HUNGARIAN ACADEMY OF SCIENCES
WIGNER RESEARCH CENTRE FOR PHYSICS

Introduction to topological superconductors – Majoranas as building blocks of topological quantum computing

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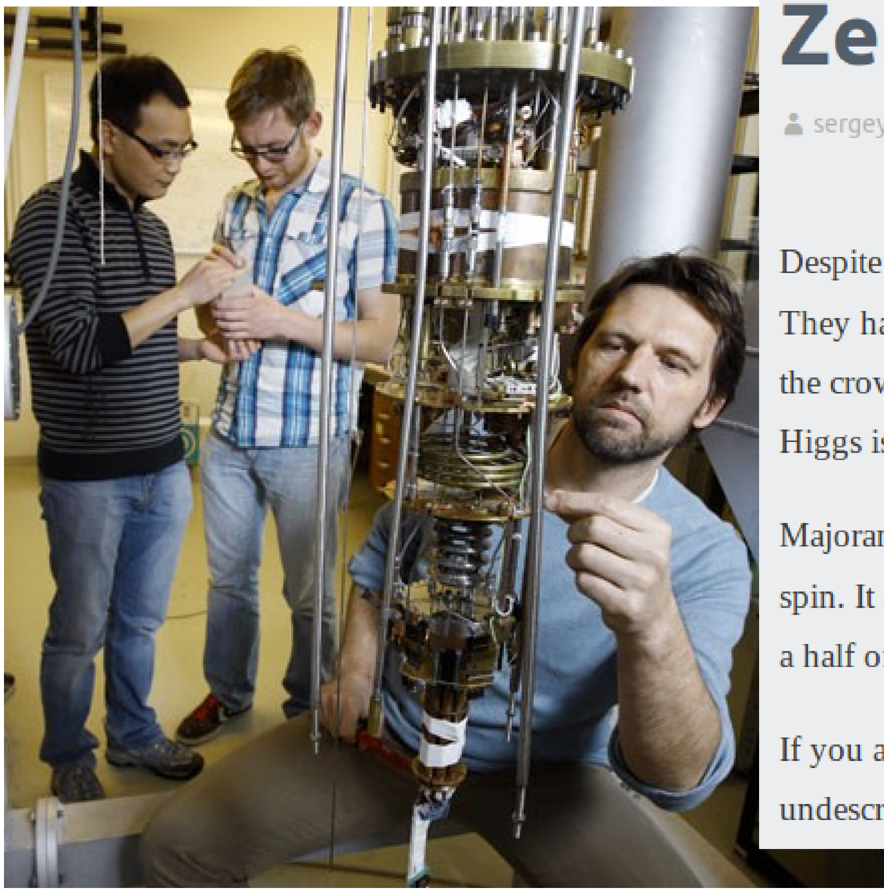
In 2012, two long-awaited particles detected…
Higgs boson = breakthrough of the year.
Majorana fermion = runner-up



Zen particle

sergeyfrolov Uncategorized April 18, 2012

Despite our best efforts it remains impossible to explain what a Majorana particle actually is. They have it much easier with the Higgs boson – it is simply the God particle. Of course, it is the crown jewel of the Standard model, and it is the reason why Everything is the way it is. Higgs is supposed to be super-massive, and they have to build tera-volt colliders to look for it.

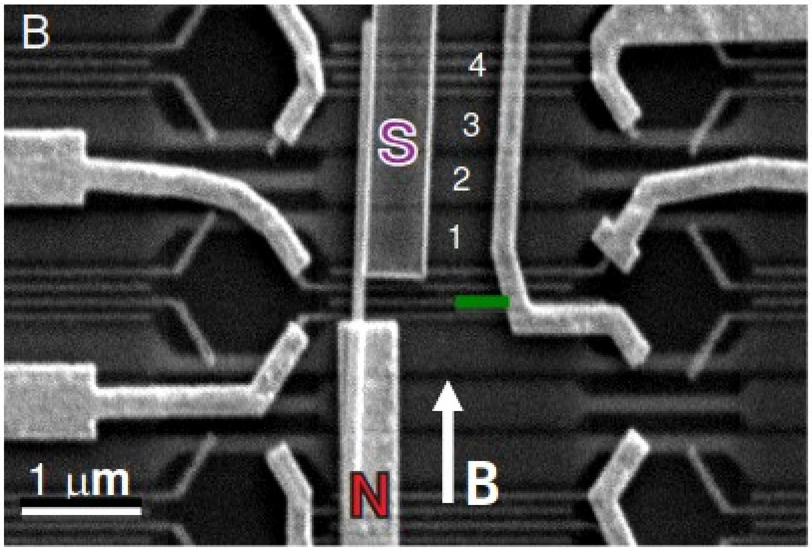
Majorana particle is just the opposite. It has no mass, it has no charge, it has no energy, and no spin. It is as close to Nothing as anything can be. It is often called a fermion, but in fact it is only a half of a real fermion. Hard to imagine this, much like it is hard to clap with one hand.

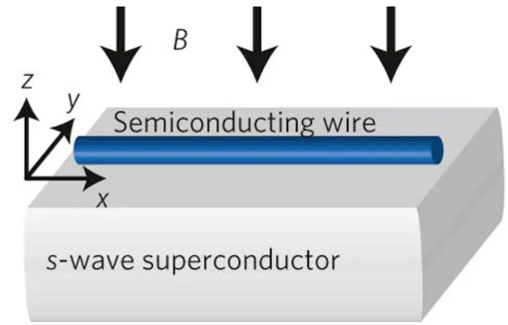
If you are looking for a single word to label the Majorana fermion it must be able to describe the undescribable. Which happens to be the essence of Zen philosophy.

