Wannier states

The Wannier state from the n_{th} band centered around site j is denoted by $|w_{(n)}(j)\rangle$.

Consider the Wannier states from different bands, $n' \neq n$, and for different positions, $j' \neq j$, i.e., $|w_{(n')}(j)\rangle$, $|w_{(n)}(j')\rangle$.

Which of the overlaps is guaranteed to be zero by construction, the one between different bands, or the one between different positions?

$$\left|\langle w^{(n')}(j)|w^{(n)}(j)\rangle\right| = 0? \qquad \left|\langle w^{(n)}(j')|w^{(n)}(j)\rangle\right| = 0?$$

a) Only the first

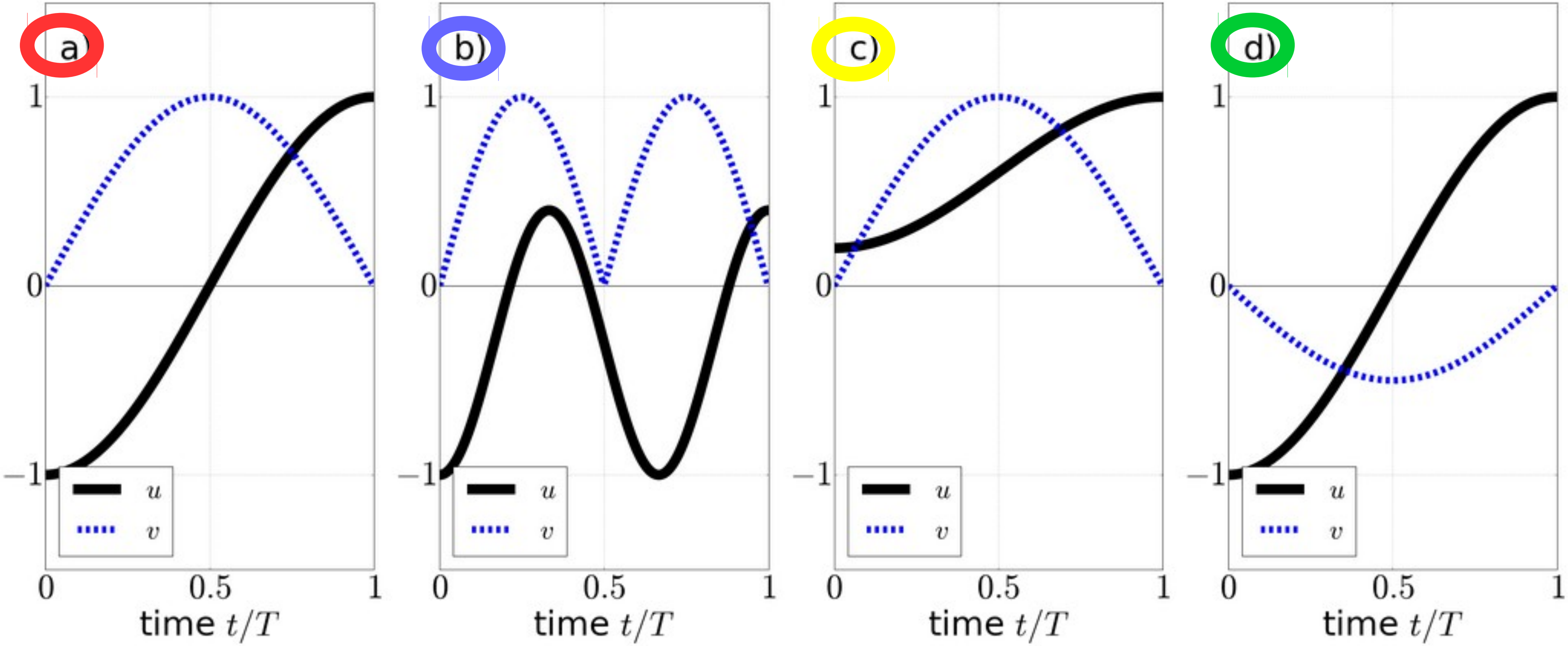
b) Only the second

c) Neither the one nor the second

d) Both

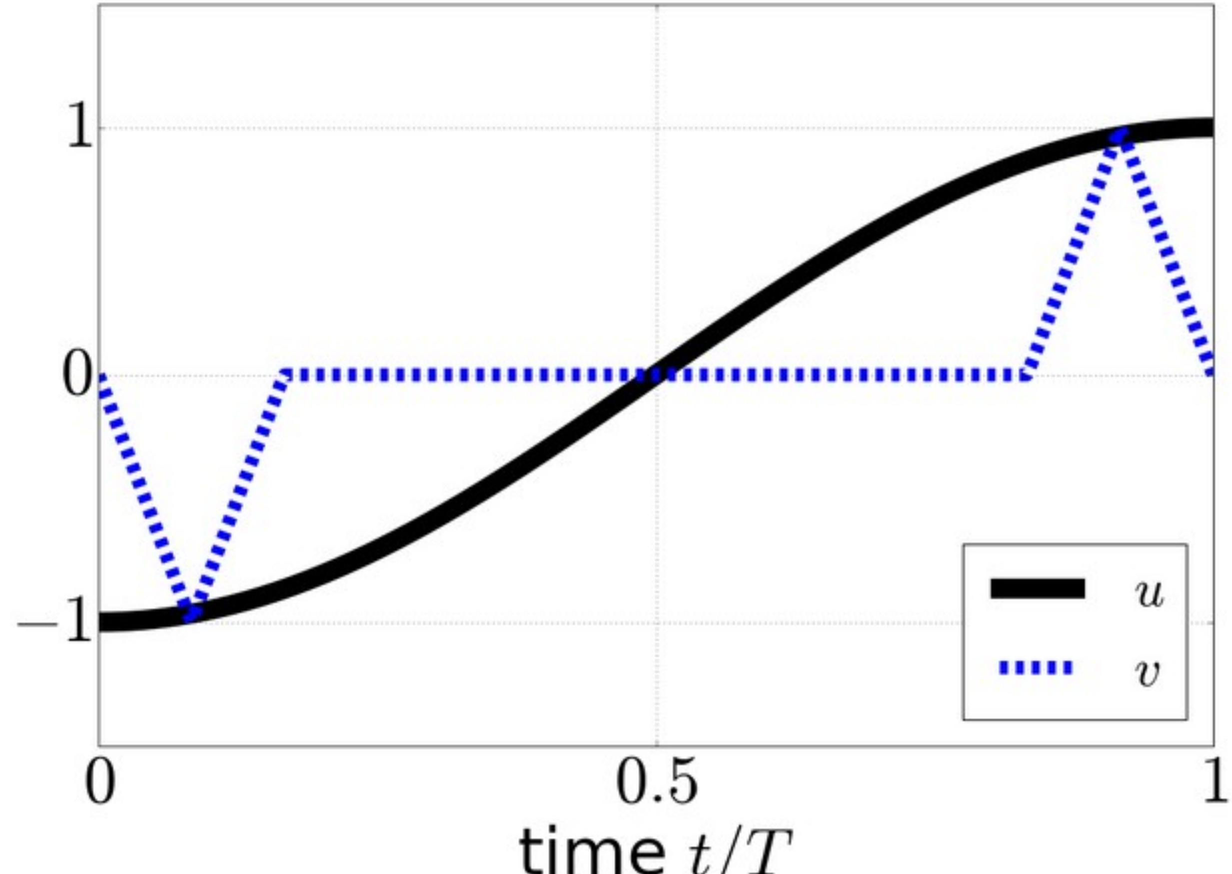
Pumping on a single dimer I.

Consider the very slow pump protocol, where $H = v(t)\sigma_x + u(t)\sigma_z$. The initial state is the ground state at $t = 0$, that is, $\psi_i = (1, 0)$. Which protocol does not shift the charge?



Pumping on a single dimer II.

