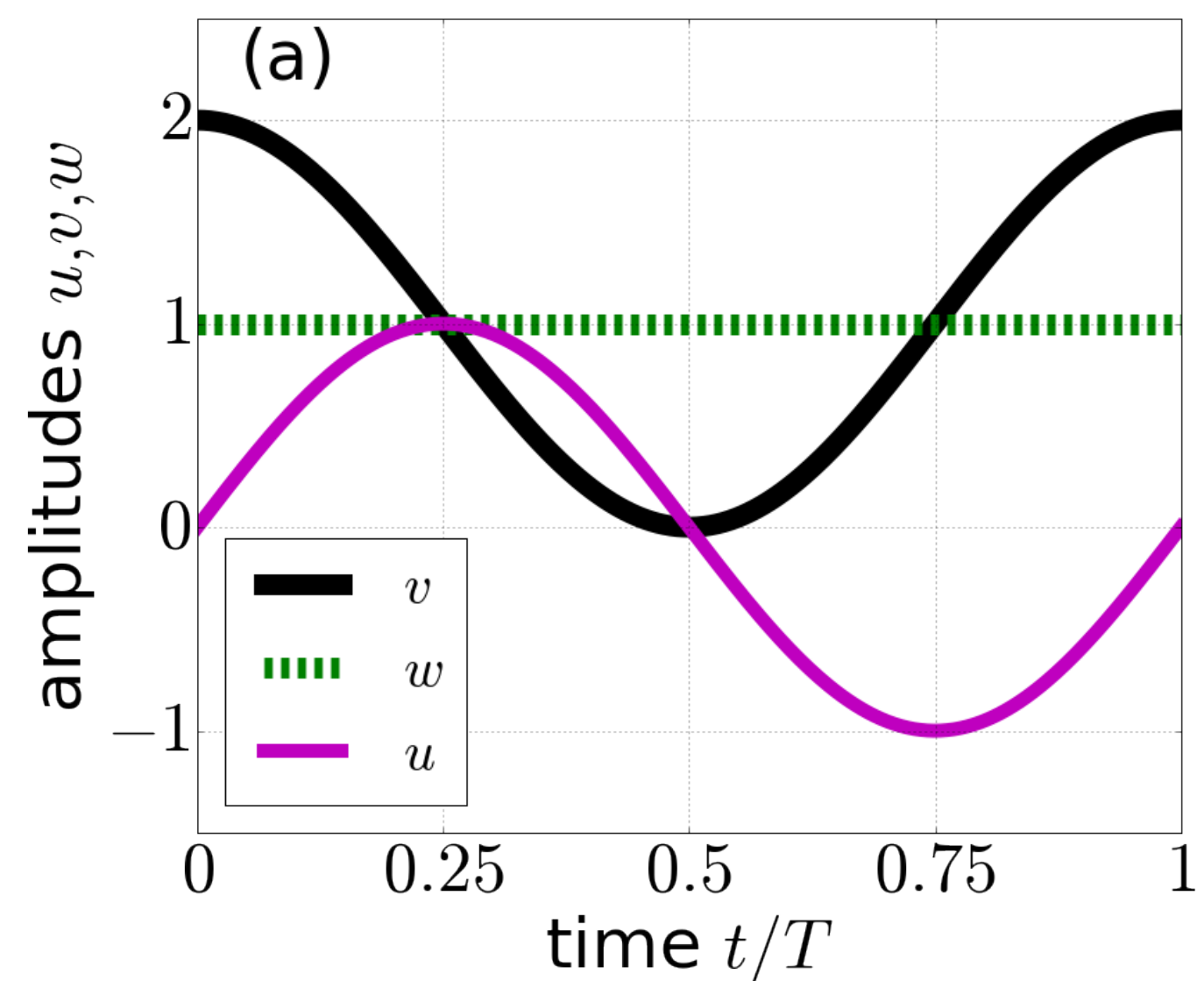


1 electron is taken from VB to CB per cycle

# Moving away from the control-freak limit

