Exam thematics:

1. Fundamentals of semiconductors, conductivity, structure, band structure, hybridization, etc.).
2. Charge carriers in intrinsic semiconductors, DOS, chemical potential, conductivity in in charge carrier mobility.
3. Charge carriers in extrinsic semiconductors, energy structure and occupation of donor le Conductivity of doped semiconductors.
4. Band structure calculation methods in semiconductors. Distinguished points of the k-spa approximation, the tight-binding method.
5. The k.p model and the envelope function aproximation. Relevance for doping.
6. Transport processes in semiconductors. Length scales, wave-packet, the semiclassical ap the relaxation time approximation.
7. Solution of the Boltzmann equation in a homogeneous electric field, correspondence to momentum relaxation, Matthiesen-rule, the Eliashberg-function. The Bloch-Grünneisen
8. Magnetotransport in semiconductors, the classical Hall effect, magnetoresistance.
9. Thermoelectric effects, reciprocal relations and coefficients, the Onsager relations, the S expression. The operation of the thermoelectric (Peltier) cooler.
10. Diffusion effects in semiconductors, minority charge carriers, charge carrier concentrati inhomogeneous semiconductors. The charge carrier diffusion length.
11. The p-n junction in biased and non-biased conditions. Rectification effect of diodes, the law.
12. Description of special diode types (avalanche breakdown, Zener effect and the Esaki dio bipolar transistor and its operation. Analogue electron tube devices.
13. Surface states, metal-semiconductor heterojunctions, the Schottky barrier. Operation of accummulation layer.
14. Fundamentals of JFET and MOSFET. CMOS based circuits, the CMOS NOT gate.

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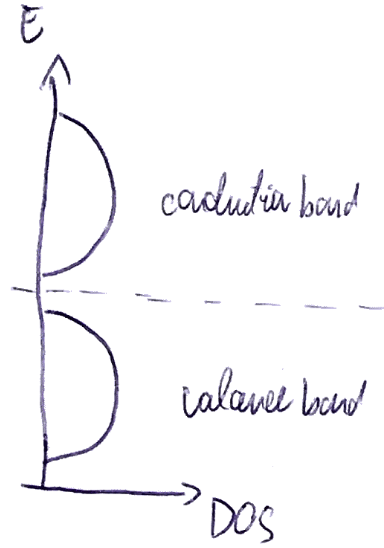
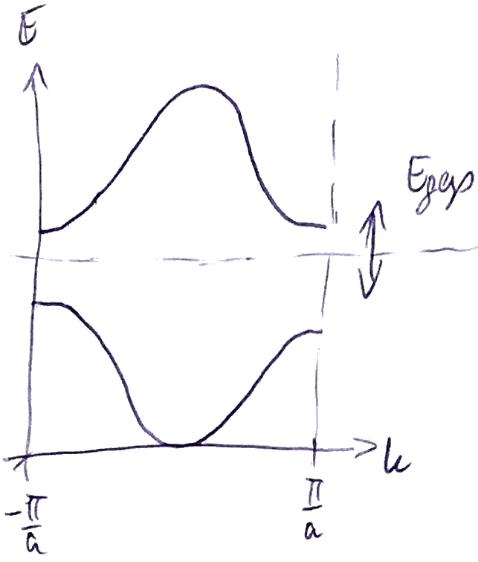
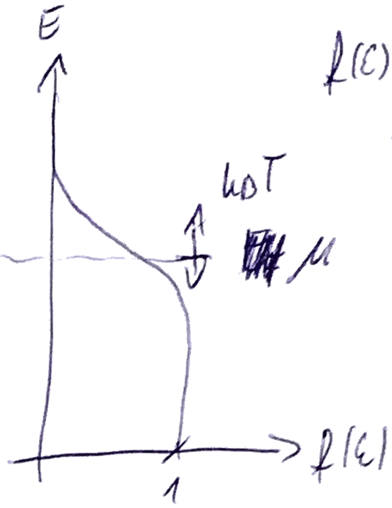






$$f(\varepsilon) = \frac{1}{1 + e^{\frac{\varepsilon - \mu}{k_B T}}}$$

$\Rightarrow$ $\hat{H}\, \psi_{n,k}(r) = E_n(k)\, \psi_{nk}(r)$ ⊕ Bloch theory: $\psi_{nk}(r) = e^{ikr} u_{nk}(r)$

⊕ $\underline{k}$ → crystal momentum ⊕ Periodic boundary condition: $\psi_{nk}(0) = \psi_{nk}(L)$

determined up to a reciprocal lattice vector ⇒ reduced zone scheme

$\Rightarrow e^{ikL} = 1 \Rightarrow kL = 2\pi n \Rightarrow k = \frac{2\pi}{L} n$

$\Rightarrow \Delta k = \frac{2\pi}{L} \Rightarrow D(k) = \frac{L}{2\pi}$ → equidistant, quasi continuous,

⊕ at zone boundaries ⇒ extrema $\Rightarrow \varepsilon_n(k) = \varepsilon_{n0} + \frac{\partial^2 \varepsilon_n(k)}{\partial k^2}(k - k_0)^2 =$

⇒ parabolic dispersion at extrema, $= \varepsilon_{n0} + \frac{\hbar^2}{2m^*}(k - k_0)^2$; $(m^*)^{-1} = \frac{1}{\hbar^2}\left(\frac{\partial^2 \varepsilon_n(k)}{\partial k^2}\right)$

DOS: 1D: $D(\varepsilon)\, d\varepsilon = D(k)\, dk \Rightarrow D(\varepsilon) = D(k)\left(\frac{d\varepsilon}{dk}\right)^{-1}$

	1D	2D	3D
$D(k)$	$\frac{L}{2\pi}$	$\frac{A}{(2\pi)^2}$	$\frac{V}{(2\pi)^3}$
$D(\varepsilon)$	$\frac{1}{\sqrt{\varepsilon}}$	const	$\sqrt{\varepsilon}$

⇐ quadratic dispersion $\Rightarrow \varepsilon = \frac{\hbar^2 k^2}{2m^*}$

2D: $D(k)\, 2\pi k\, dk = D(\varepsilon)\, d\varepsilon$

3D: $D(k)\, 4\pi k^2 dk = D(\varepsilon)\, d\varepsilon$

<u>conduction band</u>

$D(\varepsilon) \sim \sqrt{\varepsilon - \varepsilon_c}$

$\varepsilon_c(k) = \varepsilon_c + \frac{\hbar^2 (k - G)^2}{2 m_c^*}$

$m_c^* = 0.1, \ldots, 0.5 m_0$

<u>Valence band</u>

$D(\varepsilon) \sim \sqrt{\varepsilon_v - \varepsilon}$

$\varepsilon_v(k) = \varepsilon_v - \frac{\hbar^2 (k - G)^2}{2 m_v^*}$

$m_v^* = -1, \ldots, -3 m_0$

density of e^-:

$$n = \frac{1}{V}\int_{\varepsilon_0}^{\infty} D_c(\varepsilon) f(\varepsilon)\, d\varepsilon$$

density of holes: $[1-f(\varepsilon)]$

$$p = \frac{1}{V}\int_{-\infty}^{\varepsilon_V} D_V(\varepsilon)\, \overbrace{\cancel{\ \ }}^{[1-f(\varepsilon)]}\, d\varepsilon$$

$$\Rightarrow f(\varepsilon) = \frac{1}{1+e^{\frac{\varepsilon-\mu}{k_B T}}} \approx e^{-\frac{\varepsilon-\mu}{k_B T}}$$

$\uparrow$ $\varepsilon_g \gg k_B T$

$$\mu \approx \frac{\varepsilon_c+\varepsilon_V}{2}; \quad |\varepsilon-\mu| \approx \frac{\varepsilon_g}{2};$$

in a metal $\boxed{VS}$ $10^{22}\frac{1}{cm^3}$

in Ge at 300 K $\sim 10^{13}\frac{1}{cm^3}$

$$x = \frac{\varepsilon-\varepsilon_c}{k_B T}$$

$$\Rightarrow n \sim \int_{\varepsilon_c}^{\infty}\sqrt{\varepsilon-\varepsilon_c}\, e^{-\frac{\varepsilon-\mu}{k_B T}}\, d\varepsilon = \int_{\varepsilon_c}^{\infty}\sqrt{x}\, e^{-x}\, e^{-\frac{\varepsilon_c-\mu}{k_B T}}\, dx \sim \text{const}\cdot e^{-\frac{\varepsilon_c-\mu}{k_B T}}$$

$$\Rightarrow p \sim \text{const}\cdot e^{-\frac{\mu-\varepsilon_V}{k_B T}}$$

$\Rightarrow$ intrinsic SC $\Rightarrow n = p \Rightarrow \mu \approx \frac{\varepsilon_c+\varepsilon_V}{2}$

$$n\cdot p = n_i^2(T)$$

$\hookrightarrow$ Law of mass action

Drude modell:

$$ma = qE + \frac{\text{viscous}}{\text{drag}}\text{Force} \Rightarrow ma = qE - Kv \Rightarrow a = \frac{q}{m}E - \frac{K}{m}v \quad \oplus \quad \frac{K}{m} = \frac{1}{\tau}$$

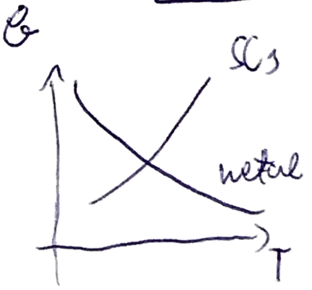
$\tau \approx 10^{-12} \ldots 10^{-15}\, s$

$$\Rightarrow \text{stationary solution} \Rightarrow 0 = \frac{q}{m}E - \frac{v}{\tau} \Rightarrow v_{drift} = \frac{qE\tau}{m}$$

$v_{drift} \approx 10^{-1} \ldots 10^{1}\,\frac{m}{s}$

$$\Rightarrow j = nev_d = \frac{ne^2\tau}{m}E \quad \oplus \quad j = \sigma E \Leftarrow \text{differential Ohm law} \Rightarrow \text{conductivity}: \sigma = \frac{ne^2\tau}{m}$$

$$\Rightarrow \text{mobility}: \frac{|v_d|}{E} = \mu = \frac{q\tau}{m}$$



	Electron	Holes
v_d	$-\frac{eE\tau_e}{m_e^*}$	$\frac{eE\tau_h}{m_h^*}$
σ	$\frac{ne^2\tau_e}{m_e^*}$	$\frac{pe^2\tau_h}{m_h^*}$
μ	$\frac{e\tau_e}{m_e^*}$	$\frac{e\tau_h}{m_h^*}$

$$\Rightarrow \sigma = ne\mu_e + pe\mu_h$$

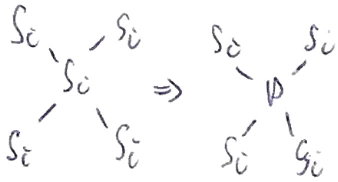
Si: $\mu_e = 1000\,\frac{cm^2}{V\cdot s}$

$\mu_h = 100\,\frac{cm^2}{V\cdot s}$

GaAs: $\mu_e = 30000\,\frac{cm^2}{V\cdot s}$

$\mu_h = 1000\,\frac{cm^2}{V\cdot s}$

extrinsic SC ⇒ doping ⇒ replacing some atoms with heteroatoms



⇒ p-doping: B:Si, Al:Si, III főcsoport

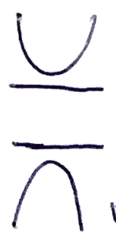
n-doping: P:Si, As:Si, V főcsoport

} ⇒ doping ⇒ extra e^- ⊕ extra p^+ → not a hole, bc it's localized

⇒ H atom problem ⇒ But: $m_0 \to m^*$, $\varepsilon_0 \to \varepsilon_0 \varepsilon_r$

↳ electron is moving in the lattice ≠ 100

↳ shielding due to the other $\bar{e}$

⇒ 

conduction band

donor level

acceptor level

valence band

} 25 meV ← much smaller than E_{gap} ⇒ thermal excitation

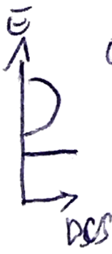
$\varepsilon_r \approx 10-20$

$m^* \approx 0.1-1$

⇒ Bohr radius: $a_B^* = \left(\frac{m}{m^*}\varepsilon_r\right) a_B \gg a_B$ 0,5 Å VS 30-100 Å

Binding energy: $E_1^* = \left(\frac{m^*}{m}\frac{1}{\varepsilon_r^2}\right) E_1 \ll E_1$ 13,6 eV VS 5-100 meV /1000

⇒ n_D → donor density ⇒ $d = \frac{1}{\sqrt[3]{n_D}}$ ⇒ average distance between the donors

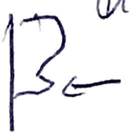


$d \ll a_B^*$ discrete energy

$\boxed{<1\text{ ppm}}$

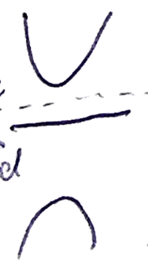
$d \lesssim a_B^*$ donor level

$\boxed{1...1000\text{ ppm}}$



$d \approx a_B^*$ ← degenerate SC ← basically a metal,

$\boxed{\text{a few \%}}$

⇒ ε_c  ε_d

μ - chemical potential

$\mu \approx \frac{1}{2}(\varepsilon_c + \varepsilon_d)$

⊕ expectation value of the occupation of the donor level,

↳ statphys ⊕ Grand Canonical ensemble

$$\langle n \rangle = \frac{\sum n e^{-\beta(E_n - \mu n)}}{\sum e^{-\beta(E_n - \mu n)}} \quad ⊕ \quad \beta = \frac{1}{k_B T}$$