Consider the very slow pump protocol, where $H = v(t)\sigma_x + u(t)\sigma_z$ The initial state is the ground state at $t = 0$, that is, $\psi_i = (1,0)$. What is the final state and why?



a $(1,0)$

b $\frac{(1,1)}{\sqrt{2}}$

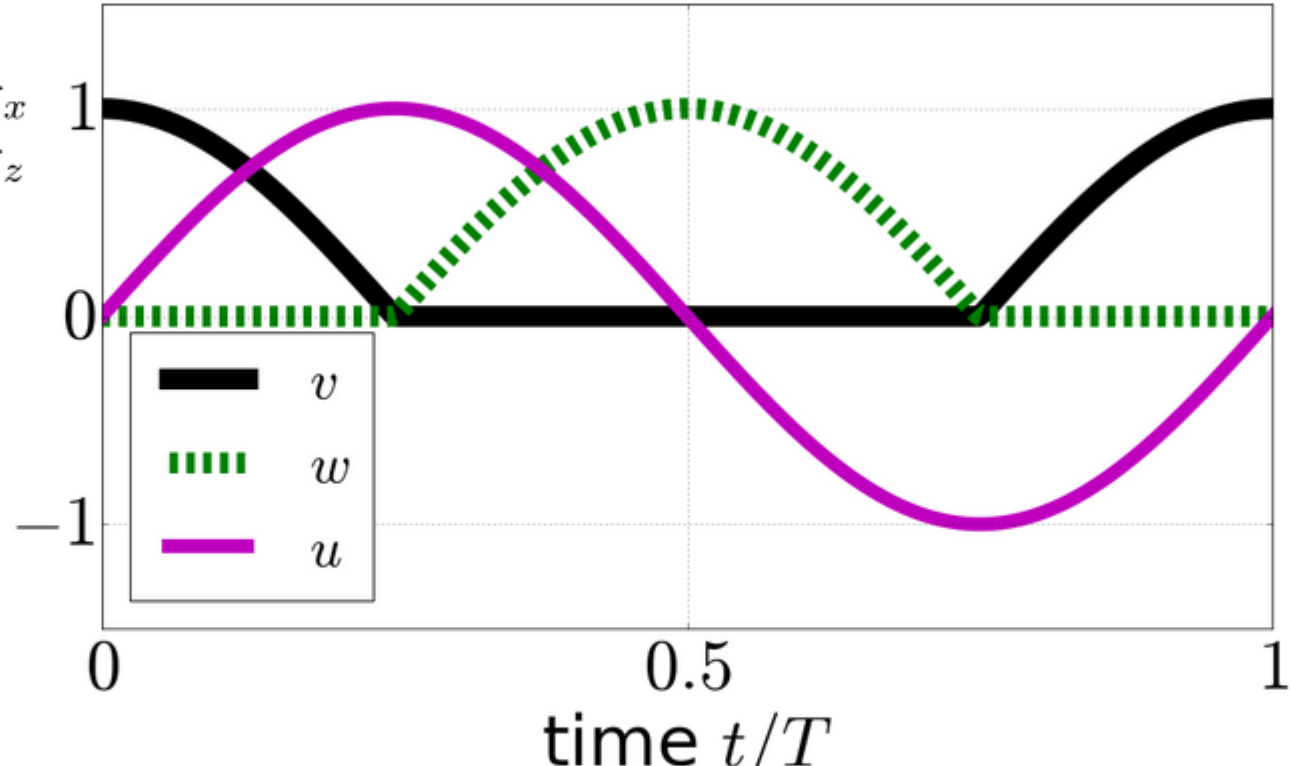
c $(0,1)$

d the question does not make sense, the energy splitting is zero at t=T/2!

Control-freak pumping II.

