MONOCROMO®

NOME
NAME Semiconductors

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1 class, 1 topic on oral exam
2 questions

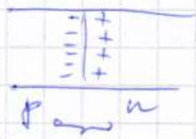
Physics of semiconductors BATÓ LILIA

semiconductors (SC) : material whose conductivity can be affected by :

- temperature
- doping (adalékolás) < intentional / unintentional (induces impurities)
  Szigorú (nem feltétlen rossz!)
- external voltage < drain (szélső fesz.) / bias (előfesz.)

V_{SD}
V_{GD}
G: gate

- another semiconductor nearby
  diffusion, charge transfer



charge diffusion ⟷ depletion layer : kiürített réteg
junction : átmenet

- light : photoconductivity
  conductivity affected by light
  sensors, photovoltaic effects (PV)

- magnetism (spintronics, magnetic SC-s)
  Mn: GaAs, MRAM (magnetoresistive random access memory)

metal's conductivity does not change significally with illumination (except low temp.)

SCs properties are good and bad (some)

Sc materials

BULK
- IV. group : C, Si, Ge, Sn (carbon is an insulator)
  ↑ has different allotropes : diamond, graphite, graphine, CNTs, fullerenes
- IV. group compounds : SiC (same structure as diamond)
- III.-V. group compounds : GaAs (arsenide), AlP, InSb and various combinations
- II.-VI. group compounds : ZnS, ZnSe, ZnO, PbS_2, FeS_2

Bulk SCs : diamond structure

Low-dimensional SCs : 1D: CNTs, nanowires GaAs
(we won't talk about these) 2D: graphene, transition metal dichalcogenite MoS_2

Molecular Scs : C_{60}

diamond : Sc or insulator ?
$E_g = 6$ eV band gap > Si_3N_4 $E_g = 4$ eV
Sc How ? insulator

diamond can be metal ?
B : C (boron doped diamond)
4% → superconductor (metal)

most innovation are happening in at the industry, physical principles are not changing, device performance is increasing (Moore - law ?)

Early History of SCs (dates are not asked on the exam)

Fundamental effects:

1833 : Faraday : AgS heated → R decreases (R: resistance, ρ: resistivity)

1839 : Becquerel : photovoltaic effect

1873 : Smith : photoconductivity Se (light → R decreases)

1874 : Braun : rectification (egyenirányítás) in metal-sulfides

Schuster : rectification in CuO (leads the a II-VI Sc)

1878 : Hall - effect

1900 : Baedeker : inverse sign Hall-effect in CuI

Hall-constant : $R_H = \frac{1}{ne}$ — n = number of charge carriers / cm^3; has a sign: − : −c.c., + : +c.c.

1910 : Weiss : Halbleiter (origin of the name) semi-conductor (metals with strange behaviour)

1928-38 : Quantum theory of SCs — Bloch, Schottky, Mott

1940 : Bardeen (2 Nobel-prizes, only one) : non-reproducing lab. results are due to about < 1 ppm unwanted doping; nobody could make SCs intentionally

1 ppm : ~~part/million~~ too pure technological limitation

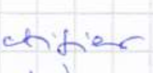
ppm : part/million (10^6)

ppb : part/billion (10^9)

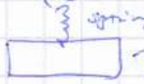
often today : 10^{-12} - 10^{-15}

Technology:

1880 : Bell voice transfer with Se, piezoelectricity (not very efficient)

1883 : Fitts : first Selencell :  (thin layer of gold (transparent), Se, metal)

1904 : J.C. Bose : sensitive radiodetector (rectifying device) (EM radiation, demodulation, AM radio)

point contact diode : spring (cat's whisker), PbS_2 galena 

1906 : Round : first LED

1920 : several people → CuO + Se rectifier AC → DC converter

1922 : Losev ZnO → amplifier (first)

1926 : Lilienfeld : first FET (field effect transistor) (térvezérelt tranzisztor)

I_{SD} depends on V_{GS}

1941 : Ohl : first Si p-n junction

1947 : Bardeen - Brattain - Schockley : bipolar junction transistor → revolutionised the industry