Leo Kouwenhoven → Microsoft StationQ Delft

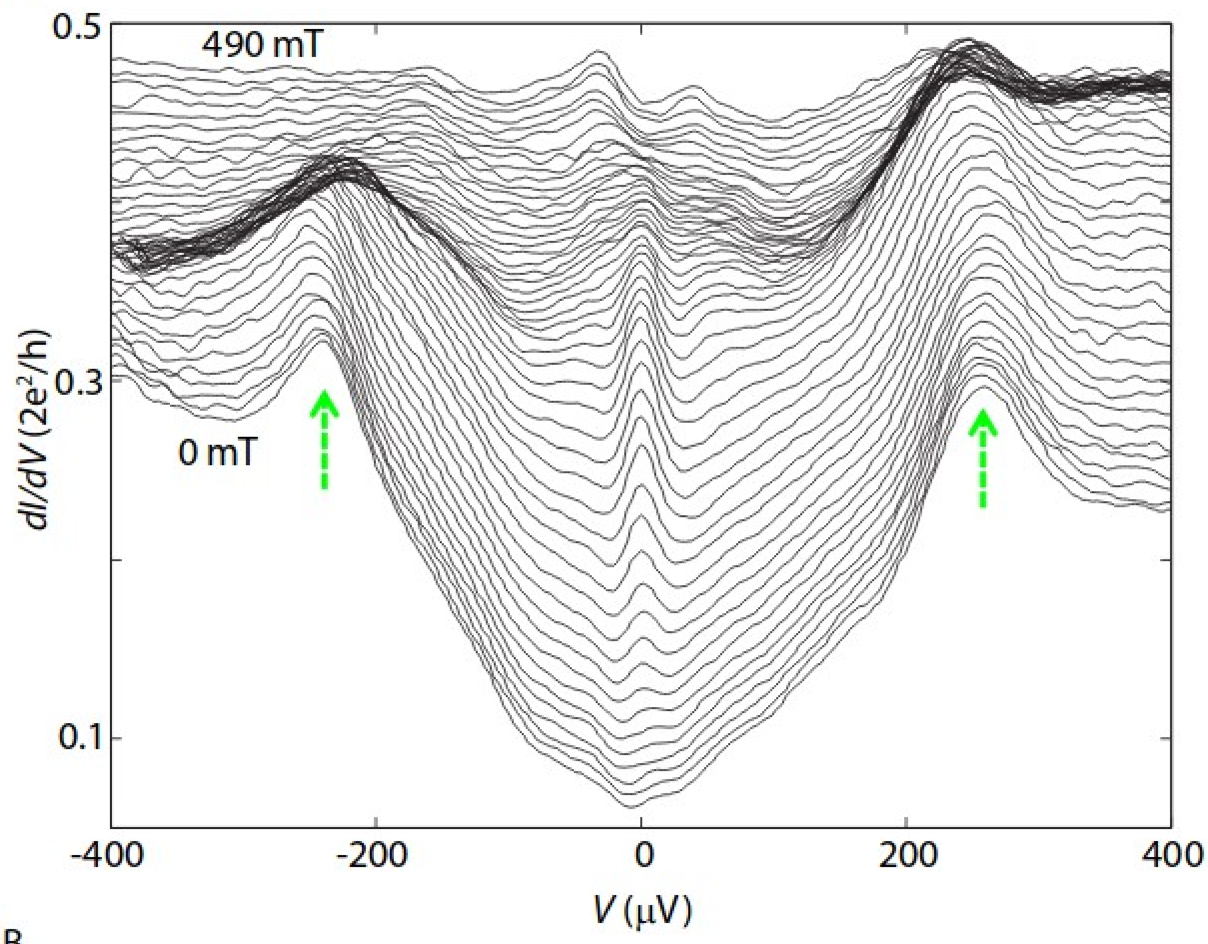
Sergey Frolov → U Pittsburgh,
(great YouTube Quantum Transport lecture series)

The experimental signature was a Zero-Bias Conductance Peak, appearing under right combination of parameters