Consider adiabatic pumping in the Rice-Mele model with the depicted time dependence of the parameters. Is this a control-freak pump?

$$\mathbf{d}(k,t)\hat{\sigma} = [v(t) + w(t)\cos(k)]\sigma_x + w(t)\sin(k)\sigma_y + u(t)\sigma_z$$



a) it is not a control-freak cycle as the graph is not assembled from straight lines

b) it is not even adiabatic as the gap closes during the cycle

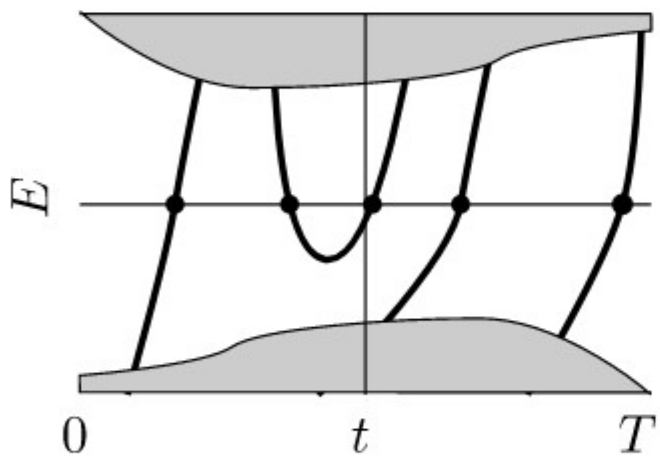
c) it is a control-freak cycle because the corresponding $\mathbf{d}(k,t)$ surface is a torus

d) it is a control-freak cycle because the energy eigenstates can be chosen to be localized to dimers

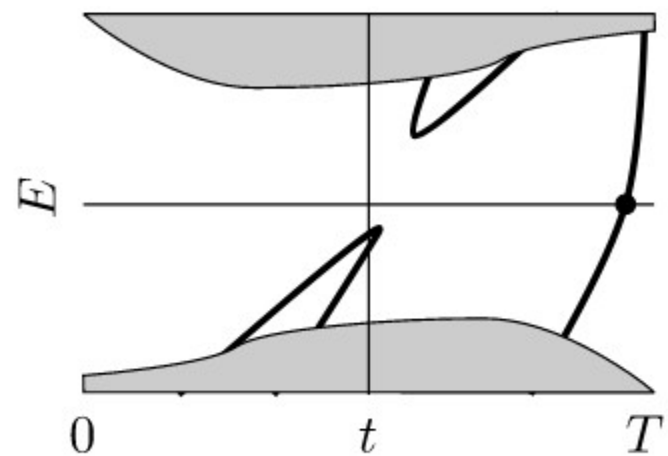
Edge states

Which of the figures below could represent the edge states at a certain edge in a pumping process?

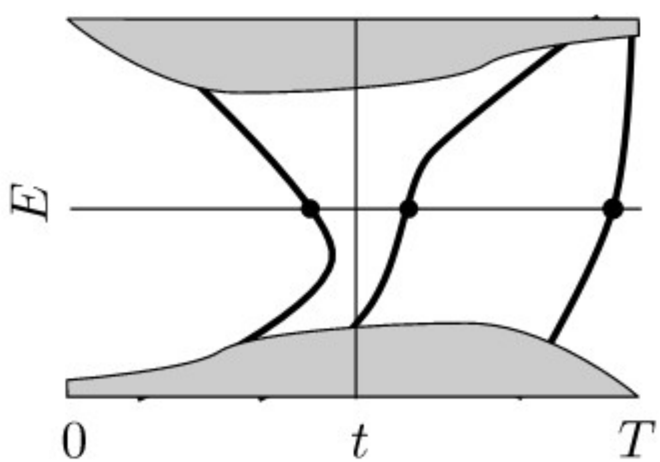
a)



b)