Nobel prizes

1956 transistor

1973 Esaki, Giaever tunnel-diode

1985 von Klitzing : quantized Hall - effect

2000 Alferov, Kroemer, Kilby — Sc laser (Alferov, Kroemer), IC : integrated circuits (Kilby)

2007 Fert, Grünberg GMR: giant-magnetoresistance
2009 Boyle, Smith CCD
2010 Geim, Novoselov graphene
2014 Akasaki, Amano, Nakamura blue LED (GaN high quality) tech. challenge

Position of SC physics

prerequisite: quantummechanics, solid state physics, statistical physics, material sciences (optics)

gives us: electronics, spintronics, laser-optics/physics

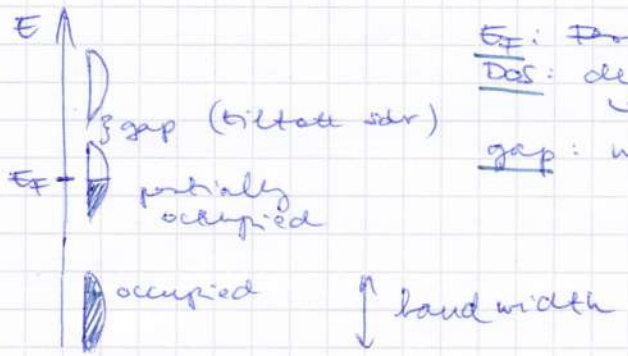
ongoing research: fundamental directions: new materials, low-dimensional SCs, optical i.r.e excitons
fundamental phenomena: quantum Hall-effect (Structural) QHE, FQHE

applied research: new devices (HEMT) /high e⁻ mobility transistors/
Size: smaller, speed (frequency): faster, power consumption: decrease

Resistivity:

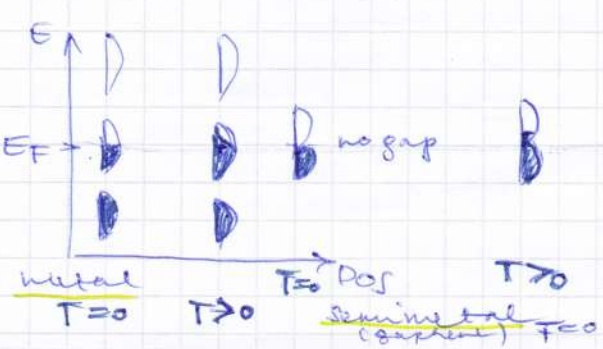
Size of the Universe: 10^{27} m
Size of nucleus: 10^{-15} m femto
42 orders of magnitude

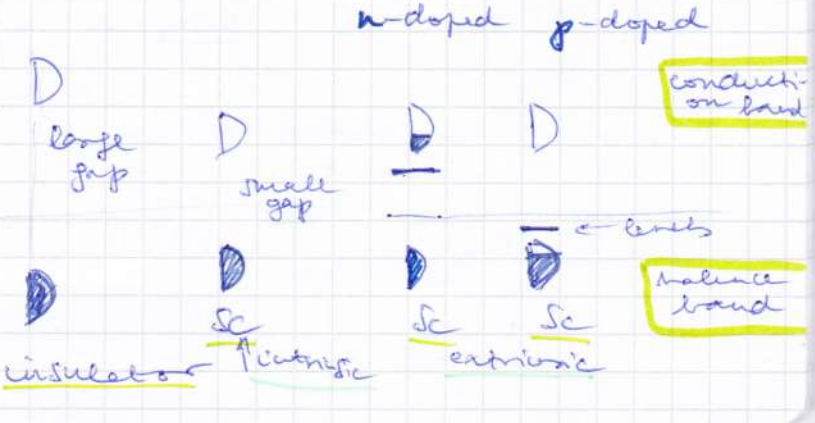
Band structure of semiconductors (generic overview)



E_F: Fermi Energy
DOS: density of states
↳ number of available states for an e⁻
gap: no available states there