⊕ n=0 — ionized

n=1 ↑ or ↓ neutral

$$\langle n \rangle = f_d(\varepsilon_d) = \frac{2e^{-\beta(\varepsilon_0+\varepsilon_d-\mu)}}{e^{-\beta\varepsilon_0} + 2e^{-\beta(\varepsilon_0+\varepsilon_d-\mu)}} = \frac{2e^{-\beta(\varepsilon_d-\mu)}}{1+2e^{-\beta(\varepsilon_d-\mu)}}$$

$$= \boxed{\frac{1}{1+\frac{1}{2}e^{\beta(\varepsilon_d-\mu)}}} \quad \text{VS} \quad \frac{1}{1+e^{\beta(\varepsilon-\mu)}}$$

$$f_d(\varepsilon_d) = \frac{1}{1+\frac{1}{2}e^{\beta(\varepsilon_d-\mu)}}$$ $\Rightarrow$ probability of ionizing the donor level

$$\Rightarrow 1-f_d(\varepsilon_d) = \frac{\frac{1}{2}e^{\beta(\varepsilon_d-\mu)}}{1+\frac{1}{2}e^{\beta(\varepsilon_d-\mu)}} = \frac{1}{1+2e^{\beta(\mu-\varepsilon_d)}} \quad \oplus \quad \mu \approx \frac{\varepsilon_c+\varepsilon_d}{2}$$

$$\Rightarrow 1-f_d(\varepsilon_d) = \frac{1}{1+2e^{\frac{\varepsilon_c-\varepsilon_d}{2k_BT}}} \Rightarrow n = n_d(1-f_d(\varepsilon_d))$$

intrinsic ⟷ extrinsic ⟷

depletion ⟷

intrinsic ⇓ usually never reached ⇓ rock-melt

depletion ↳ doesn't depend on T → ideal for devices

$\sigma = n e \mu_e$

I $T \to 0$, $n = 0$,

II $k_BT \approx \varepsilon_c - \varepsilon_d \Rightarrow n \approx n_d$,

III $k_BT \gtrsim \varepsilon_g$ from the valence band,

$\varepsilon_g = \varepsilon_c - \varepsilon_v$

Law of mass action

<u>intrinsic:</u>

$$\left.\begin{array}{l} n = p = n_i(T) \text{ or } np = n_i^2(T) \\ n = n_c e^{-\frac{\varepsilon_c-\mu}{k_BT}} \quad p = n_v e^{-\frac{\mu-\varepsilon_v}{k_BT}} \end{array}\right\} n\cdot p \sim e^{-\frac{\varepsilon_g}{k_BT}}$$

<u>extrinsic:</u>

$n \cdot p = n_i^2(T)$ is valid here too $\oplus$ $n = n_d \Rightarrow n_d p = n_i^2(T) \Rightarrow p = \frac{n_i^2(T)}{n_d} \ll n_i(T)$

$$\Rightarrow \mu \text{ shift} \Rightarrow \left.\begin{array}{l} p \text{ drops} \\ n \text{ increase} \end{array}\right\} \text{their product remains the same}$$

④

Band structure calculation methods,

① approx: calculate only with the valence $\bar{e}$ s

② approx: Born-Oppenheimer: ion cores (core $\bar{e}$'s + nucleus) are much havier
$\Rightarrow$ they are not moving

③ approx: Every $\bar{e}$ feels the same lattice periodic potential $(V(\underline{r}) = V(\underline{r}+\underline{R}))$

$\}\Rightarrow$ Schrödinger eq: $H\psi = E\psi$, $H = \frac{\hat{p}^2}{2m} + U(\underline{r})$, $\hat{p} = -i\hbar\nabla$,

$-\frac{\hbar^2}{2m}\Delta + U(\underline{r}) = H$, 1 $\bar{e}$ electron Schrödinger equation,

⊕ potential has the same periodicity as the lattice $\Rightarrow U(\underline{r}) = U(\underline{r}+\underline{R})$

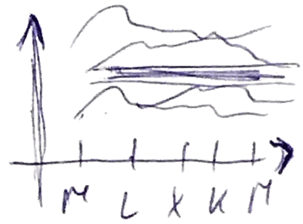
⊕ Bloch theorem: $\psi_{n\underline{k}}(\underline{r}) = e^{i\underline{k}\underline{r}}\, u_{n\underline{k}}(\underline{r})$, $u_{\underline{k}}(\underline{r}) = u_{\underline{k}}(\underline{r}+\underline{R})$, $\Leftarrow$ because of the discrete translational symmetry

$\Rightarrow$ $\underline{k}$ ~ crystal momentum $\Rightarrow$ determined up to a reciprocal lattice vector
$\Rightarrow$ reduced zone scheme $\Rightarrow$ n - band index, $\underline{k}$ - cryst. momentum, $\Rightarrow$ define a state,

⊕ periodic boundary condition: $\Rightarrow \psi(0) = \psi(L) \Rightarrow e^{ikL} = 1$
$\Rightarrow kL = 2\pi n \Rightarrow k = \frac{2\pi}{L} n \Rightarrow \Delta k = \frac{2\pi}{L}$ ⊕ $k = \frac{2\pi}{a}\frac{n}{N}$

⊕ $v = \frac{1}{\hbar}\frac{\partial \varepsilon(k)}{\partial k}$; $(m^*)^{-1} = \frac{1}{\hbar^2}\frac{\partial^2 \varepsilon(k)}{\partial k^2}$; $F_{extend} = \hbar \dot{\underline{k}}$;

Band diagram

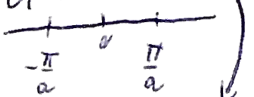


Distinguished points in k-space,

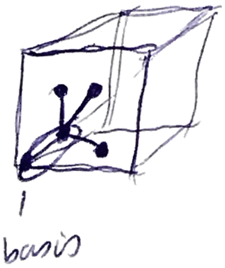
3D diamond lattice = FCC + 2 same atom (Zinc Blende different atoms)

1D

Wigner Seitz cell of the reciprocal lattice

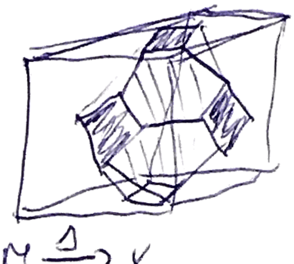


Brillouin zone



basis

$\Rightarrow$

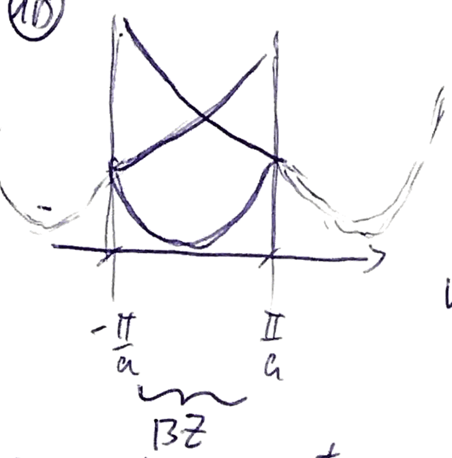


100: $\Gamma \xrightarrow{\Delta} X$
111: $\Gamma \xrightarrow{\Lambda} L$ 110: $\Gamma \xrightarrow{\Sigma} K$

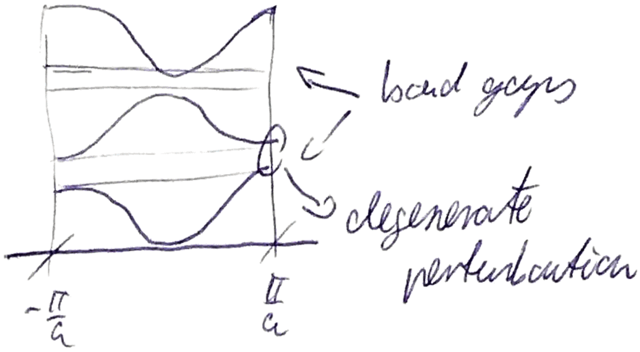
Γ: zone centre
L: middle of the hexagon
X: middle of the square
K: middle point of two bordering hexagon

Empty lattice and quasi free e^-;

(1D)



$\Longrightarrow$ $U(x)$ weak periodic potential



Reduced zone picture,

$$H = -\frac{\hbar^2}{2m}\partial_x^2, \quad \psi_k = \frac{1}{\sqrt{V}} e^{ikr}$$

$$\varepsilon(k) = \frac{\hbar^2 k^2}{2m},$$

$$H = -\frac{\hbar^2}{2m}\partial_x^2 + U(x), \quad U(x) = U(x + na)$$

$\Rightarrow H = H^0 + U(x) \Rightarrow U(x)$ perturbation,

$$\Rightarrow \varepsilon^0_{\underline{G}}(\underline{k}) = \frac{\hbar^2(\underline{k}+\underline{G})^2}{2m} \quad \psi^0_{\underline{G},\underline{k}}(\underline{r}) = \frac{1}{\sqrt{V}} e^{i(\underline{k}+\underline{G})\underline{r}}$$

<u>2nd order non-degenerate perturbation</u> $\Rightarrow \varepsilon_{\underline{G}}(\underline{k}) = \varepsilon^0_{\underline{G}}(\underline{k}) + \langle \underline{k}\underline{G}|U|\underline{k}\underline{G}\rangle + \sum_{\underline{G}' \neq \underline{G}} \frac{|\langle \underline{k}\underline{G}|U|\underline{k}\underline{G}'\rangle|^2}{\varepsilon^0_{\underline{G}}(\underline{k}) - \varepsilon^0_{\underline{G}'}(\underline{k})}$

$$\langle \underline{k}\underline{G}|U|\underline{k}\underline{G}'\rangle = \int d^3r \frac{1}{\sqrt{V}}\frac{1}{\sqrt{V}} e^{-i(\underline{k}+\underline{G})\underline{r}} V(\underline{r}) e^{i(\underline{k}+\underline{G}')\underline{r}} = \tilde{V}(\underline{G}-\underline{G}') \leftarrow \text{Fourier transform}$$

$$\Rightarrow \varepsilon_{\underline{G}}(\underline{k}) = \varepsilon^0_{\underline{G}}(\underline{k}) + \underbrace{\tilde{V}(\underline{G}=0)}_{\text{constant shift}} + \sum_{\underline{G}' \neq \underline{G}} \frac{|\tilde{V}(\underline{G}-\underline{G}')|^2}{\varepsilon^0_{\underline{G}}(\underline{k}) - \varepsilon^0_{\underline{G}'}(\underline{k})}$$

not good at the zone boundary

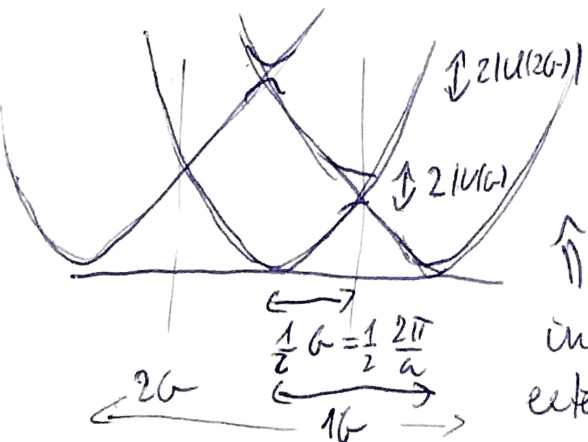
<u>Degenerate perturbation</u>: $\Rightarrow \varepsilon^0_{\underline{G}}(\underline{k}_0) = \varepsilon^0_{\underline{G}'}(\underline{k}_0), \quad k_0 = 0, \pm\frac{\pi}{a}$ 1D-ben

$\Rightarrow H$ generaten a degenerált altéren kifejtve $|\underline{k}\underline{G}\rangle, |\underline{k}\underline{G}'\rangle$,

$$\Rightarrow H = \begin{pmatrix} \varepsilon^0_{\underline{G}}(\underline{k}) + \tilde{V}(0) & \tilde{V}(\underline{G}-\underline{G}') \\ \tilde{V}(\underline{G}'-\underline{G}) & \varepsilon^0_{\underline{G}'}(\underline{k}) + \tilde{V}(0) \end{pmatrix} \Rightarrow$$

eigen values gives the energies

$$\Rightarrow \Delta = 2|\tilde{V}(\underline{G}-\underline{G}')|$$



$\Uparrow$ in the extended zone scheme,

Tight binding approximation:

1D, 1 atom, 1 orbit, $H\psi = E\psi$, $H = -\frac{\hbar^2}{2m}\Delta + U(\underline{r})$, $U(\underline{r}) = \sum_{\underline{R}} V_{atom}(\underline{r}-\underline{R})$

$$\psi_{\underline{k}}(\underline{r}) = \frac{1}{\sqrt{N}} \sum_{\underline{R}} e^{i\underline{k}\underline{R}} \varphi_{atom}(\underline{r}-\underline{R}), \quad \left[-\frac{\hbar^2}{2m} + V_{atom}\right]\varphi_a = \varepsilon_a \varphi_a$$

$$\Rightarrow \psi_{\underline{k}}(\underline{r}) = \frac{1}{\sqrt{N}} \sum_n e^{ikna} \varphi_a(r-na) = \frac{1}{\sqrt{N}} \sum_n e^{ikna} |n\rangle$$