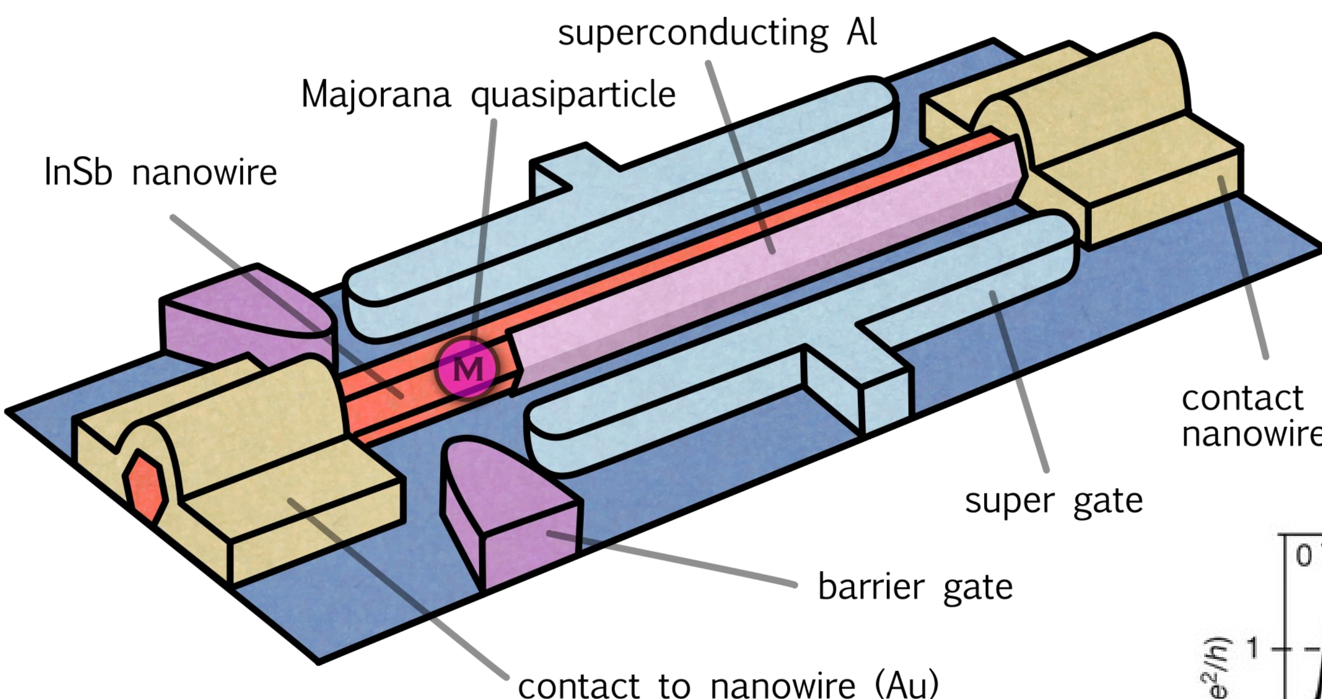


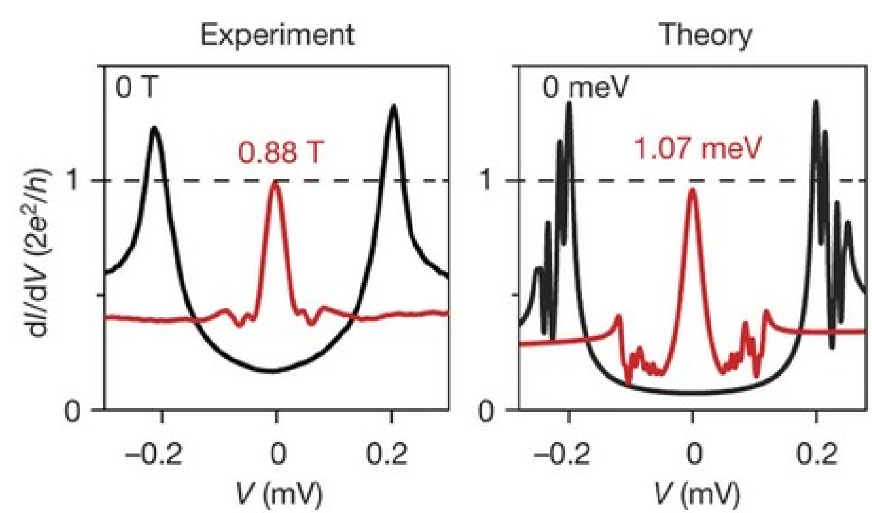
Semiconducting nanowire + superconductivity + magnetic field = topological superconductor



Since then, Higgs boson has been awarded Nobel prize. Majorana fermion?

- Not really a particle, Majorana ~~fermion~~ Zero Mode (bound state)
- Lots of measurement results, but still no clear signature





Zhang et al, Quantized Majorana Conductance, Nature (2018)

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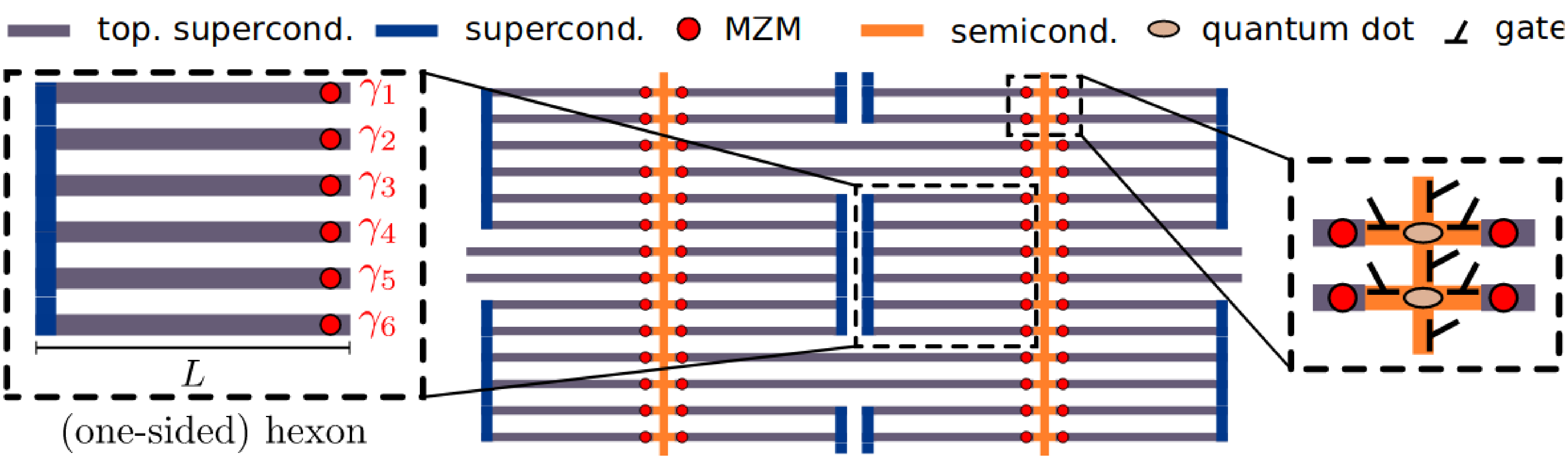
- Microsoft trying to build quantum computer with it (~1B$ in Delft, Copenhagen, Santa Barbara)



Karzig et al, Scalable Designs for Quasiparticle-Poisoning-Protected Topological Quantum Computation with Majorana Zero Modes, PRB (2017)

Program of next semester: Topological Superconductors, Majorana Zero Modes, Quantum Computing

Use a Bogoliubov-de Gennes, mean field, description of superconductors
→ single-particle Hamiltonian for fermionic excitations
→ apply topological insulator formalism
→ interpret for many-body superconducting state