Fermi function describes



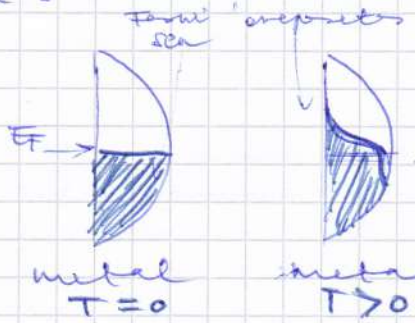


Fermi energy ⟷ chemical potential

E_F — μ

$T=0$ property — final temperature quantity

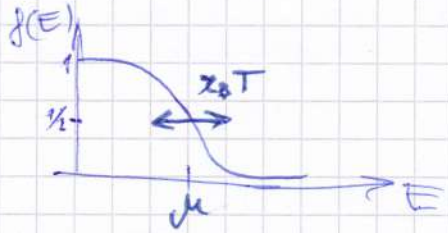
cut-off E of e⁻s — average E of particles



$$f(E) = \frac{1}{1+e^{\frac{E-\mu}{k_BT}}}$$

$\mu(T \to 0) = E_F$

$\mu \neq E_F$



$\frac{1}{1+e^{\frac{E-\mu}{k_BT}}}$

$T=0 \Rightarrow E<\mu \Rightarrow f(E)=$

$E>\mu \Rightarrow f(E)=$

becomes the step function (Haag-side)

k_BT at $T=300K \rightarrow 26\,meV$

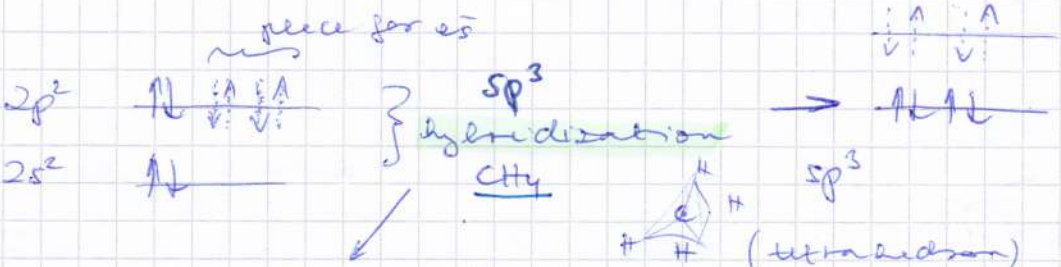
(broadening is not very large) bands ~ eV

Na (sodium): $1s^2 2s^2 2p^6 3s^1$

atom — metal

Pauli principle → degeneracy is lifted (e⁻ do not want to be at the same energy)

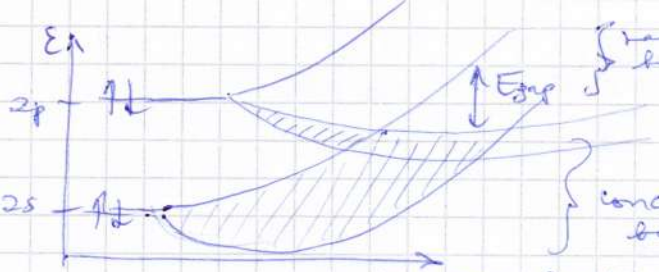
Diamond $1s^2 2s^2 2p^2$ — Si $1s^2 2s^2 2p^6 3s^2 3p^2$ — same band both



in order to gain E

needs: s and three p orbitals

(IV elements)



8 levels / atoms (with spin)
only 4 e^--s

E_g > E_g diamond Si → smaller d

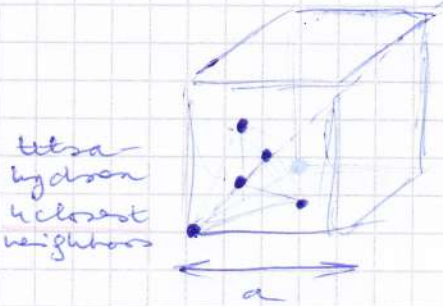
hybrid band

gaps of metals cannot be completely used (due to hybridization)

diamond structure is a consequence of hybridization

structure → hybr ? or hybr → structure (?) which dictates which (?)