$\varphi_{atom}(\underline{r}) \to$ Wannier function exponentially vanishing

$$\Rightarrow H\psi = \sum_n e^{ikna}\left[-\frac{\hbar^2}{2m}\Delta + \sum_{n'} V_{atom}(r-n'a)\right]|n\rangle = E\sum_n e^{ikna}|n\rangle \quad /\langle n''|$$

$$\Rightarrow \langle n''|n\rangle = \delta_{nn''} \oplus \sum_{n'} V_{atom}(r-n'a) = V_{atom}(r-na) + \sum_{n'\neq n} V_{atom}(r-n'a) \qquad e^{ikn''a}$$

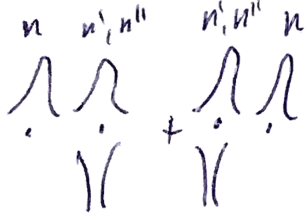
$$\Rightarrow \sum_n e^{ikna}\left[\varepsilon_a \delta_{nn''} + \sum_{n'\neq n} \langle n''|V_{atom}(r-n'a)|n\rangle\right] = E\sum_n e^{ikna}\delta_{nn''} \quad / e^{-ikn''a}$$

$$\Rightarrow E(k) = \varepsilon_c + \sum_{n, n'\neq n} e^{ik(n-n'')a} \langle n''|V_{atom}(r-n'a)|n\rangle$$

$\Rightarrow$ Most of the term are vanishing $\Rightarrow |n\rangle$ decay fast

Nearest neighbour:

$$\sum_n e^{ik(n-n'')a}\langle n''|\sum_{n'\neq n} V_{atom}(r-n'a)|n\rangle =$$



$$= \sum_n e^{ik(n-n'')a}\langle n''|\Delta U|n\rangle \to -|t|, \text{ if } n''=n\pm 1$$

$$\Rightarrow E(k) = \varepsilon_c - |t|\left(e^{ika} + e^{-ika}\right) = \varepsilon_c - 2|t|\cos(ka)$$

p orbitals

$$\Rightarrow E(k) = \varepsilon_c + 2|t|\cos(ka)$$

2nd nearest neighbour

$$E(k) = \varepsilon_a - 2|t_{nn}|\cos ka - 2|t_{2nd\,nn}|\cos(2ka)$$

↳ semi empirical model $\Rightarrow |t|$ is from an experiment, ③

$\underline{k}\cdot p$ model, envelope function, relevance for doping,

⇒ band structure calculation ⇒ particularly effective mass calculation

⇒ perturbation based method

⇒ semi empirical ⇒ fitting with measured data

canonical impulse

Schrödinger eq: $H\psi = E\psi$; $H = \frac{\hat{p}^2}{2m} + V = -\frac{\hbar^2}{2m}\Delta + V$, $p = -i\hbar\nabla$

Bloch theorem: ⇒ Discrete translational symmetry $\psi_{n,\underline{k}}(\underline{r}) = e^{i\underline{k}\underline{r}} u_{n,\underline{k}}(\underline{r})$,

⇒ $\underline{k}$ ⇒ crystal momentum ⇒ $\hbar\underline{k} \neq p$ (it's only for plane waves)

$$\Rightarrow \Delta\psi_{n,\underline{k}}(\underline{r}) = \nabla^2\psi_{n\underline{k}}(\underline{r}) = \nabla\left[i\underline{k}\, e^{i\underline{k}\underline{r}} u_{n\underline{k}}(\underline{r}) + e^{i\underline{k}\underline{r}}\nabla u_{n,\underline{k}}(\underline{r})\right] =$$

$$= -\underline{k}^2 e^{i\underline{k}\underline{r}} u_{n\underline{k}}(\underline{r}) + i\underline{k}\, e^{i\underline{k}\underline{r}}\nabla u_{n\underline{k}}(\underline{r}) + i\underline{k}\, e^{i\underline{k}\underline{r}}\nabla u_{n,\underline{k}}(\underline{r}) + e^{i\underline{k}\underline{r}}\Delta u_{n\underline{k}}(\underline{r}) =$$

$$= -\underline{k}^2 e^{i\underline{k}\underline{r}} u_{n,\underline{k}}(\underline{r}) + 2(i\underline{k})e^{i\underline{k}\underline{r}}\nabla u_{n,\underline{k}}(\underline{r}) + e^{i\underline{k}\underline{r}}\Delta u_{n,\underline{k}}(\underline{r}),$$

$$\Rightarrow \left[\frac{\hbar^2 k^2}{2m} - \frac{\hbar^2}{2m}2(i\underline{k})\nabla - \frac{\hbar^2}{2m}\Delta + V(\underline{r})\right]u_{n,\underline{k}}(\underline{r}) = E_{n,\underline{k}}\, u_{n\underline{k}}(\underline{r})$$

$$\Rightarrow \left[\frac{\hbar^2 k^2}{2m} + \frac{\hbar\underline{k}\cdot p}{m} + \frac{p^2}{2m} + V(\underline{r})\right]u_{n,\underline{k}}(\underline{r}) = E_{n,\underline{k}}\, u_{n,\underline{k}}(\underline{r})$$

treat as a perturbation

$$\Rightarrow \text{at } \Gamma \Rightarrow \underline{k} = 0 \Rightarrow \left[\frac{p^2}{2m} + V(\underline{r})\right]u_{n,0}(\underline{r}) = E_{n,0}\, u_{n,0}(\underline{r}) \oplus \overbrace{\frac{\hbar^2 k^2}{2m} + \frac{\hbar}{m}\underline{k}\cdot p},$$

⇒ E_n, $|n\rangle$ original solution ⊕ 2nd order, non degenerate perturbation,

$$E_n = \varepsilon_n + \langle n|V|n\rangle + \sum_{n'\neq n}\frac{|\langle n'|V|n\rangle|^2}{\varepsilon_{n'} - \varepsilon_n} \oplus \psi_n = |n\rangle + \sum_{n'\neq n}\frac{\langle n'|V|n\rangle}{\varepsilon_{n'}^0 - \varepsilon_n}|n'\rangle$$

$$u_{n,\underline{k}} = u_{n,0} + \frac{\hbar}{m} \sum_{n' \neq n} \frac{\langle u_{n',0} | \underline{k} \cdot \underline{p} | u_{n,0} \rangle}{\varepsilon_{n',0} - \varepsilon_{n,0}} | u_{n',0} \rangle$$

$\frac{\hbar^2 k^2}{2m} \langle n' | n \rangle \sim \delta_{nn'} = 0$

$$\varepsilon_{n,\underline{k}} = \varepsilon_{n,0} + \frac{\hbar^2 k^2}{2m} + \frac{\hbar^2}{m^2} \sum_{n' \neq n} \frac{\langle u_{n',0} | \underline{k} \cdot \underline{p} | u_{n,0} \rangle^2}{\varepsilon_{n',0} - \varepsilon_{n,0}} \quad (1)$$

$\langle n | \frac{\hbar}{m} \underline{k} \cdot \underline{p} | n \rangle = 0$ linear in k

$\Rightarrow$ Effective mass approximation $\quad \varepsilon_{n,\underline{k}} = \varepsilon_{n,0} + \frac{\hbar^2 k^2}{2m^*} \quad (2)$

$$(m^*)^{-1} = \frac{1}{\hbar^2} \frac{\partial^2 E(k)}{\partial k^2};$$

$$\Rightarrow (1) \text{ VS } (2) \Rightarrow \frac{1}{m^*} = \frac{1}{m} + \frac{2}{m^2 k^2} \sum_{n' \neq n} \overbrace{\frac{\langle u_{n',0} | \underline{k} \cdot \underline{p} | u_{n,0} \rangle}{\varepsilon_{n',0} - \varepsilon_{n,0}}}^{\sim p^2} \quad (+)$$

look at the bottom of the conduction band, top of the vallence band,

$$\Rightarrow \frac{1}{m^*} = \frac{1}{m} + \frac{2p^2}{m^2 E_g}$$

$u_{n0} = u_{\text{valence}\,0}$, $u_{n'0} = u_{\text{conduction}\,0}$;

$\Rightarrow \frac{2p^2}{m} \approx 20\,\text{eV}$ a legtöbb IV, III–V, félvezetőre $\Rightarrow$ mérésből,

Envelope function method

↳ slowly varying external fields ($\underline{B}, \underline{E}$, doping, hetero structures)
↳ characteristic length is larger than the lattice constant,
↳ goal is to get the effect of the external field,
↳ perturbation based method ⊕ Semi-empiric without external field,

$\Rightarrow H_0 \psi = E \psi$, $\psi_{n\underline{k}}(\underline{r}) = e^{i\underline{k}\underline{r}} u_{n\underline{k}}(\underline{r})$, $\Rightarrow$ we know this,

$\Rightarrow H = H_0 + V(\underline{r})$ ↳ perturbation ⊕ $V(\underline{k}) = \int e^{i\underline{k}\underline{r}} V(\underline{r}) d\underline{r}^3 \approx \delta_{\underline{k}} V_0$

↳ slowly varying $\Rightarrow$ simple FFT

$\Rightarrow H \Phi = E \Phi$ ⊕ Ansatz: $\Phi(\underline{r}) = \sum_{n,\underline{k}} F_n(\underline{k}) \psi_{n,\underline{k}}(\underline{r}) \Leftarrow$ Linear Combination of Atomic orbitals (LCAO)

↳ envelope function,

$$\Rightarrow \sum_{n,\underline{k}} \psi_{n,\underline{k}} \left[E_{n\underline{k}} - E + V(\underline{r}) \right] F_n(\underline{k}) = 0 \qquad / \langle \psi_{n',\underline{k}'}(\underline{r}) |$$

$$\Rightarrow \sum_{n,\underline{k}} \left[(E_{n,\underline{k}} - E) \delta_{nn'} \delta_{\underline{k}\underline{k}'} + \langle \psi_{n'\underline{k}'} | V(\underline{r}) | \psi_{n,\underline{k}} \rangle \right] F_n(\underline{k}) = 0$$

$$\langle \psi_{n'\underline{k}'} | V(\underline{r}) | \psi_{n,\underline{k}} \rangle = \int \underbrace{u^*_{n'\underline{k}'}(\underline{r}) u_{n,\underline{k}}(\underline{r})}_{\text{rapidly varying}} \underbrace{e^{i(\underline{k}-\underline{k}')\underline{r}} V(\underline{r})}_{\text{slowly varying}} d\underline{r}^3 \approx$$

$$\approx \int u^*_{n'\underline{k}'}(\underline{r}) u_{n\underline{k}}(\underline{r}) d\underline{r}^3 \int e^{i(\underline{k}-\underline{k}')\underline{r}} V(\underline{r}) d\underline{r}^3 = \delta_{nn'} V(\underline{k}-\underline{k}')$$

$$\Rightarrow \underbrace{\sum_{\underline{k}} \left[(E_{n\underline{k}} - E) \delta_{nn'} + V(\underline{k}-\underline{k}') \right] F_n(\underline{k}) = 0}_{} \Leftarrow \text{Schrödinger eq for } F_n(\underline{k})$$

E.g.:

$$E_n(k) = E_c + \frac{\hbar^2 k^2}{2m_c^*}$$ ⇐ conduction band, effective mass approximation,

Fourier space

$$\underbrace{\frac{\hbar^2 k^2}{2m_c^*} F_c(k) + \sum_{k'} V(k-k') F_c(k) = (E - E_c) F_c(k)}$$

$\Rightarrow FT(f'(x)) = ik f(k), \quad FT(V(x) F_c(x)) = \int V(k-k') F_c(k) dk^2$

⊕

$\Rightarrow -i\nabla \Rightarrow k$

$$\underbrace{\left[-\frac{\hbar^2}{2m_c^*} \Delta + E_c + V(r) \right] F_c(r) = E F_c(r)},$$

Real space

⇒ n-doped Si, ⇒ extra potential ⇒ extra proton ⇒ Coulomb potential,

$$\Rightarrow n(r) = \sum_i |\psi_i(r)|^2 f(E_i) = n_{c\phi} \sum_i F_i(r) f(E_i)$$

original charge density

modulation,

⑥

<u>Transport processes, length scales,</u> $l < \delta_\phi < \delta_s < \delta_c$

$\sim 1-10\,nm$ $\sim 10\,nm - 1\,\mu m$ $\sim 1-100\,\mu m$ $\sim 100\,\mu - 1\,cm$

l - mean free path

Ballistic transport

$\tau \sim 10^{-13} - 10^{-15}\,s$

mesoscopic physics — makroskopic physics, Ohm's law, Boltzmann eq.

$l = v_F \cdot \tau$, τ ~ momentum relaxation time

$\sim 1\mu s \ldots 10\,ms$

δ_c ~ charge carrier diffusion length $\rightarrow \tau_c$

δ_s ~ spin diffusion length $\rightarrow \tau_s$

δ_ϕ ~ phase coherence length τ_ϕ

} diffusion

$\xrightarrow{\Delta t}$

$j = -D\nabla n$

$\delta = \sqrt{D\tau}$ ← length ↔ time

Makroscopic physics ⇒ > 1 μm ⇒ diffusion, drift ⊕ equilibrium because of collisions

↳ spontaneous ↳ external force

<u>Free electrons</u>

quantum number $\underline{k}$ ← periodic b.c.