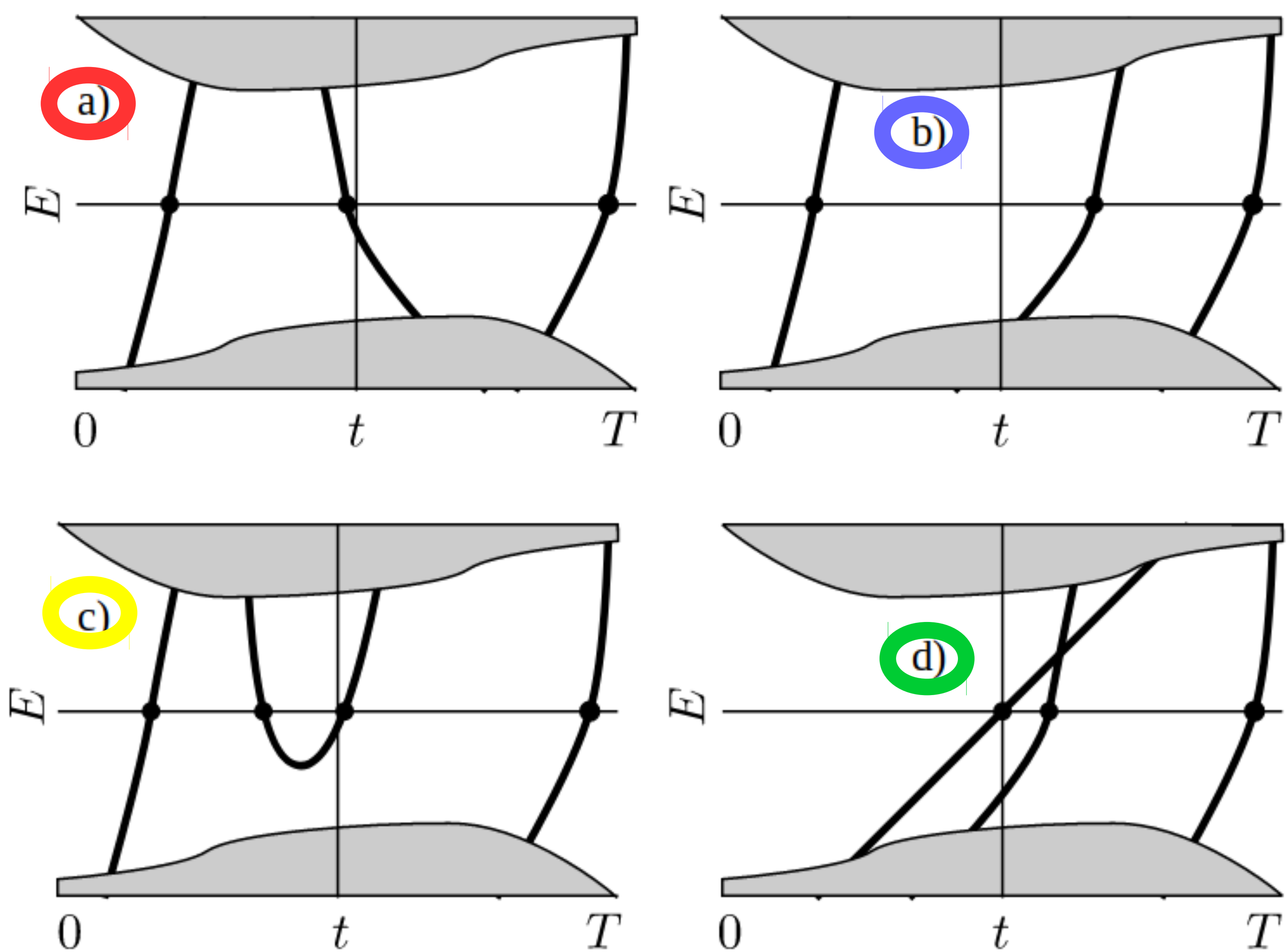
c)