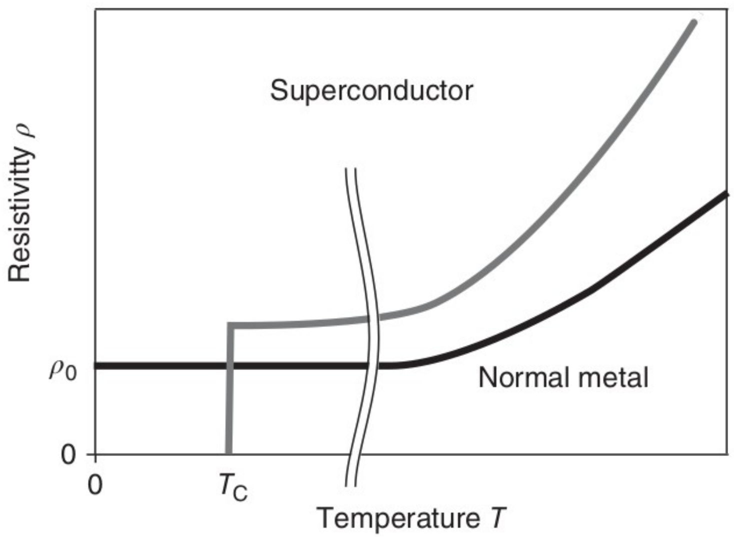
What is a topological superconductor?
A superconductor, but with bulk topology like in 1st semester

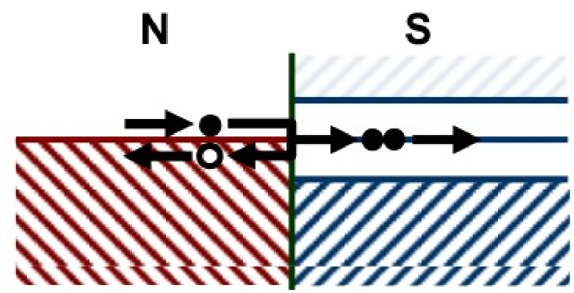
What are Majorana Fermions and Zero Modes?
The topologically protected states of superconductors. Half a fermion.

How can Majorana Zero Modes be used for quantum computing?
Hide (encode) quantum information in them, process, measure

What is the experimental status, signatures, goals?
Conductance peaks, 4π Josephson effect, fusion statistics

Much of what is interesting about superconductors can be understood at a mean-field, single-particle level





Put in Cooper pair formation below Tc “by hand”
→ Andreev reflection → dissipationless current

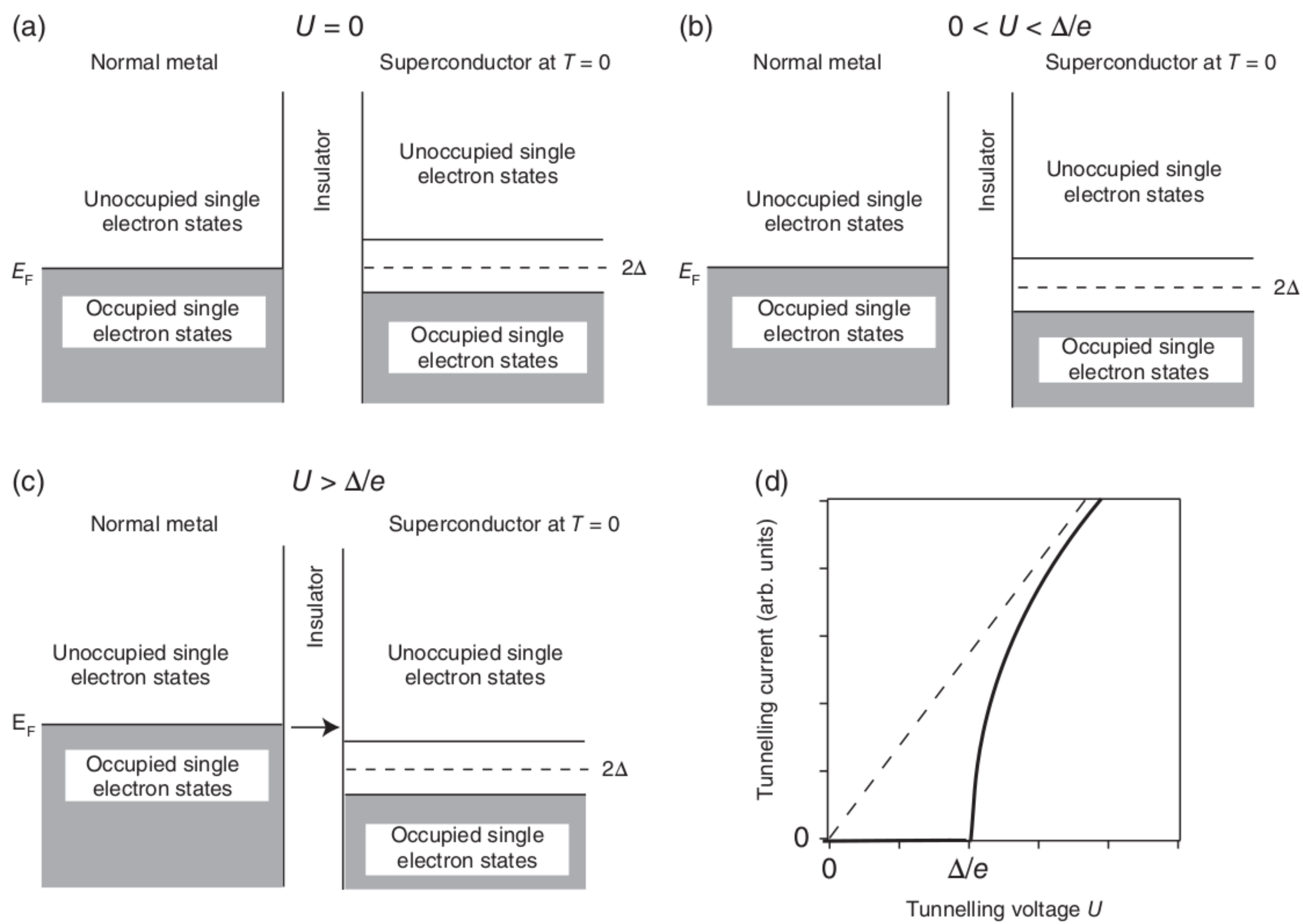
n.n. hopping chemical potential

$$\hat{H}_s = \sum_{m=1}^{N} \Big(\sum_{\sigma=\uparrow,\downarrow} \left[-\hat{c}^\dagger_{m,\sigma}\hat{c}_{m+1,\sigma} - \hat{c}^\dagger_{m+1,\sigma}\hat{c}_{m,\sigma} - \mu \hat{c}^\dagger_{m,\sigma}\hat{c}_{m,\sigma} \right] + \Delta \hat{c}^\dagger_{m,\uparrow}\hat{c}^\dagger_{m,\downarrow} + \Delta^* \hat{c}_{m,\downarrow}\hat{c}_{m,\uparrow} \Big)$$