Structure



diamond structure

Bravais-lattice
FCC: face centered cubic

basis: (0,0,0)
$(\frac{1}{4},\frac{1}{4},\frac{1}{4})a$
1 type of atom

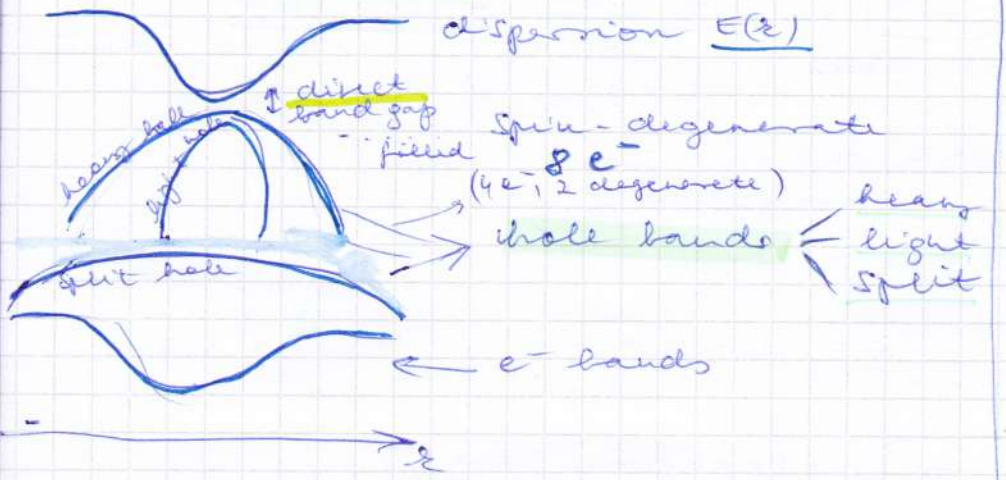
moved by a quarter of body diag

basis:
case of GaAs → (0,0,0) Ga
$(\frac{1}{4},\frac{1}{4},\frac{1}{4})a$ As
2 type of atoms

alloy: no underlying structure

name of structure: Zincblende (2 type interstructure)

Band Structure GaAs



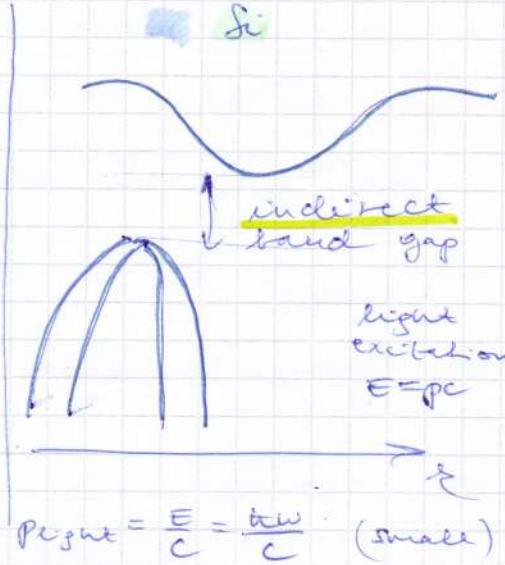
can absorb or emit light
→ can make a laser, LED

valid for GaAs unit, not individually (atoms)
→ double atom basis

8 e⁻ basis based bandstr.
(number of bands?)

curvature ~ effective mass

Si



$p_{light} = \frac{E}{c} = \frac{\hbar\omega}{c}$ (small)

light cannot give or take large momentum
cannot emit or absorb light (k change)