$\varepsilon(\underline{k}) = \frac{\hbar^2 k^2}{2m}$

$v = \frac{\hbar k}{m}$

$\psi_k(\underline{r}) = \frac{1}{\sqrt{V}} e^{i\underline{k}\underline{r}}$

$m^* = m$

<u>Bloch electrons</u>

$\underline{k}$ ← periodic boundary, n ← band index

$\varepsilon_n(\underline{k})$ ~ dispersion relation $\rightarrow \varepsilon_n(\underline{k}) = \varepsilon_n(\underline{k} + \underline{G})$

$v_n(\underline{k}) = \frac{1}{\hbar} \frac{\partial \varepsilon_n(\underline{k})}{\partial \underline{k}}$

$\psi_{n\underline{k}}(\underline{r}) = e^{i\underline{k}\underline{r}} u_{n\underline{k}}(\underline{r}) \rightarrow u_{n\underline{k}}(\underline{r} + \underline{R}) = u_{n\underline{k}}(\underline{r})$

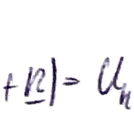
$m^* = \frac{1}{\hbar^2} \frac{\partial^2 \varepsilon}{\partial k^2}$ ⇒ at band edges ⇒ quasi free electron approx $\varepsilon_n(\underline{k}) = \varepsilon_{n0} + \frac{\hbar^2}{2m^*}(\underline{k} - \underline{k}_0)^2$

⇒ Drude model : collision with ions ← resistivity

⇒ Bloch formalism: ⇒ perfect lattice with ions → $\rho = 0$, $\sigma = \infty$

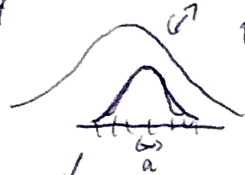
⇒ canonical impulse = $\hat{p} = -i\hbar\nabla$ ⇒ $\hat{p} \neq \hbar \underline{k}$

⇒ $\hbar \dot{\underline{k}} = F_{ext}$ ← Semiclassical approx ← crystal momentum



well localised in k

wave packet picture



external force

not localised on a site,

Closed bands and transport => concept of holes:

$\underline{j} = -e \int dk^3 \, \underline{v}(\underline{k}) \, N(\underline{k}) \, f(\underline{k})$

$\underline{j} = -e \underbrace{\int}_{\text{closed band}} \frac{dk^3}{4\pi^3} \frac{1}{\hbar} \frac{\partial \varepsilon}{\partial \underline{k}} = \phi$ not trivial

$\underbrace{-e \int_{\text{until } \varepsilon} \frac{dk^3}{4\pi^3} \frac{1}{\hbar} \frac{\partial \varepsilon}{\partial \underline{k}}}_{\text{current of } e^-} = \underbrace{e \int_{\text{above } \varepsilon} \frac{dk^3}{4\pi^3} \frac{1}{\hbar} \frac{\partial \varepsilon}{\partial \underline{k}}}_{\text{current of } h^+}$

if describe the movement of holes is equivalent to describing the electrons

Boltzmann equation: => $f(t, \underline{r}, \underline{k})$ => distribution function

stationary solution, equilibrium => $f^0(\underline{k}) = \frac{1}{1 + e^{\frac{\varepsilon(\underline{k}) - \mu}{k_B T}}}$

non-equilibrium $f(\underline{k}, t) = f^0(\underline{k}) + g(\underline{k}, t)$ -> linearization g <- deviation from equilibrium

$\frac{df}{dt} = \frac{\partial f}{\partial t} + \nabla_{\underline{r}} f \, \dot{\underline{r}} + \nabla_{\underline{k}} f \, \dot{\underline{k}} = \frac{\partial f}{\partial t}\Big|_{\text{collision}}$ -> Semiclassical approximation

=> equilibrium bc of collisions

$\dot{\underline{k}} \nabla_{\underline{k}} f$ ~ force term $\dot{\underline{k}} = \frac{1}{\hbar} F_{ext}$

$\dot{\underline{r}} \nabla_{\underline{r}} f$ ~ diffusion term

=> relaxation time approximation: $\frac{\partial f}{\partial t}\Big|_{coll} = -\frac{g}{\tau}$

-> proportional with the deviation

-> relaxation time, probability of scattering from k to k' (same as Drude

↳ no force, no diffusion

$\frac{df}{dt} = \frac{\partial f^0}{\partial t} + \frac{\partial g}{\partial t} = -\frac{g}{\tau}$ => $g(t) = g(t=0) e^{-t/\tau}$ => $f \to f^0$ after τ time, ⑦

$n = \int d^3k\, D(k) f(k)$, 3D: $D(k) = 2\, \frac{1}{(2\pi)^3} = \frac{1}{4\pi^3}$

spin   normalized to V

$j = -e\int d^3k\, v_k\, D(k) f(k)$,

$j_Q = \int d^3k\, (\varepsilon_k - \mu)\, v_k\, D(k) f(k)$, $v_k = \frac{1}{\hbar}\left(\frac{\partial \varepsilon}{\partial k}\right)$, $f_0(k) = \frac{1}{1+e^{\frac{\varepsilon-\mu}{k_B T}}}$,

Boltzmann equation, ⇒ Semiclassical approximation
⇒ describe the time changing of the distribution function

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \nabla_r f\cdot \dot{r} + \nabla_k f\cdot \dot{k} = \frac{\partial f}{\partial t}\Big|_{collision}$$

describe the collision in the system, heuristically

$\dot{k}\nabla_k f$: force term, $\hbar \dot{k} = F_{ext}$

$\dot{r}\nabla_r f$: diffusion term,

⊕ linearization of the problem, $f(k,t) = \underbrace{f^0(k)}_{equilibrium} + \underbrace{g(k,t)}_{non\ equilibrium}$ ⇒ $\frac{\partial f}{\partial t} = \frac{\partial g}{\partial t}$

⊕ relaxation time approx, $\frac{\partial f}{\partial t}\Big|_{collision} = \frac{f^0 - f}{\tau} = -\frac{g(k,t)}{\tau}$; τ – relaxation time (momentum)

$\frac{1}{\tau}$ – probability of scattering from $k \to k'$

⇒ zero external forces: $\frac{\partial g}{\partial t} = -\frac{g(k,t)}{\tau}$

$g(t) = g(t=0)\, e^{-t/\tau}$ → exponential decay to equilibrium,

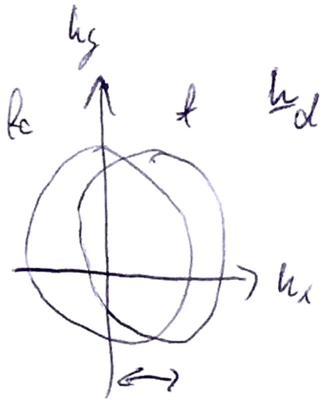
⊕ no diffusion, $F_{ext} = -eE$,

stationary case: $\frac{\partial f}{\partial t} = 0$, ⇒ $\underline{\nabla_k f\, \dot{k} = -\frac{g}{\tau}}$, ⊕ $\hbar\dot{k} = F_{ext} = -eE$ ⇒ $\dot{k} = -\frac{eE}{\hbar}$

⊕ $\nabla_k f = \frac{\partial f}{\partial \varepsilon}\frac{\partial \varepsilon}{\partial k} = \frac{\partial f}{\partial \varepsilon}\hbar v(k)$, $v(k) = \frac{1}{\hbar}\frac{\partial \varepsilon}{\partial k}$ } ⇒ $\frac{\partial f}{\partial \varepsilon}\hbar v(k)\left(-e\frac{E}{\hbar}\right) = -\frac{g}{\tau}$

⇒ $\underline{g(k,t) = \frac{\partial f}{\partial \varepsilon} E\, v(k)\, e\tau}$ ⊕ approx $\frac{\partial f}{\partial \varepsilon} \approx \frac{\partial f^0}{\partial \varepsilon}$ } ⇒ $\underline{f(k) = f^0(k) + \frac{\partial f^0}{\partial \varepsilon} E\, v\, e\tau}$,

⇒ $\underline{f(k) \approx f^0\left(k + \frac{eE\tau}{\hbar}\right) \approx f^0(k) + \frac{eE\tau}{\hbar}\frac{\partial f^0}{\partial k} = f^0(k) + \frac{\partial f}{\partial \varepsilon} E\, v(k)\, e\tau}$,



$\underline{k}_d = -\frac{e\underline{E}\tau}{\hbar}$ ⇒ displacement of the distribution function,

⊕ Drude model: $ma = qE - Kv \Rightarrow a = \frac{q}{m}E - \frac{K}{m}v \quad \frac{K}{m} = \frac{1}{\tau}$

$\Rightarrow a = 0 \Rightarrow \frac{q}{m}E = \frac{v}{\tau} \Rightarrow v_d = \frac{qE\tau}{m}$

$\Rightarrow \hbar \underline{k}_d = m^* \underline{v}_D \Rightarrow v_D = \hbar \frac{k_d}{m^*} = \frac{eE\tau}{m^*}$ ⇒ we get the same result

Drude

→ all of the electrons are moving slowly

$v = v_d \ll v_F$

$j = nev_d$

Bloch (Boltzmann) model

→ just a few e^- near the Fermi surface are moving with the Fermi velocity

$v = v_F$

$j = nev_d = \overbrace{\frac{nv_d}{v_F}}^{\text{number of moving } e^-}\, e\, v_F$

Momentum scattering time, Matthiesen's rule,

$\tau \Rightarrow$  , $\ell = v_F \tau$, $\tau = 10^{-13} \ldots 10^{-15}\,s$ ⇒ a lot of scattering

$\tau_c > \tau$

charge carrier lifetime ← $\tau_c \sim 1\mu s \ldots 1ms$

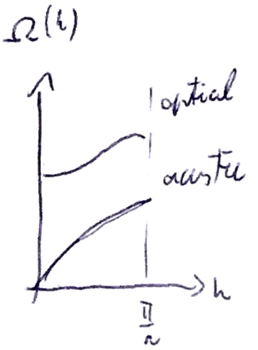
$\Rightarrow \frac{1}{\tau} = \frac{1}{\tau_{e-e}} + \frac{1}{\tau_{e-phonon}} + \frac{1}{\tau_{impurities}}$ ⇒ phonons, impurities, defects ⇒ shortening τ

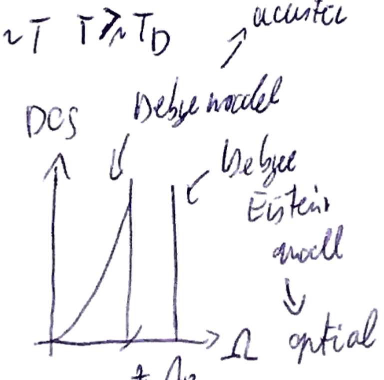
doping ⇒ low T → neutral dopants; high T → ionized dopants

$\frac{1}{\tau_{e-e}} \to T^2$

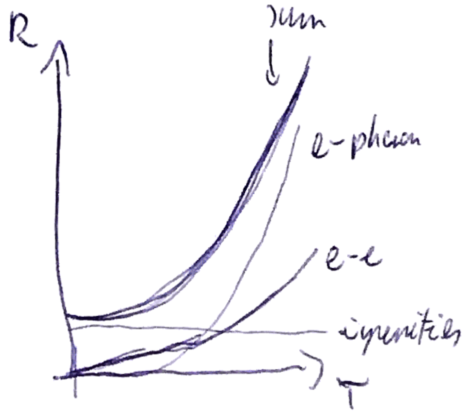
$\frac{1}{\tau_{e-phonon}} \to \sim T^5 \; T \ll T_D$; $\sim T \; T \gtrsim T_D$

$\frac{1}{\tau_{impurities}} \to$ const





$T_D = \frac{\hbar \Omega_D}{k_B}$



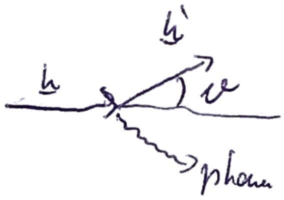
Eliashberg function:

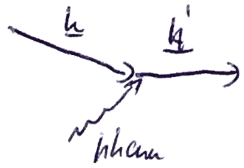
$$\frac{1}{\tau_{e\text{-phon}}} = 2\pi \frac{k_B}{\hbar} T \cdot \lambda \rightarrow \text{e-phonon coupling} = 0.1-1$$

$$\boxed{1-\cos\vartheta}$$

$$\lambda = 2 \cdot \int \frac{d\Omega}{\Omega} \underbrace{\alpha^2 F(\Omega)}_{} \quad , F(\Omega): \text{phonon DOS}$$

Eliashberg function





$\vartheta = 0$ forward scattering

$\vartheta = \pi$ backward scattering

$\Downarrow$

large contribution to τ

Bloch-Grüneisen formula:

$$\frac{1}{\tau_{e\text{-phon}}} = 2\pi \frac{k_B}{\hbar} T \lambda \underbrace{\int_0^{\Omega_D} \frac{d\Omega}{\Omega} \left(\frac{\Omega}{\Omega_D}\right)^4 \left[\frac{\hbar\Omega / k_B T}{\sinh\left(\frac{\hbar\Omega}{k_B T}\right)}\right]^2}_{} \quad \begin{cases} T \to 0 & \sim T^5 \\ T \to \infty & \sim T \end{cases}$$

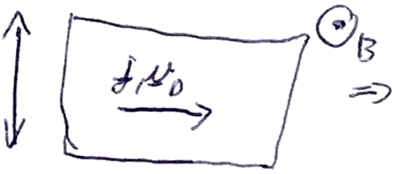
$T \to \infty \approx 1$

$T \to 0 \sim T^4$

(18.)