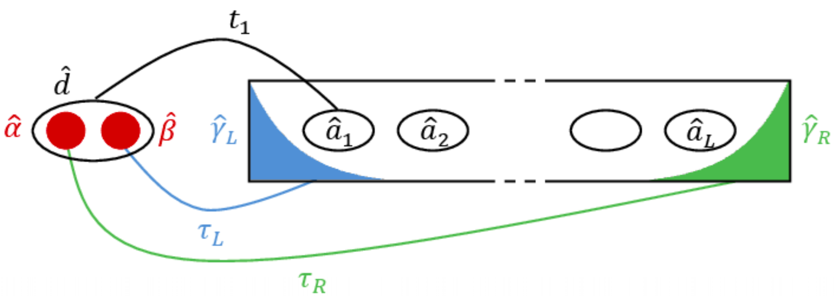
Cooper pair breaking + creation

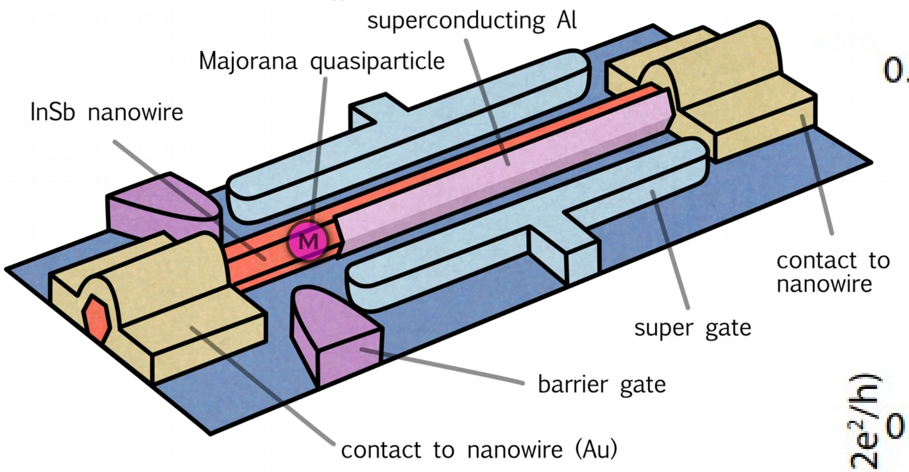
Dynamics of Cooper pairs not tracked (no self-consistent calculation of e.g. supercurrent)

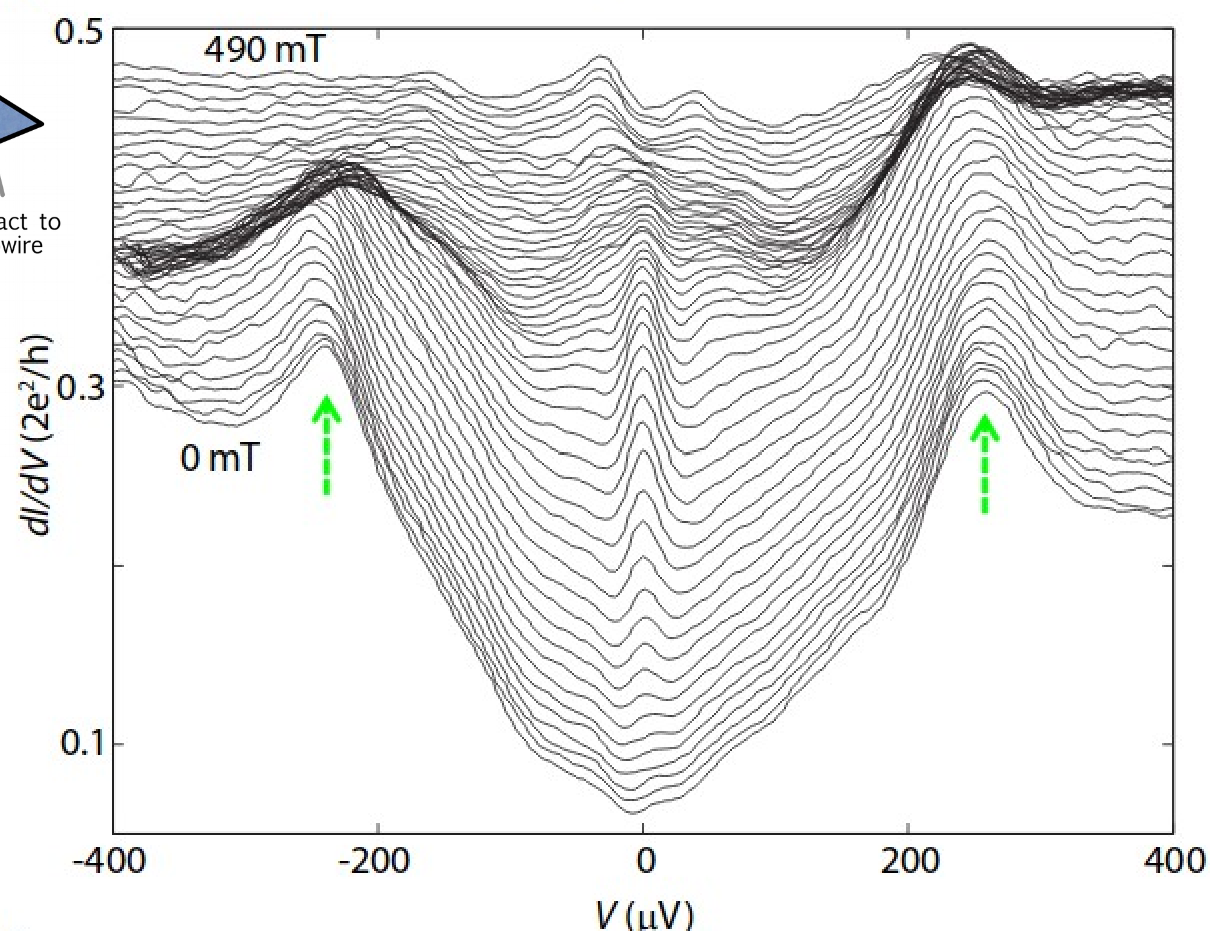
Single-particle level intuition also good enough for tunneling experiments to measure gap