Si:
(2 atom basis)
GaAs

Band gap: C Si Ge GaAs AlAs
5eV 1,1eV 0,7eV 1,5eV 2,2eV

small a
⇒ gap large

general consequences for technology

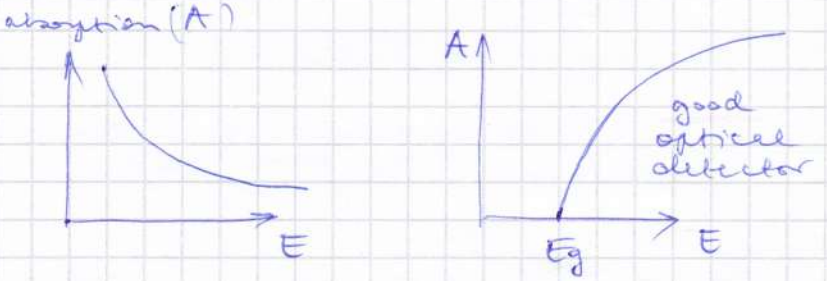
Ens: 2eV E_{12}: < 1eV light detecting

for high power transistor Si or AlAs or C
(high E, high T)

larger gap → T does not influence it like smaller gap mate

Optical properties:

metal SC



change a lot
/ selective abs. (?) /

Charge carriers in SCs

n: concentration of c.c.s

in a metal $n = 10^{22} \frac{1}{cm^3}$ / Ge (300k) $n = 2 \cdot 10^{13} \frac{1}{cm^3}$
↳ low amount at room

Drude-model: conductivity ↔ n

$ma = F = qE$ plus a viscous force (drag-force)

$ma = qE - Kv$

stationary (a=0) solution: $0 = qE - Kv$

we introduce: $\frac{K}{m} = \frac{1}{\tau}$ relaxation time

$ma = qE - Kv \Rightarrow a = \frac{qE}{m} - \frac{K}{m} \cdot v$ $\left[\frac{K}{m}\right] = \frac{1}{s}$

phenomenological model

$qE = Kv \rightarrow v_{drift} = \frac{qE\tau}{m}$ $\tau = 10^{-17} - 10^{-15}$ sec typically

$v_{drift} \approx 10^{-1} \ldots 10^{1} \frac{m}{s}$

/ $v_{signal} \approx$ speed of light /

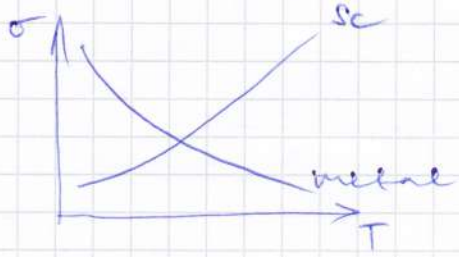
current density: $j = n \cdot e \cdot v_D$

$j \sim n$

$[j] = \frac{C}{m^2 s} = [n e v_D] = \frac{1}{m^3} C \frac{m}{s} = \frac{A}{m^2}$

$j = \sigma E$ differential Ohm's law

$j = n e v_D = n e^2 \frac{\tau}{m} E \Rightarrow \sigma = \frac{n e^2 \tau}{m}$ conductivity $\left[\frac{1}{\Omega m}\right]$



metal: $n \approx$ const.
σ is dominated by τ shortening (ρ increases)

SC: n is strongly dependent on T
ΔT dominates σ

Solid state physics session

1 dimensional free e^- gas (particle in the box)

$\psi(x,t) = \frac{1}{\sqrt{L}} e^{ikx}$ plane wave (wave function)

$p = \hbar k$ ← crystal momentum
↑ canonical impulse

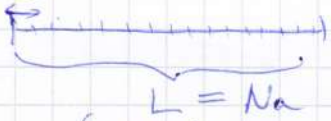
Energy dispersion

$E(k) = \frac{\hbar^2 k^2}{2m}$ (Sch-eq.) $H\psi = E\psi$

$H\psi = E\psi$ free: $V = 0$ (no potential)

$-\frac{\hbar^2}{2m} \Delta \psi = E\psi$

In a solid k is quantized by periodic boundary conditions.
↓ math tool to make solving problems easier.