Classical Hall-effect:

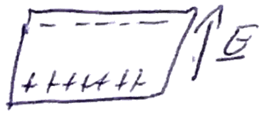
transversal



longitudinal

$\Rightarrow$ Lorentz force: $F = q\, v \times B$ ⊕ internal fields builds up

charge accumulation ↑



$\Rightarrow$ stationary state: $e E_{trans} = e v_D B \Rightarrow E_{trans} = v_D B$

⊕

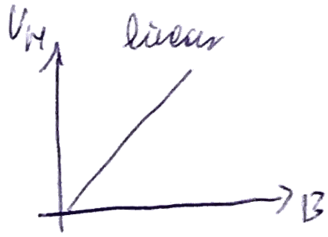
Drude model: $v_D = \frac{e\tau}{m^*} E_{long} = \mu E_{long}$

microscopic Ohm's law, // $j = \sigma E$, $j\rho = E$ //

$\Rightarrow$ Hall voltage: $V_H = d\, v_D B = \frac{d e \tau}{m^*} E_{long} B$

$\Rightarrow$ Hall resistivity: $R_H = \frac{E_{trans}}{j_{long}} = \frac{v_D B}{n e v_D} = \frac{1}{ne} B$ $\Rightarrow$ Hall constant: $R \equiv \frac{R_H}{B} = \frac{1}{ne}$

$\Rightarrow$ has a sign $\Rightarrow$ depends on the charge carrier $\Rightarrow$ $n < 0$ for e^-, $n > 0$ for h^+



$$\Rightarrow \begin{pmatrix} j_x \\ j_y \end{pmatrix} = \underbrace{\begin{pmatrix} \sigma_{xx} & -\sigma_{xy} \\ \sigma_{xy} & \sigma_{xx} \end{pmatrix}}_{\text{isotrop material}} \begin{pmatrix} E_x \\ E_y \end{pmatrix} \Rightarrow \underline{\underline{\sigma}}^{-1} = \underline{\underline{\rho}}$$

$v_D = \frac{e\tau}{m^*} E = \mu E$, $j = n e v_D = n e \mu E \Rightarrow \rho = \frac{1}{n e \mu}$

Hall resistivity ↓

$$\Rightarrow \rho_{xx} = \frac{\sigma_{xx}}{\sigma_{xx}^2 + \sigma_{xy}^2} = \frac{1}{n\mu e} \quad ⊕ \quad \rho_{xy} = \frac{\sigma_{xy}}{\sigma_{xx}^2 + \sigma_{xy}^2} = \frac{B}{|e| n}$$

from measurement of $\underline{\rho}$ ↙ ↓

$$\Rightarrow n, \mu \text{ measurement} \Rightarrow \left(\frac{\partial \rho_{xy}}{\partial B}\right)_{B=0} = \frac{1}{|e| n} \Rightarrow n = \frac{1}{|e| \left(\frac{\partial \rho_{xy}}{\partial B}\right)_{B=0}}, \quad \mu = \frac{\left(\frac{\partial \rho_{xy}}{\partial B}\right)}{\rho_{xx}}$$

Magnetoresistance:

$\Rightarrow v = \mu(\underbrace{E}_{\text{statatic field}} + \underbrace{v \times B}_{\text{Lorentz force}})$ $\Rightarrow$ solve for $v = \frac{\mu}{1 + (\mu B)^2}(\dots)$

↳ μ mobility is decreasing if increase the B $\Rightarrow$ longitudinal component of v decreasing.

(9.)

$\int dk^3\, N(k) f(k)$ OR $\int d\varepsilon\, D(\varepsilon) f(\varepsilon)$

$j_n = \int dk^3\, v(k) N(k) f(k)$

$j = -e \int dk^3\, v(k) N(k) f(k)$

$j_E = \int dk^3\, \varepsilon(k)\, v(k) N(k) f(k)$

$j_Q = \int dk^3\, (\varepsilon(k)-\mu)\, v(k) N(k) f(k)$ // $dU = \delta Q + \delta W = TdS + \mu dn \Rightarrow \delta Q = TdS = dU - \mu dn$ //

same integrals with $D(\varepsilon)$

$j_n = \int d\varepsilon\, v\, D(\varepsilon) f(\varepsilon)$

$j = -e \int d\varepsilon\, v\, D(\varepsilon) f(\varepsilon)$

⋮

$$\underline{\text{Boltzmann eq.:}}\quad \frac{df}{dt} = \frac{\partial f}{\partial t} + \underbrace{\dot{k} \nabla_k f}_{\text{Force term}} + \underbrace{\dot{r} \nabla_r f}_{\text{Diffusion term}} = -\frac{g}{\tau}\Big|_{\text{collision}}$$

(1) $f(r,k,t) = f^0(k) + g(k,r,t)$ (2) $\mu(r),\ T(r) \to \nabla\mu, \nabla T$

(3) neglect diffusion (4) $f^0 = \dfrac{1}{1+e^{\frac{\varepsilon-\mu}{k_B T}}} = \dfrac{1}{1+e^x}$, $x = \dfrac{\varepsilon-\mu}{k_B T} = \beta(\varepsilon-\mu)$

(5) $\hbar \dot{k} = F_{ext}$, $\dfrac{\partial f}{\partial k} = \dfrac{\partial f}{\partial \varepsilon}\dfrac{\partial \varepsilon}{\partial k} = \dfrac{\partial f}{\partial \varepsilon}\hbar v$ $\Big\} \Rightarrow \dot{k}\nabla_k f = v \dfrac{\partial f}{\partial \varepsilon} F_{ext}$

$\dot{k} = \dfrac{1}{\hbar} F_{ext}$,

(6) $$+ \dot{r} \frac{\partial f}{\partial x}\frac{\partial x}{\partial \mu}\nabla\mu + \dot{r}\frac{\partial f}{\partial x}\frac{\partial x}{\partial T}\nabla T$$ (7) $\dfrac{\partial f}{\partial x} = \dfrac{\partial f}{\partial \varepsilon}\dfrac{\partial \varepsilon}{\partial x} = \dfrac{\partial f}{\partial \varepsilon}\dfrac{1}{\beta}$

$\dfrac{\partial x}{\partial \mu} = -\beta$ $\quad \dfrac{\partial x}{\partial T} = (\varepsilon-\mu)\beta\left(-\dfrac{1}{T}\right)$

$$= v\frac{\partial f}{\partial \varepsilon}\left(\frac{1}{\beta}(-\beta)\nabla\mu\right) + v\frac{\partial f}{\partial \varepsilon}\left(\frac{1}{\beta}(\varepsilon-\mu)\beta\left(-\frac{1}{T}\right)\nabla T\right) = v\frac{\partial f}{\partial \varepsilon}\left(-\nabla\mu - (\varepsilon-\mu)\frac{1}{T}\nabla T\right)$$

$$\Big\} \Rightarrow \frac{df}{dt} = \frac{\partial f}{\partial t} + v\frac{\partial f}{\partial \varepsilon}\left[F_{ext} - \nabla\mu - (\varepsilon-\mu)\frac{\nabla T}{T}\right] = -\frac{g}{\tau}$$ (8) $\dfrac{\partial f}{\partial \varepsilon} \approx \dfrac{\partial f^0}{\partial \varepsilon}$, $\dfrac{\partial f}{\partial t} = 0$,

$$\Rightarrow f = f^0 - \tau v \frac{\partial f^0}{\partial \varepsilon}\left[\underbrace{F_{ext} - \nabla\mu}_{} - \underbrace{(\varepsilon-\mu)\frac{\nabla T}{T}}_{}\right]$$ $\Rightarrow$ appear in the integrals above.

Peltier effect $\leftarrow j$ $\quad$ ∇T ~~[illegible]~~ (Seebeck effect)

$\downarrow$ $\quad$ $\downarrow$

j_Q $\quad$ j

③

$\bar{j} = K_0\left(E + \frac{\nabla\mu}{e}\right) - K_1\left(-\frac{\nabla T}{T}\right)$; $j = -e\int dk^3\, v(k)\, N(k)\, f(k)$

$\bar{j}_Q = -K_1\left(E + \frac{\nabla\mu}{e}\right) + K_2\left(-\frac{\nabla T}{T}\right)$; $j_Q = \int dk^3\, (\varepsilon - \mu)\, v(k)\, N(k)\, f(k)$

Onsager relation ⇒ K_1 cross terms are the same

⇒ in equilibrium ⇒ process and inverse process have the same probability,

Heat conductivity: $\underline{j_Q \triangleq -\kappa \nabla T}$, $\bar{j} = \emptyset$, ⇒ $K_0\left(E + \frac{\nabla\mu}{e}\right) = K_1\left(-\frac{\nabla T}{T}\right)$

⇒ $-K_1\left(E + \frac{\nabla\mu}{e}\right) = -\frac{K_1^2}{K_0}\left(-\frac{\nabla T}{T}\right)$ ⇒ $\bar{j}_Q = -\left(\frac{K_2}{T} - \frac{K_1^2}{K_0}\frac{1}{T}\right)\nabla T$ $\quad \underline{\kappa = \frac{K_2}{T} - \frac{K_1^2}{K_0}}$

Electric conductivity: $\underline{\bar{j} \triangleq \sigma\left(E + \frac{\nabla\mu}{e}\right)}$, $\bar{j}_Q = \emptyset$ ⇒ $K_1\left(E + \frac{\nabla\mu}{e}\right) = K_2\left(-\frac{\nabla T}{T}\right)$

$-K_1\left(-\frac{\nabla T}{T}\right) = -\frac{K_1^2}{K_2}\left(E + \frac{\nabla\mu}{e}\right)$ ⇒ $\bar{j} = \left(K_0 - \frac{K_1^2}{K_2}\right)\left(E + \frac{\nabla\mu}{e}\right)$, $\underline{\sigma = K_0 - \frac{K_1^2}{K_2}}$

Seebeck effect: $E = \emptyset$, $\nabla T \neq \emptyset$ } $j \neq 0$ OR $E \neq \emptyset$, $\nabla T \neq \emptyset$ } $\bar{j} = \emptyset$

short circuit $\qquad$ open circuit

↳ thermometer,

$\underline{E + \frac{\nabla\mu}{e} \triangleq S\nabla T}$,

$j = \emptyset$ ⇒ $K_1\left(-\frac{\nabla T}{T}\right) = K_0\left(E + \frac{\nabla\mu}{e}\right)$ ⇒ $-\frac{K_1}{K_0}\frac{1}{T}\nabla T = E + \frac{\nabla\mu}{e}$, $\underline{S = -\frac{K_1}{K_0}\frac{1}{T}}$

Peltier effect: $\nabla T = \emptyset$, $j \neq \emptyset$ } $j_Q \neq 0$

$\bar{j}_Q = \Pi \bar{j}$ ⇒ $\nabla T = \emptyset$ $\quad \bar{j} = K_0\left(E + \frac{\nabla\mu}{e}\right)$, $\bar{j}_Q = -K_1\left(E + \frac{\nabla\mu}{e}\right)$

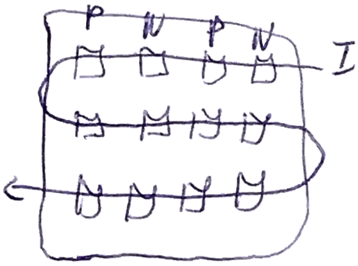
$\bar{j}_Q = -\frac{K_1}{K_0} j$ ⇒ $\underline{\Pi = -\frac{K_1}{K_0}}$

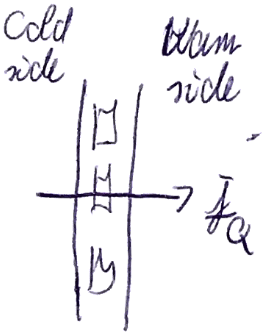
Kelvin relation: Seebeck VS Peltier: $\frac{\Pi}{S} = T$ ← Special case of Onsager,

↓

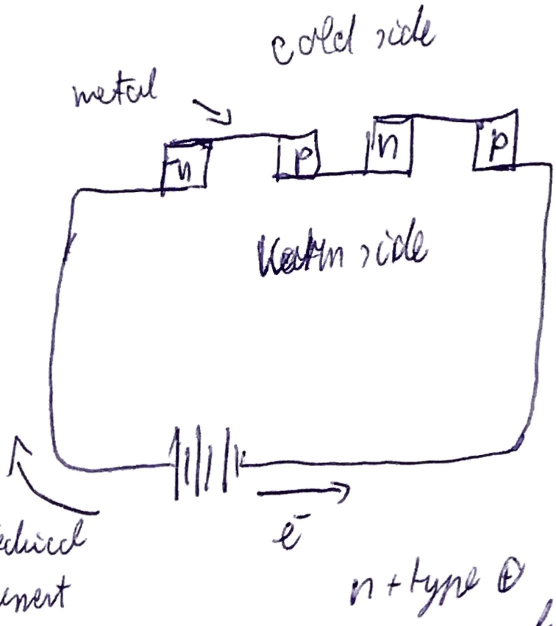
Wiedemann Franz law: Heat VS Electric conductivity: $\frac{\kappa}{\sigma} = \frac{1}{T}\frac{K_2}{K_0}$

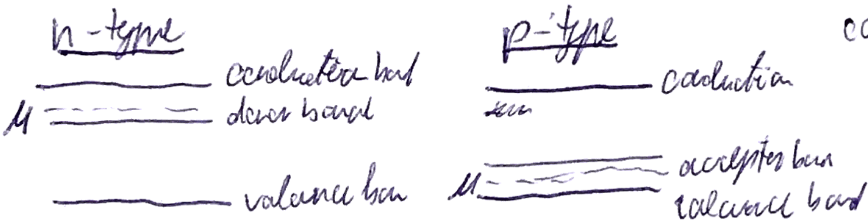
Peltier cooler:



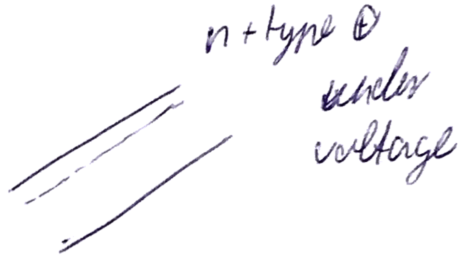


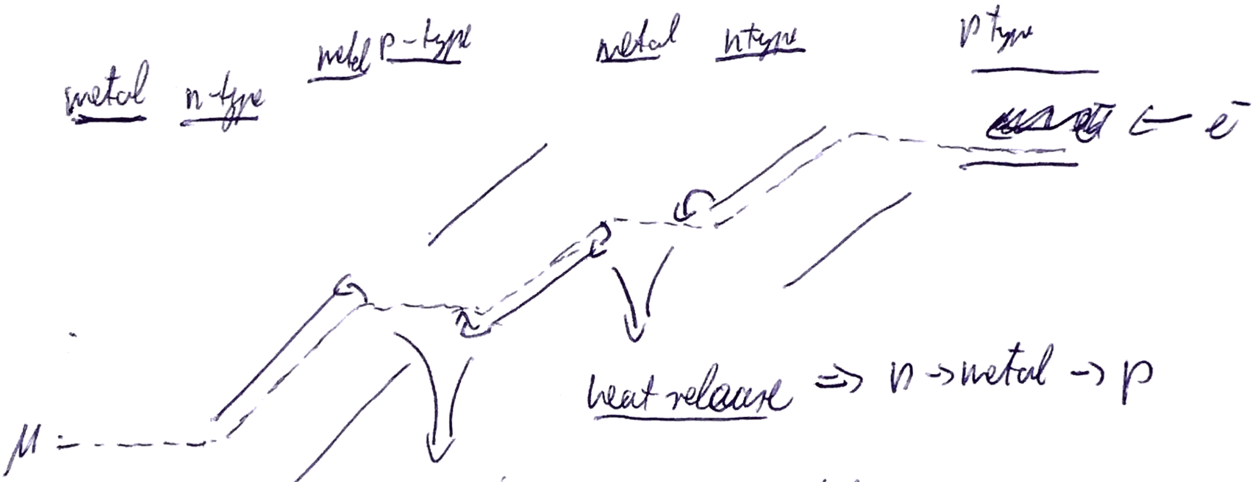
- dissipate the heat from the cold side





+





heat release => n -> metal -> p

heat absorbtion => p -> metal -> n

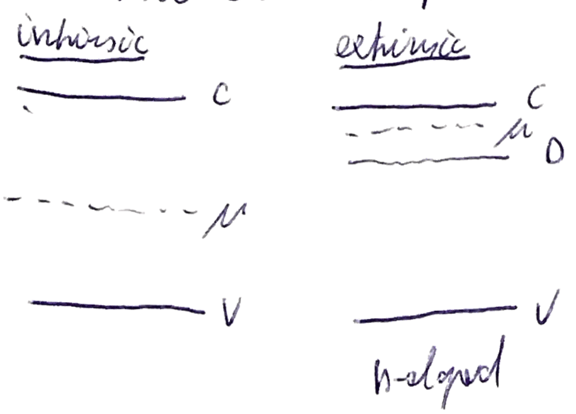
=> single $\boxed{n}$ could do the job too

=> best operation $\boxed{n}$ $\boxed{p}$ in series + $\boxed{n}$ in series wouldn't work

(10) difusion <=> inhomogeneous charge distribution

mass action law $n \cdot p = n_i^2(T)$ => valid for intrinsic, extrinsic SC-

=> intrinsic SCs => $n = p = n_i(T) \sim \text{const}(T, m_e^*, m_p^*)\, e^{-\frac{E_g}{k_B T}}$



=> majority | charge carriers
minority |

=> n-type: $n \approx n_d \gg p$ if T high enough to ionise every dopants
=> p-type: $p \approx n_a \gg n$

Non equilibrium: => charge carriers can be injected, excited, pumped etc.

=> equilibrium: n_0, p_0 => $n_0 p_0 = n_i^2(T)$ => excitation rate $G = C \cdot n_0 p_0$

=> non-equilibrium: n, p => recombination rate $R = C \cdot n p$

}=> net recombination rate: $U = R - G = C(n_0 p_0 - n p) = C[(n-n_0)(p-p_0) + n_0(p-p_0) + p_0(n-n_0)]$

$\approx C[n_0(p-p_0) + p_0(n-n_0)]$ => n-type: $U = C n_0 (p-p_0)$ } recombination is given by the minority charge carriers
p-type: $U = C p_0 (n - n_0)$

=> charge carrier lifetime: $\tau_p, \tau_n \gg \tau$: momentum relaxation time
$\sim 10^{-6} - 10^{-3}$ s $\qquad \sim 10^{-13} - 10^{-13}$ s

Continuity equations:

n-type

$$\frac{\partial n}{\partial t} - \frac{1}{e} \nabla j_n = -\frac{p-p_0}{\tau_p}$$

$$\frac{\partial p}{\partial t} + \frac{1}{e} \nabla j_p = -\frac{p-p_0}{\tau_p}$$

p-type

$$\frac{\partial n}{\partial t} - \frac{1}{e} \nabla j_n = -\frac{n-n_0}{\tau_n}$$

$$\frac{\partial p}{\partial t} + \frac{1}{e} \nabla j_p = -\frac{n-n_0}{\tau_p}$$

(+) $j_n = -D \nabla n$ <- particle current

$j_{diff} = -e j_n = e D \nabla n$

↳ diffusion current

VS

Drift: $j_{drift} = n e v_d = n e \mu E$

$j_{drift} = -n e \mu \nabla V$

equilibrium: $j_{drift} + j_{diff} = 0$; $-n e \mu \nabla V = e D \nabla n$

$\nabla n \to n(x) \to n \sim f = \frac{1}{1+e^{\frac{E-\mu}{k_B T}}}$ => $f(x) = \frac{1}{1+e^{\frac{E-eV-\mu}{k_B T}}} \approx e^{-\frac{E-eV-\mu}{k_B T}}$

=> $\nabla n = n \frac{e}{k_B T} \nabla V$ => $-n e \mu \nabla V = e D n \frac{e}{k_B T} \nabla V$ $\quad \boxed{\mu = \frac{e}{k_B T} D}$

Charge inhomogeneity:

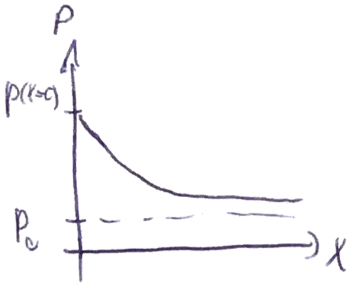
n-type: $\Rightarrow \frac{\partial p}{\partial t} + \frac{1}{e}\nabla j_p = -\frac{p-p_0}{\tau_p}$

$\Rightarrow \frac{\partial p}{\partial t} = 0 \Rightarrow \frac{1}{e}\nabla j_p = -\frac{p-p_0}{\tau_p}$ ⊕ $j_p = -eD\nabla p$

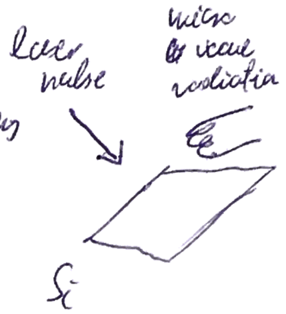
$\Rightarrow \Delta p = \frac{p-p_0}{D\tau_p} \Rightarrow$ 1D ⊕ $x=0$: charge carriers injected

$\Rightarrow \frac{\partial^2 p}{\partial x^2} = \frac{p-p_0}{D\tau_p} \Rightarrow p(x) = p_0 + (p(x=0)-p_0)\,e^{-\frac{x}{L_p}} \Rightarrow L_p = \sqrt{D\tau_p}$ ~ 10µm^1cm

↳ charge diffusion length



"it's important for solar cells ⇒ light excite new charge carriers ⇒ they have to get out the panel,"

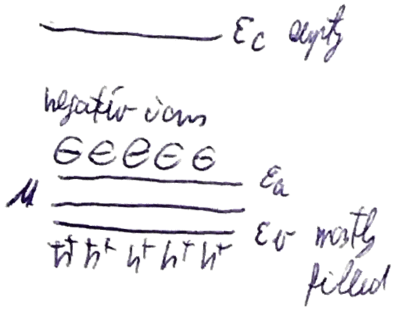


//

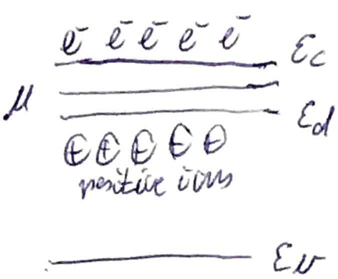
(11)

p-n junction

p-doped

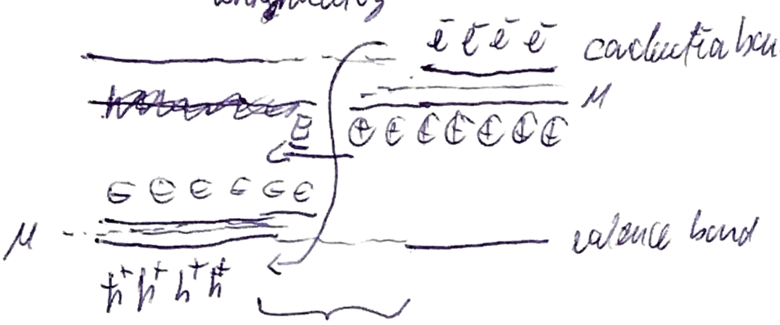


n-doped



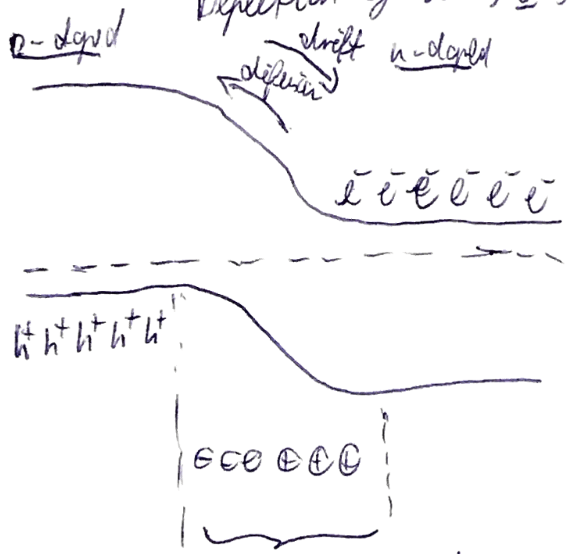
$\Rightarrow$ make a junction

annihilation



$\Rightarrow$ equilibrium

Depletion region $\Rightarrow$ $\underline{E}$ builds bc of the ions



$\rightarrow \Delta E = eV_D \leftarrow$ contact potential $\lesssim E_{gap}$

diffusion

pure chemical potential

Depletion region = no free charge carriers

diffusion

drift

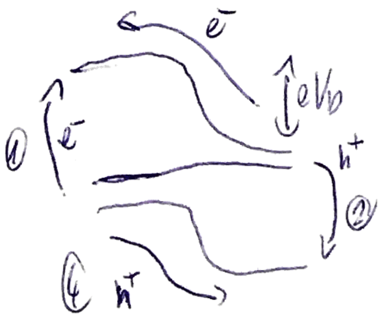
Shockley law

$\Rightarrow$ in equilibrium diffusion + drift = 0

$\Rightarrow -E\, n\, v_D\, e + e\, D \nabla n = \phi$

$\hookrightarrow$ built in $\underline{E}$

small, always there, doesn't depend on V



$① + ② \Rightarrow I_{left} \sim e^{-\frac{E_g}{k_B T}}$

$③ + ④ \Rightarrow I_{right} \sim e^{-\frac{(E_g + eV)}{k_B T}}$

$V = 0 \Rightarrow I_{left} + I_{right} = 0$

$V \neq 0 \Rightarrow I_{tot} = I_{right} - I_{left} =$

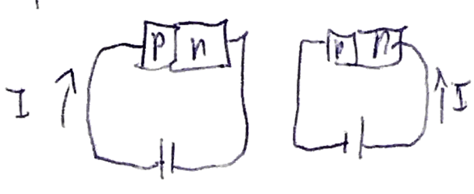
$\Downarrow$

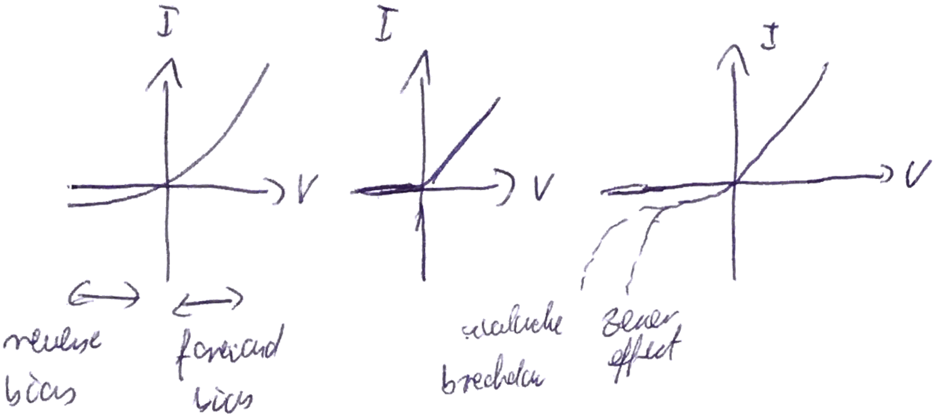
$= I_0 \left(e^{-\frac{eV}{k_B T}} - 1 \right)$