d) None of the above

Edge states and Chern number

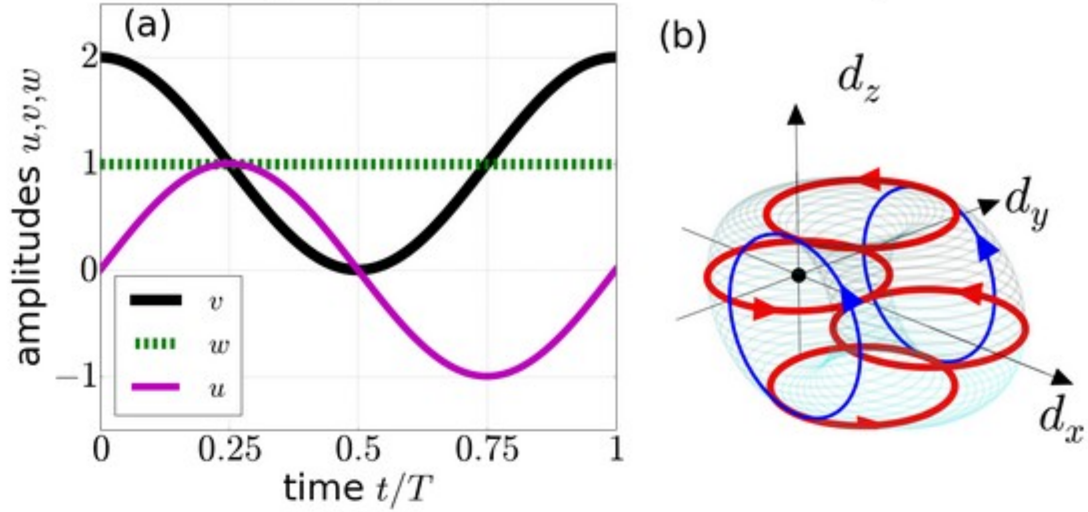
Which of the figures below could represent the edge states of a pump sequence with Chern number 2?



Adiabatic pumping in a finite chain I.

The figures represent the $\bar{v} = 1$ case of the pump sequence defined by

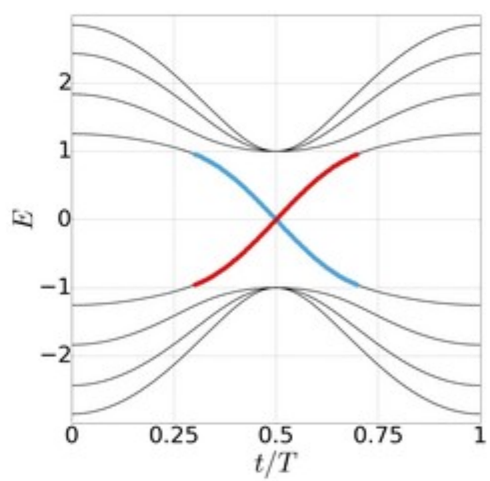
$$
\begin{aligned}
u(t) &= \sin(2\pi t/T), \\
v(t) &= \bar{v} + \cos(2\pi t/T), \\
w(t) &= 1,
\end{aligned}
$$



in the finite-sized Rice-Mele model with N=4 unit cells.

Assume that initially the electronic system is in its ground state: the four electrons occupy the negative-energy states. At least how many cycles should be completed to arrive to this ground state again?