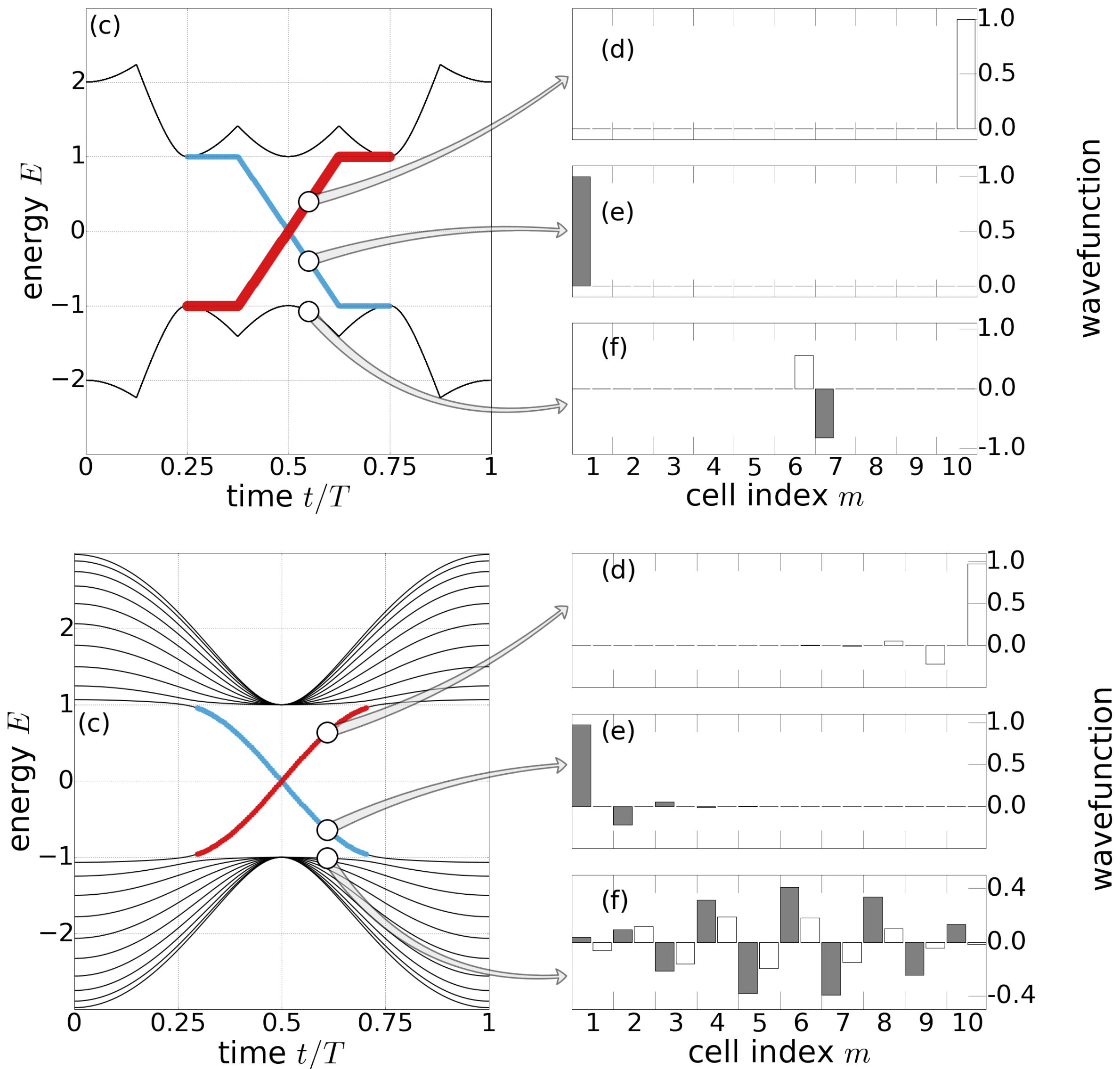


$$\mathbf{d}(k, t) = \begin{pmatrix} v(t) + w \cos(k) \\ w(t) \sin(k) \\ u(t) \end{pmatrix}$$