Zero Bias Conductance Peaks are evidence of 0-energy bound states near interface (like in SSH model)







Technically, single-particle description of superconductors in mean-field approximation is by doubling degrees of freedom, Bogoliubov-de Gennes Hamiltonians

$$\hat{H} = \sum_{m,l=1}^{N} \hat{c}_m^\dagger h_{ml} \hat{c}_l + \frac{1}{2} \sum_{m,l=1}^{N} \hat{c}_m^\dagger \Delta_{ml} \hat{c}_l^\dagger - \frac{1}{2} \sum_{m,l=1}^{N} \hat{c}_m \Delta_{ml}^* \hat{c}_l$$

Double degrees of freedom for better numerics:

$$\hat{H} = \frac{1}{2} \begin{pmatrix} \hat{\mathbf{c}}^\dagger & \hat{\mathbf{c}} \end{pmatrix} \mathcal{H} \begin{pmatrix} \hat{\mathbf{c}} \\ \hat{\mathbf{c}}^\dagger \end{pmatrix} + \frac{1}{2} \mathrm{Tr} h; \qquad \mathcal{H} = \begin{pmatrix} h & \Delta \\ -\Delta^* & -h^* \end{pmatrix}$$

Diagonalize single-particle Bogoliubov-de Gennes Hamiltonian:

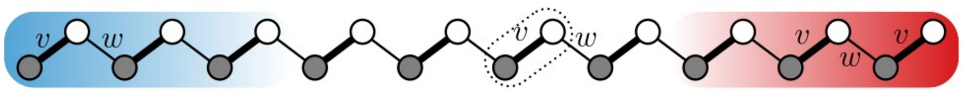
$$\mathcal{H} \begin{pmatrix} u_n^* \\ v_n^* \end{pmatrix} = E_n \begin{pmatrix} u_n^* \\ v_n^* \end{pmatrix}, \quad \text{with } E_n \geq 0 \text{ for } n = 1, \ldots, N;$$

$$\mathcal{H} \begin{pmatrix} v_n \\ u_n \end{pmatrix} = -E_n \begin{pmatrix} v_n \\ u_n \end{pmatrix}, \quad \text{for } n = 1, \ldots, N,$$

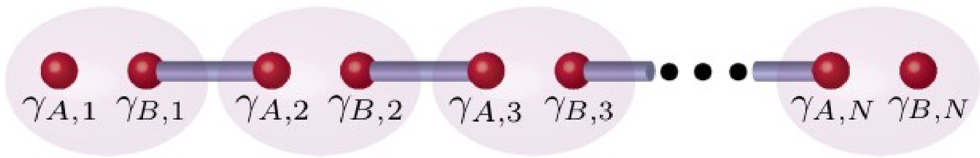
Obtain eigenmodes of system (particle-hole superpositions):

$$\hat{H} = \sum_{n=1}^{N} E_n \hat{d}_n^\dagger \hat{d}_n + \text{const} \qquad \hat{d}_n = \sum_m u_{nm} \hat{c}_m + v_{nm} \hat{c}_m^\dagger$$

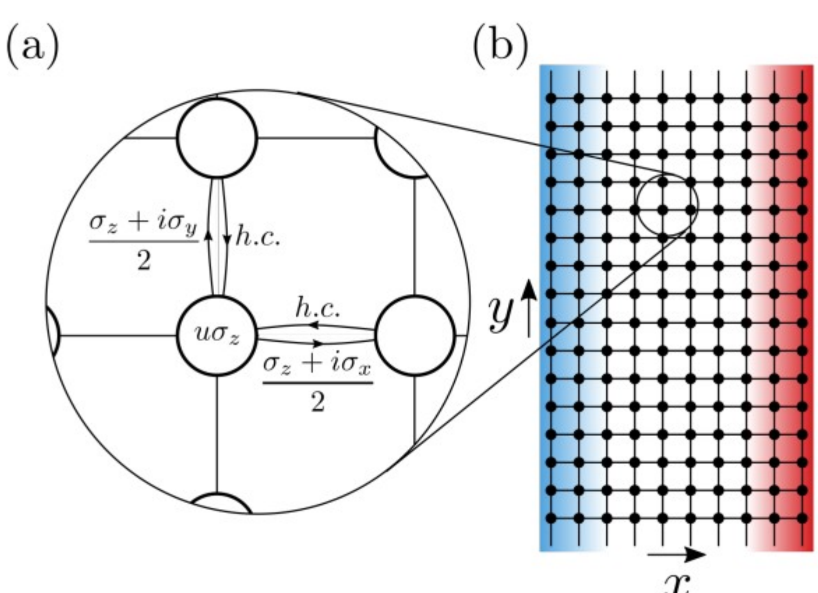
Single-particle topological Hamiltonians 1st semester can be understood as BdG Hamiltonians