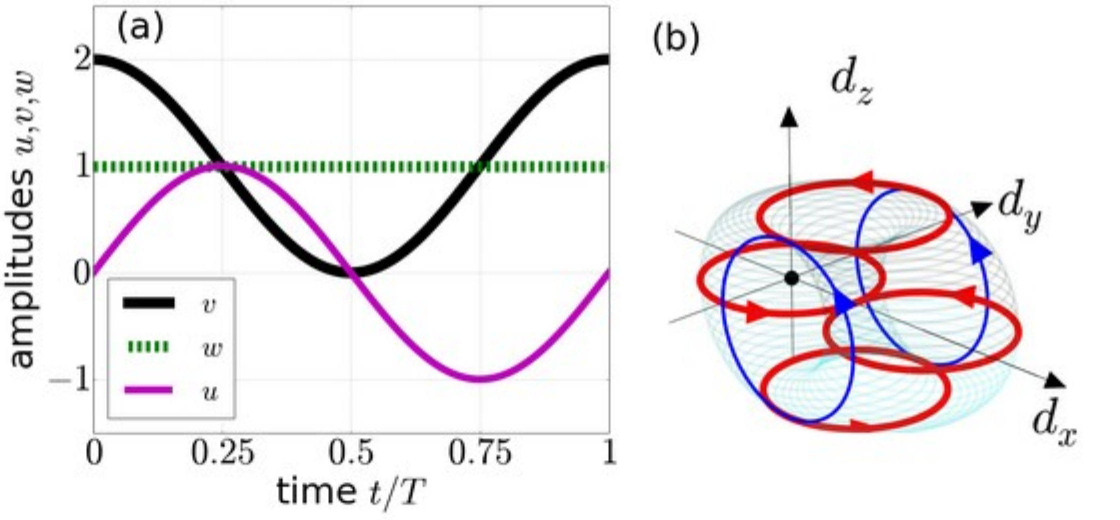
a) 1
b) 2
c) 4
d) 8



Smooth pumping sequence

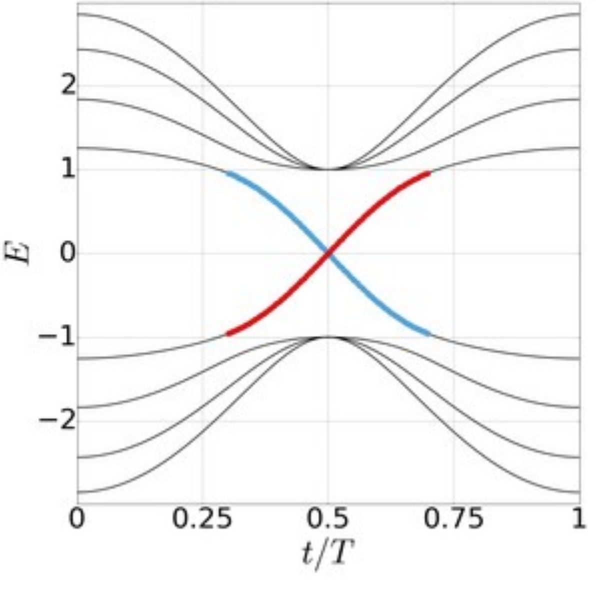
The figures represent the $\bar{v} = 1$ case of the pump sequence defined by

$$\begin{aligned} u(t) &= \sin(2\pi t/T), \\ v(t) &= \bar{v} + \cos(2\pi t/T), \\ w(t) &= 1, \end{aligned}$$



in the finite-sized Rice-Mele model with N=4 unit cells.

Do you expect to see any qualitative difference in the energy-vs-time graph, if $\bar{v} = 1$ is changed to $\bar{v} = 1.5$?