Continuous deformation can't change the topological invariant

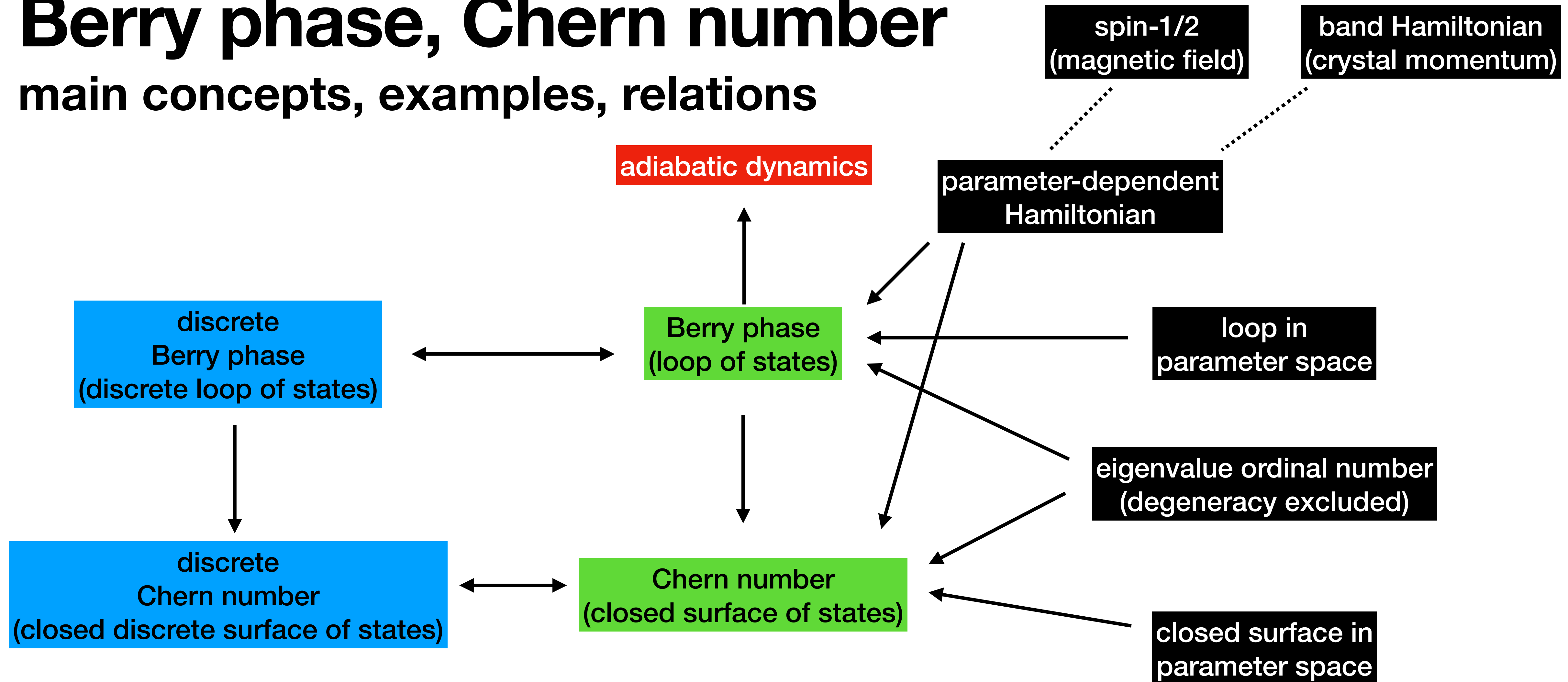
# Moving away from the control-freak limit



Continuous deformation can't change the # of edge states

# Berry phase, Chern number

## main concepts, examples, relations



remark: terminology: *Berry phase factor* lives on the complex unit circle, *Berry phase* lives in  $]-\pi, \pi]$  or  $[0, 2\pi[$

# Polarization and Berry phase

start with concepts from last week, think of 1D and 2-band tight binding models