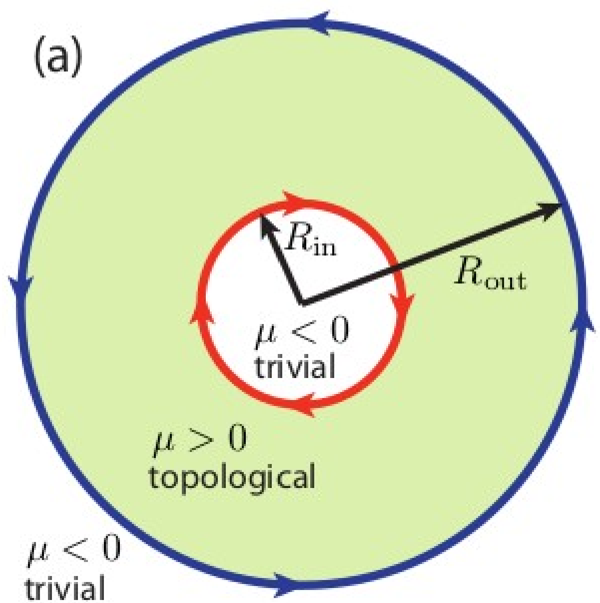
Su-Schrieffer-Heeger model
topological nanowire



Kitaev model for topological
superconductor nanowire
with / without chiral symmetry



Qi-Wu-Zhang model
2D Chern insulator

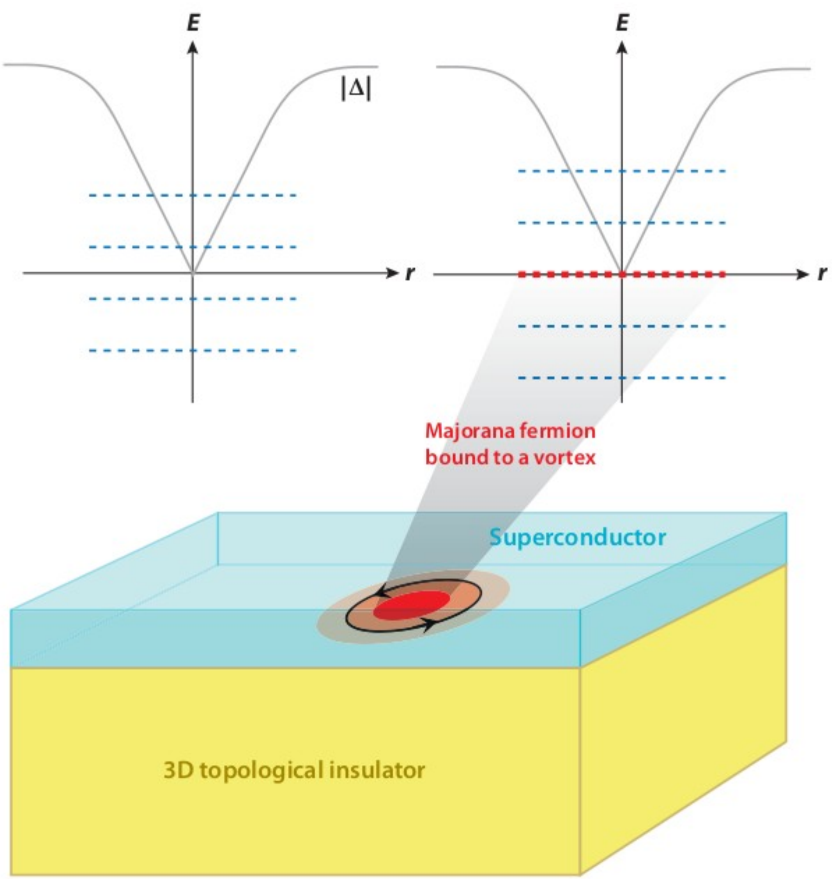


Lattice model for p+ip topological
superconductor

Topologically protected 0-energy bound states of BdG Hamiltonians are Majorana Zero Modes – each of them is half of a nonlocal fermion

Zero modes at end of nanowire

Zero modes also in centers of vortices of type II superconductors



Single Majorana Zero Mode:
single mode of doubled BdG Hamiltonian
→ particle-hole symmetry eigenstate
→ its own symmetry partner
→ pinned to 0 energy,
→ fermion parity switch operator
→ no charge, no spin, no mass

$$\hat{\gamma}_j^2 = 1$$

Any two Majoranas can be combined into a complex fermion, which represents a 0-energy excitation of the system.

$$\frac{1}{2}(\hat{\gamma}_1 + i\hat{\gamma}_2) = \hat{d}_{12}$$

Pairs of topological defects in topological superconductors host nonlocal zero-energy fermions

Electrons torn apart into half fermions

Quantum information written into fermions can be protected by splitting the fermions into Majorana Zero Modes

We will investigate schemes for writing, reading, manipulating qubits in Majorana Zero Modes