$\downarrow$

reverse, saturation current

forward bias   reversed bias



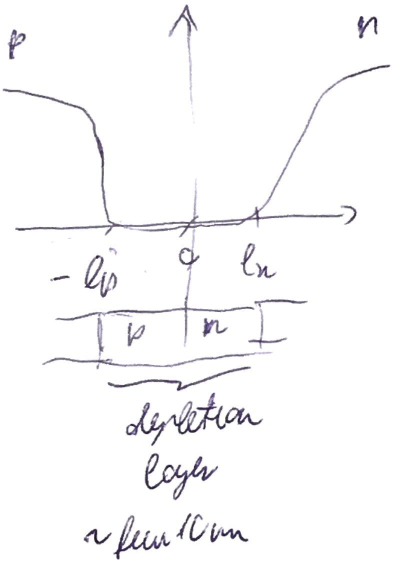


=> rectification

=> forward => open direction

=> reverse => current

Schottky - approximation



$$p \cdot l_p = n\, l_n$$

$$p \approx n_a \quad n \approx n_d$$

$$l_p\, n_a = n_d\, l_n$$

heavy doping => small depletion depth

light doped => big depletion length

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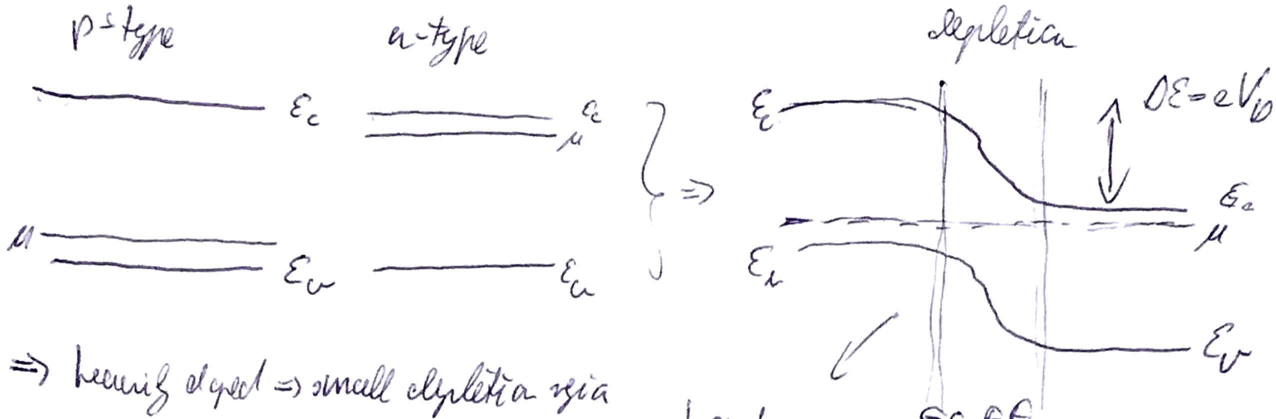


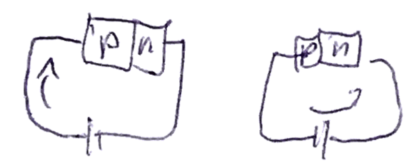
$\Delta E = eV_0$ ~ contact/difference potential,

⇒ heavily doped ⇒ small depletion region

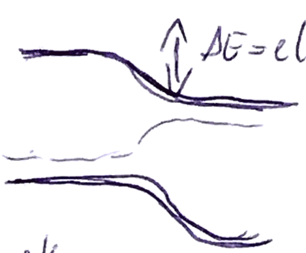
⇒ lightly doped ⇒ wide depletion region

band bending

↳ space charge ⇒ ions



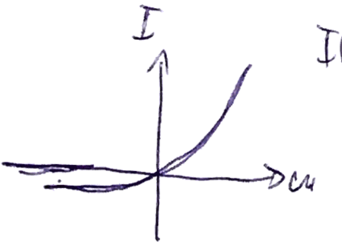
forward bias   reversed bias



$\Delta E = e(V_0 - V)$

$\overleftarrow{E}$ ↳ build in potential

↳ smaller barrier



$I(V) = I_{reverse}(e^{\frac{eV}{k_0 T}} - 1)$

⇒

big reverse voltage

⇒ breakdown

avalanche breakdown   Zener effect

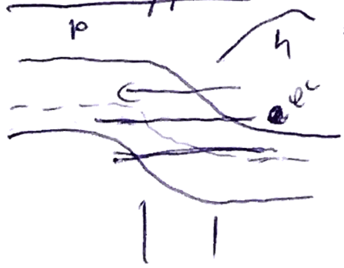
<u>avalanche breakdown</u>

↳ bad ⇒ destroy the diode

↳ lightly doped ⇒ wide depletion

↳ minority c-c. accelerate ⇒ generate new ones

<u>Zener-effect</u>:



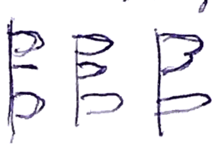
tunneling effect

small depletion ⇒ heavily doped

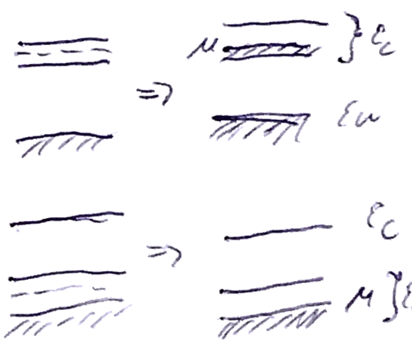
⇒ doesn't destroy the diode

<u>Esaki diode</u> ~ tunneling diode $\boxed{p^{++} \mid n^{++}}$

1-2 bonding



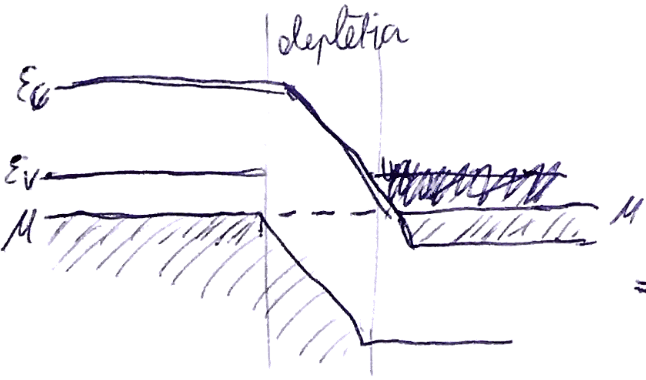
"semi metal"



μ in the bands

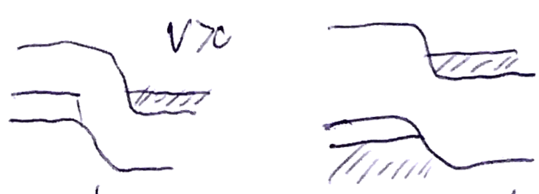
⇒ partially filled band

$V > 0$

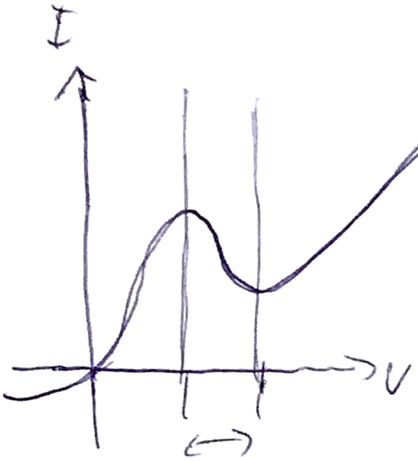


$V = 0$ ⇒ no current

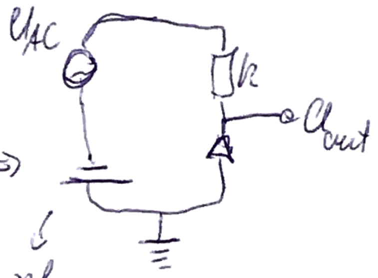
⇒



max tunneling current   normal diode operation



weak point

=> 

↳ ref the weakpoint

=> $\frac{U_{out}}{-r} = \frac{U_{AC}}{R-r}$

=> $U_{out} = U_{AC}\frac{r}{r-R} > U_{AC}$ if $r > R$

=> a negative resistance $\frac{dI}{dV} = -\frac{1}{r}$

=> V↑ => I↓

=> amplifier or oscillator

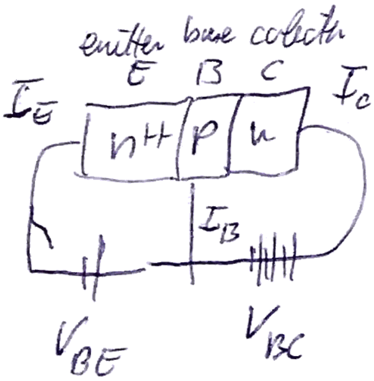
"unipolar – FET"

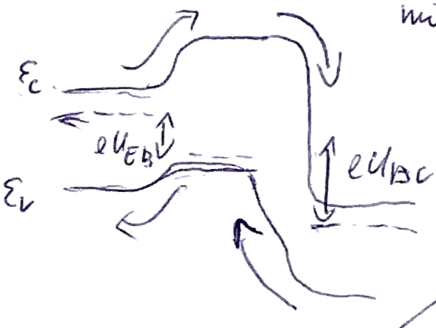
Bipolar transistor: bipolar – e^-, h^+ takes part in the conduction

=> 

=> $p^{++}np$ => emitter is heavily doped } not two p-n junction,

$n^{++}pn$ base is very thin

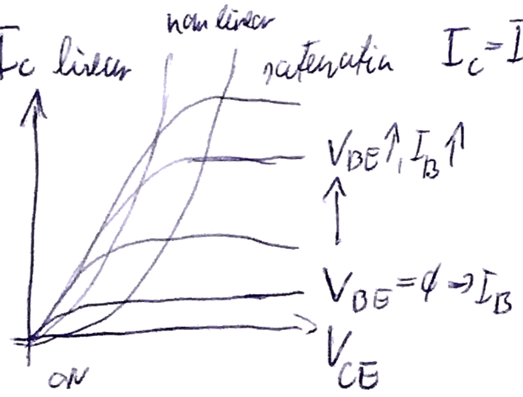




V_{BE} → forward bias => put ~~minority~~ e^- in the base => base < diffusion length => most e^- reach BC where => strong built-in field bc of V_{BC}

minority

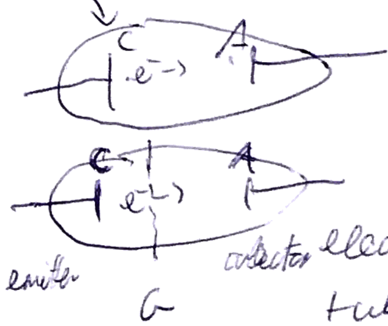
}=> $I_C \propto V_{BE} \propto I_B$ => $\frac{I_C}{I_B} \sim 10-100$



$I_C = I_E - I_B$

↳ recombined

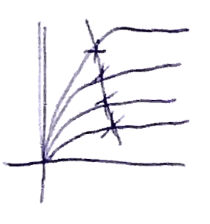
=> it's a loss



electron tube,

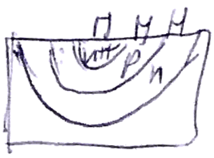
digital device

logic



analog

amplifier



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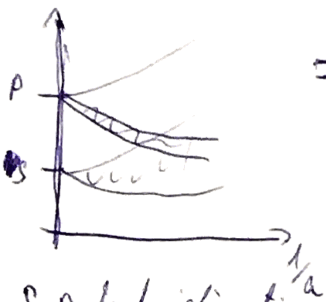
Surface states



$\Rightarrow$ at the surface $\Rightarrow$ sp^3 bonds $\oplus$ dangling bonds $\Rightarrow$ missing atoms $\Rightarrow$ these form midgap states

$\Rightarrow$ they are important in devices where surfaces is important

(So not that much, GaAs has a lot)