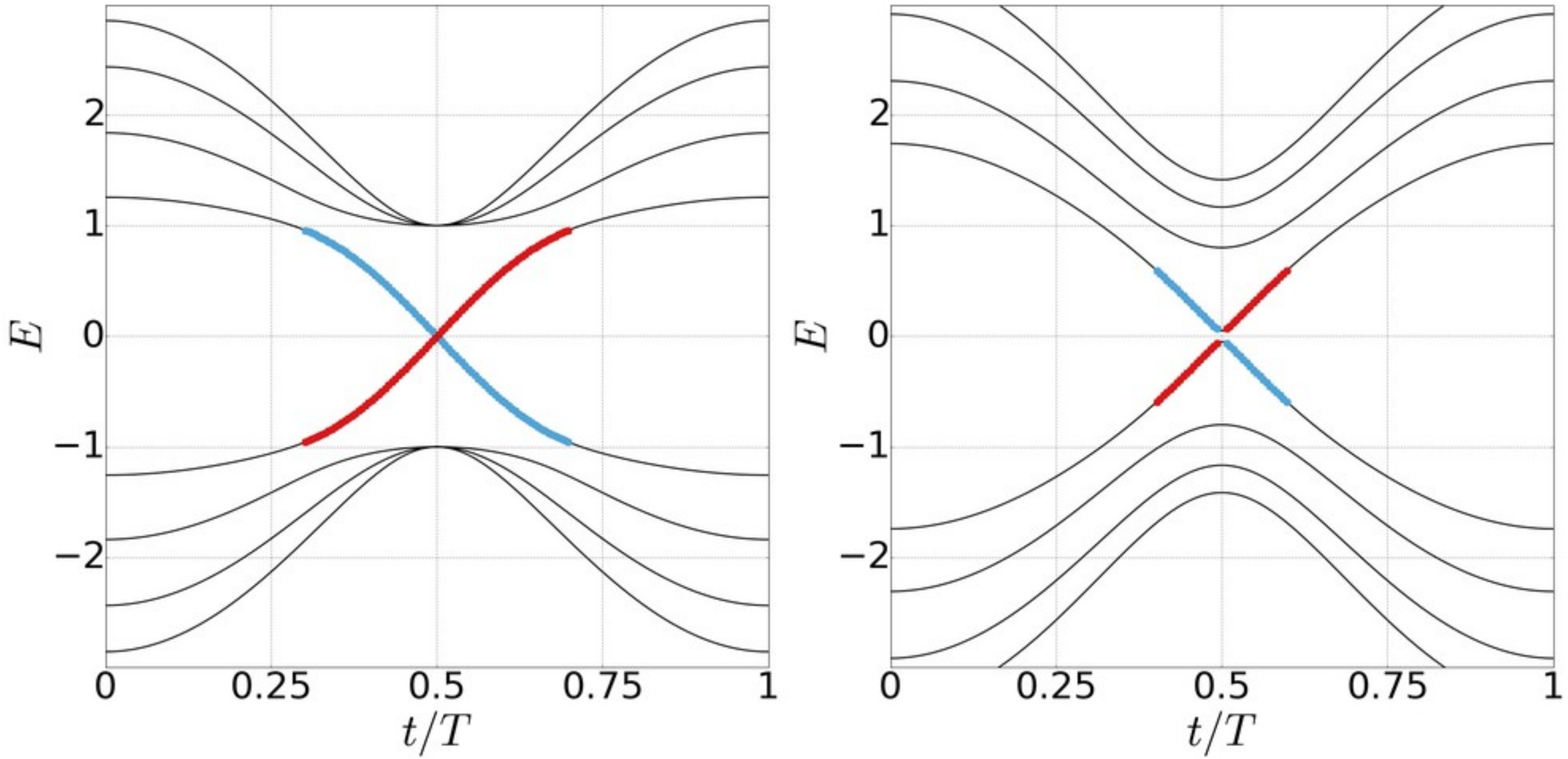
a) No

b) Yes: bulk states become degenerate

c) Yes: all degeneracies are lifted at t=0.5 T

d) Yes: two edge states appear on both edges

What really happens



$\bar{v} = 1$ $\bar{v} = 1.5$

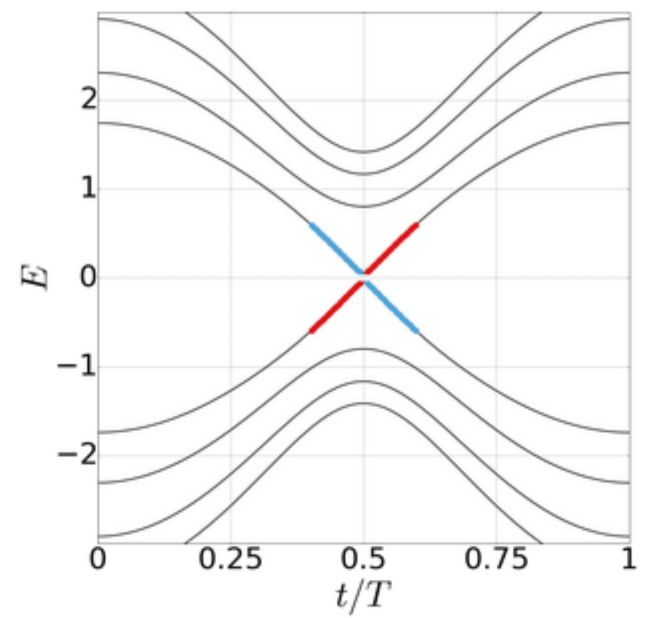
Adiabatic pumping in a finite chain II.

The figure represent the $\bar{v} = 1.5$ case of the pump sequence defined by

$$\begin{aligned} u(t) &= \sin(2\pi t/T), \\ v(t) &= \bar{v} + \cos(2\pi t/T), \\ w(t) &= 1, \end{aligned}$$

in the finite-sized Rice-Mele model with N=4 unit cells.

Assume that initially the electronic system is in its ground state: the four electrons occupy the negative-energy states. At least how many cycles should be completed to arrive to this ground state again?



a) 1

b) 2

c) 4

d) 8