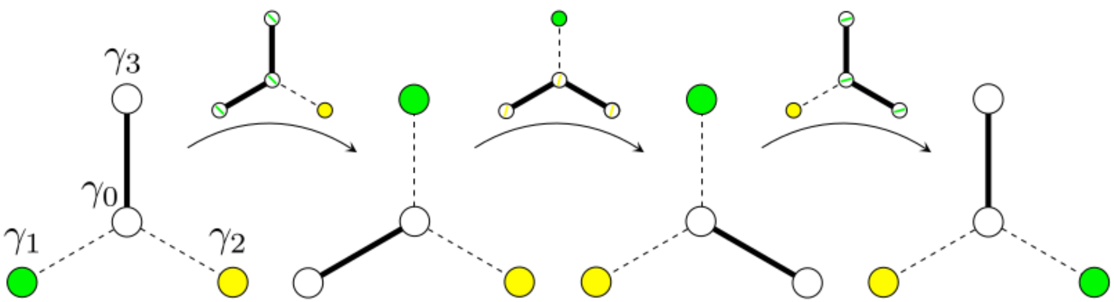
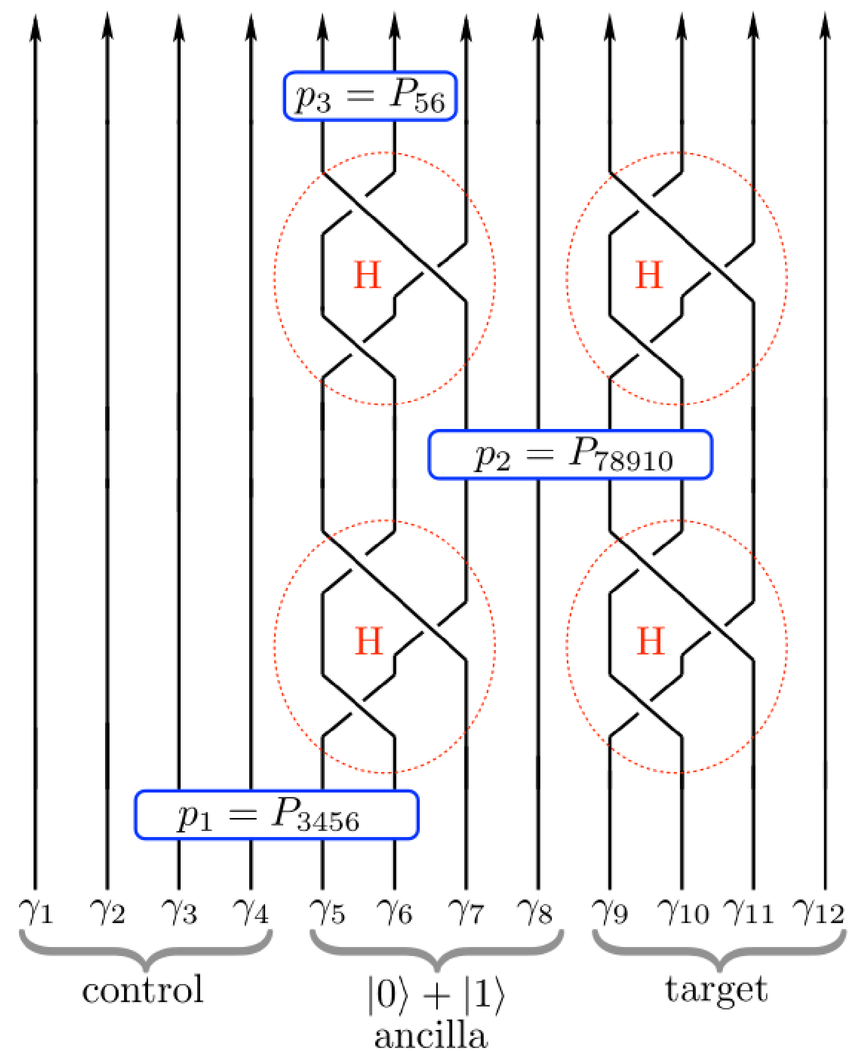
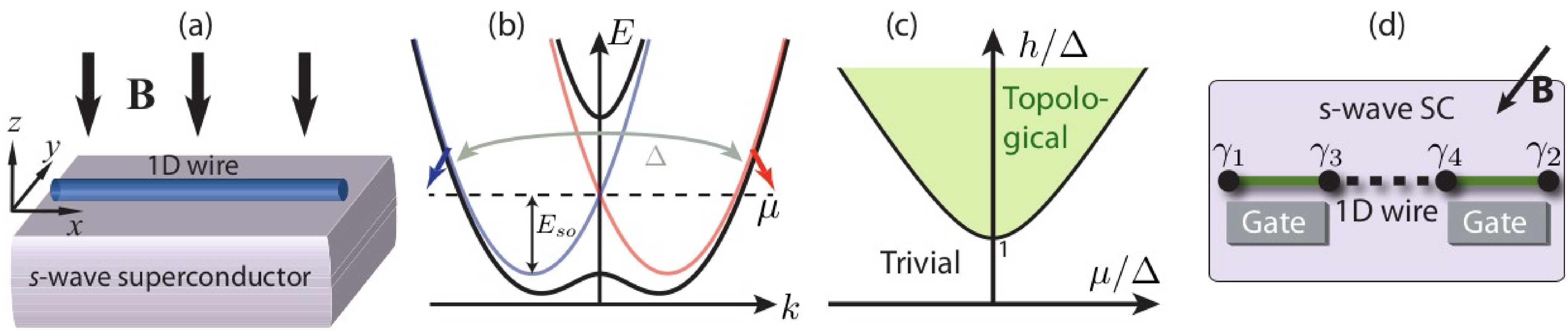


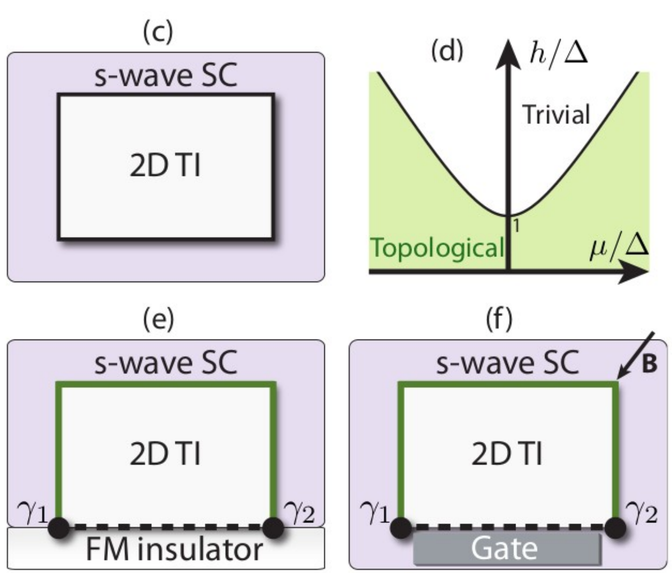
FIG. 6. Exchange of Majorana zero-modes via the three-point turn [20]. Three nanowires meet at a tri-junction, where

CNOT using braiding and collective parity measurements



We will discuss schemes to realize topological superconductors





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Write us an email if you are planning to do this course!
janos.asboth@gmail.com