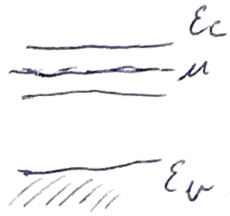
s-p hybridization

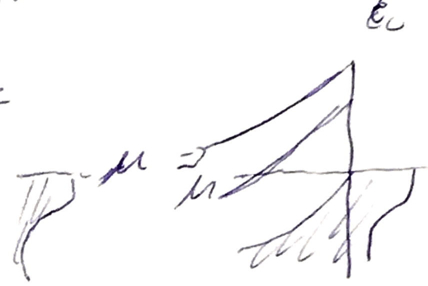
$\Rightarrow$ sp^3 orbits

$\Rightarrow$ Fermi level pinning

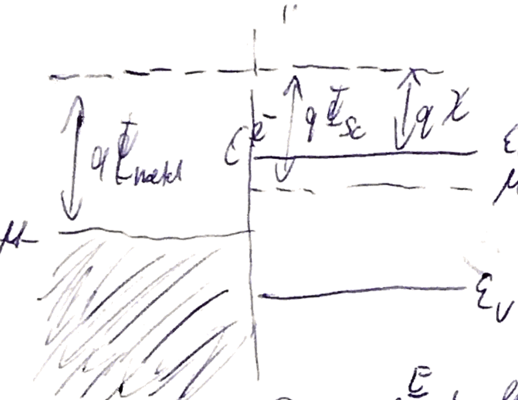
$\Rightarrow$ # surface states traps charges $\Rightarrow$ prevent band bending

density of surface states





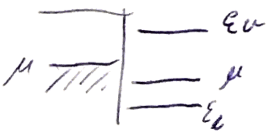
Metal-Semiconductor heterojunction



$q\Phi_m$ – work function

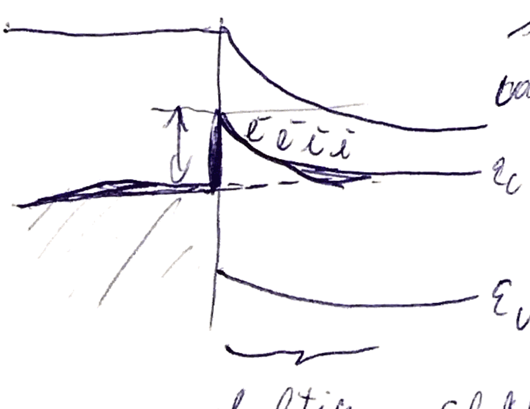
χ – electron affinity

~~$q\Phi_{metal} - q\chi$ = Schottky barrier~~



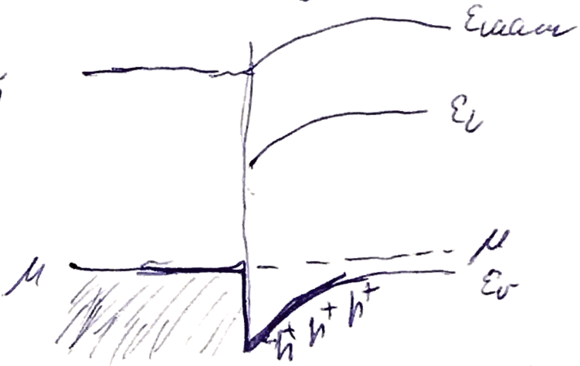
$\ominus$ $\oplus$ $\Rightarrow$ $\vec{E}$ built in potential

$\Rightarrow$ band bending



depletion region

Schottky barrier $= q\Phi_{metal} - q\chi$



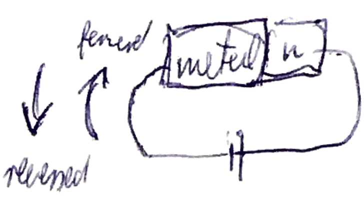
metal p-type $\Rightarrow \Phi < \Phi_{sc}$

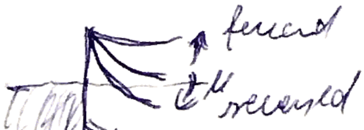
Schottky barrier $= E_g - (q\Phi_m - q\Phi_{sc})$

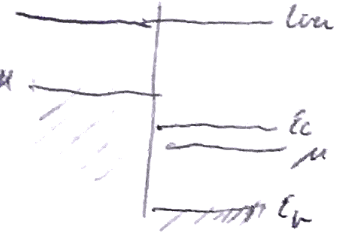
metal n-type $\Rightarrow \Phi_{metal} > \Phi_{sc}$

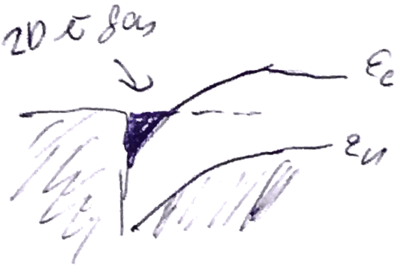
$\Rightarrow$ rectifying behaviour like the p-n junction

$\Rightarrow$ looks like a half of it









metal + n-type
$\Phi_{metal} < \Phi_{SC}$

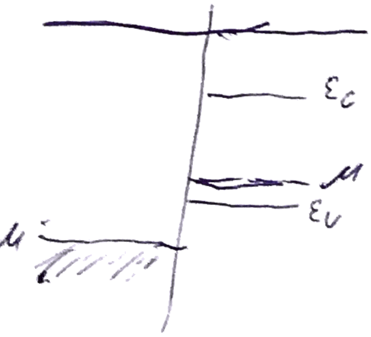
↳ Ohm like behavior
=> chanel in FETs,

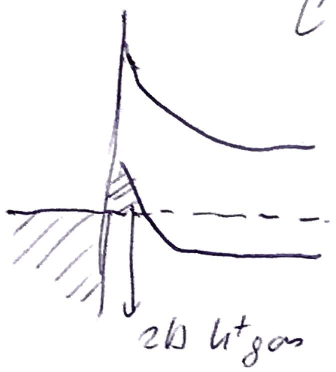
Schottky diode

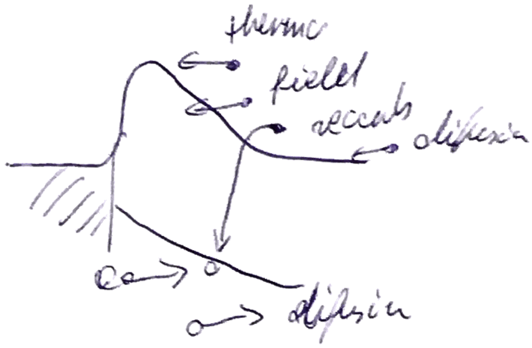
↳ tunable Schottky barrier with coating the metal

↳ V_B -> opening voltage is small
-> small heat consumption

↳ no minority c-carriers
=> faster (because of the metal)
↳ no recombination



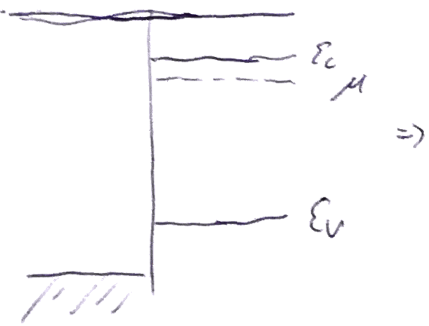


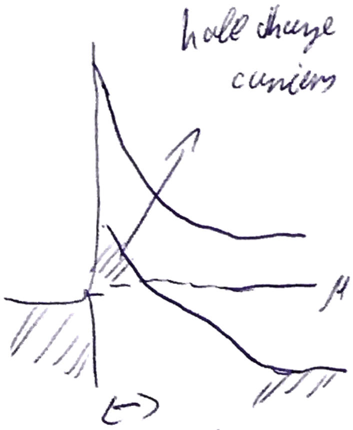


metal + p-type
$\Phi_{metal} > \Phi_{SC}$

heavily doped
=> small depletion => field emission
=> through tunneling

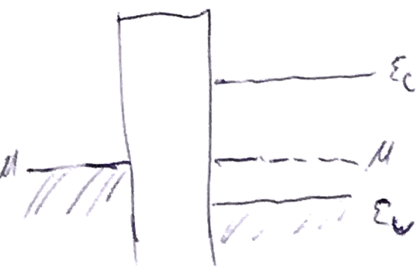
lightly doped
=> big depletion => no tunneling



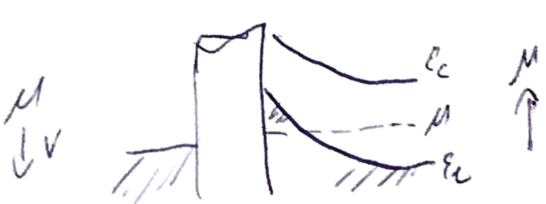


inversion layer => minority becomes the majority

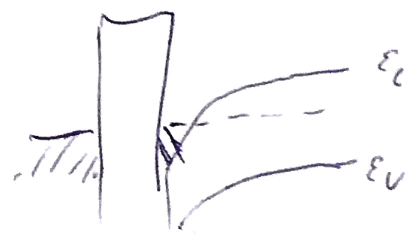
Metal Insulator p-type



Accumulation



Inversion



(14.)

JFET: ~ junction field effect transistor

↳ depletion region

↳ V_G controls the size of the depletion region

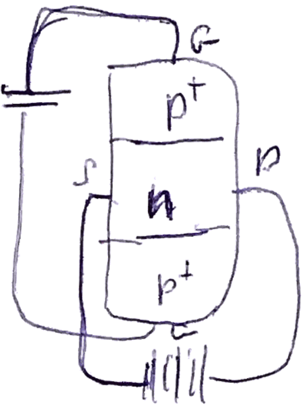
=> V_{GS}: forward on the pn junction => I_{SD} can be large

reverse on the pn -"- => I_{SD} small

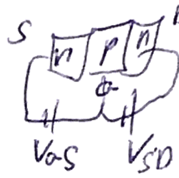
=> $I_{SD} \Leftrightarrow V_{GS}$

=> big gate impedance $I_G \approx 0$

=> good for digital logic



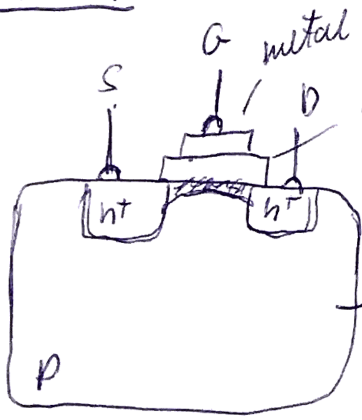
// BJT:



=> small impedance

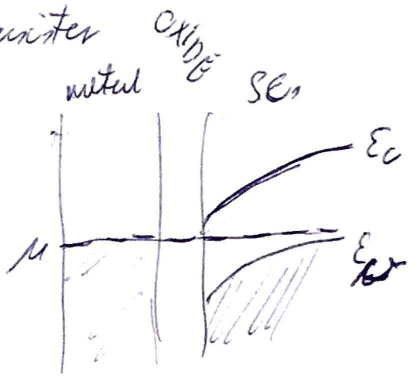
=> analog amplifier //

MOSFET: ~ Metal-Oxide-Sc Field effect transistor



$V_G = 0$ no conduction between SD => $I_{SD} = 0$ "4D"

like two diode facing each other



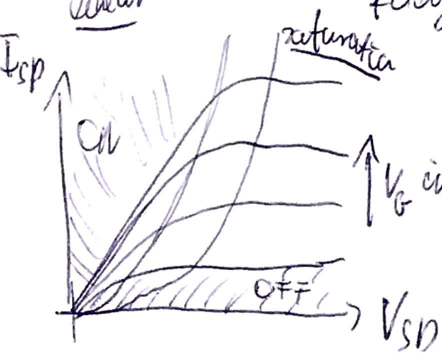
$V_G > 0$ => μ moves up

e^- chanel forms

$I_{SD} \neq 0$

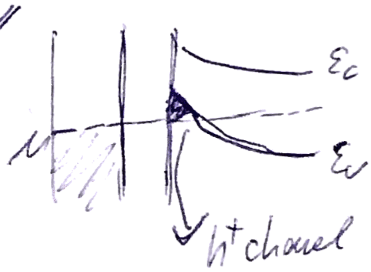
non linear

linear



$V_G < 0$

//

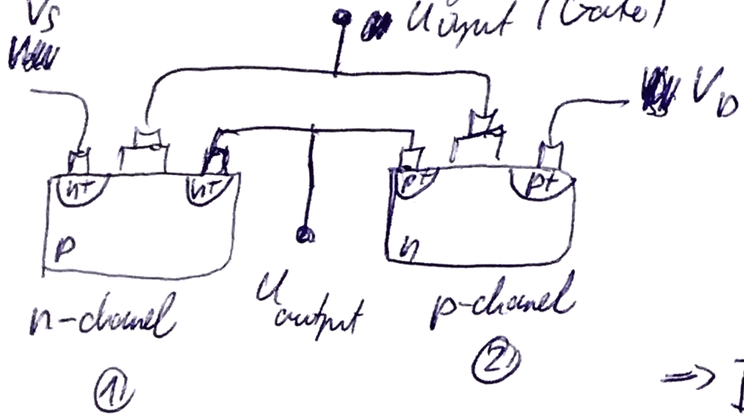


h^+ chanel forms //

=> digital logic

-> analog amplifier

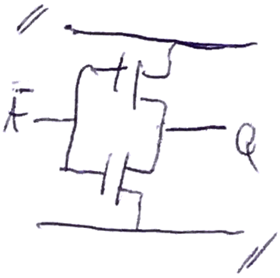
CMOS ~ Complementary MOS



$\Rightarrow U_{input} > 0 \Rightarrow$ ① open $\Rightarrow U_{out} = V_S$
② closed

$\Rightarrow U_{input} < 0 \Rightarrow$ ② open $\rightarrow U_{out} = V_D$
① closed

$\Rightarrow I_{SD} = 0$, except when doing the switching



Not gate

$V_S = 0$, ground

$V_{dd} > 0$,

$\Rightarrow$ logic true $\rightarrow$ high voltage
logic false $\Rightarrow$ small voltage $\Rightarrow$ ground

$\Rightarrow$ ON-OFF OFF-ON

$\Rightarrow$ current never flows
$\Rightarrow$ important because of the power consumption,

PMOS $\Rightarrow$ p-channel MOS
NMOS $\Rightarrow$ n channel MOS