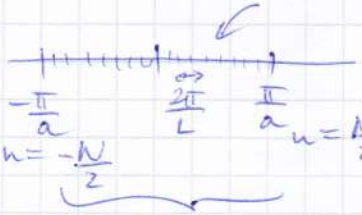


$\psi(0) = \psi(Na)$ single valued

$\hookrightarrow e^{ikL} = 1 \Rightarrow kL = 2\pi n$ $n \in 0, \dots N-1$ or $n \in -\frac{N}{2}, \dots +\frac{N}{2}+1$

$k = \frac{2\pi n}{Na} = \frac{2\pi}{a} \cdot \frac{n}{N}$

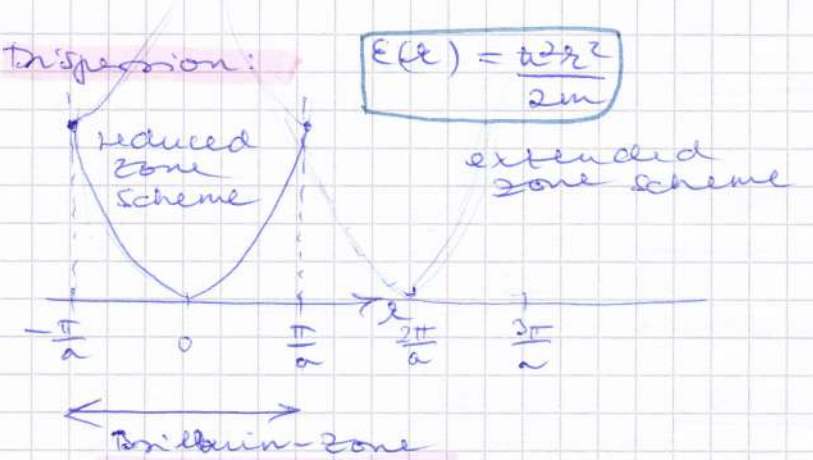
1D



$[k] = \frac{1}{m}$

↓ quasi-continuous (small spacing)

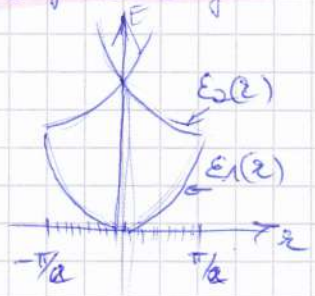
Dispersion: $E(k) = \frac{\hbar^2 k^2}{2m}$



We want sth. practical: e.g. compute the n (charge carriers)

Charge carriers in SCs

1D free e⁻ gas



$E_n(k)$ ← band index

$\mathcal{E}(k) = \frac{\hbar^2 k^2}{2m}$ quadratic band dispersion $E(k)$

periodic boundary conditions:

$k = \frac{2\pi}{L} n \quad n = -\frac{N}{2} \dots \frac{N}{2} \qquad \Delta k = \frac{2\pi}{L}$ (separation (quantization))

$(m^*)^{-1} = \frac{1}{\hbar^2} \frac{\partial^2 \mathcal{E}(k)}{\partial k^2}$ band effective mass (result of the lattice) (cc.s mass is different from e⁻)

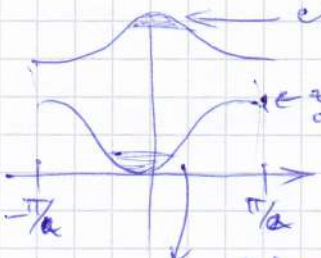
$v = \frac{1}{\hbar} \frac{\partial \mathcal{E}(k)}{\partial k}$ velocity of e⁻

for 1D: $\frac{\hbar k}{m} = v \quad | \quad p = \hbar k \qquad \left[\frac{\hbar^2}{m}\right] = \left[\frac{k_F m^2}{m s^2}\right]$

for free e⁻ gas: $(m^*)^{-1} = \frac{1}{m} \rightarrow \boxed{m^* = m}$ in this case

$F_{ext.} = \hbar \dot{k}$ (external forces) ↑ crystal momentum (next lecture: + periodic potential)

1D free e⁻ gas + periodic potential



$m^* < 0$ charge carriers are holes (negative mass(?))

$ma = F$ redefine the force → m

$m^* > 0$ charge carriers: e⁻

GaAs

hole → depending on m^* the curvature

typical values:

$m_c^* = 0{,}1 \dots 0{,}5\ m_0$ electrons
$m_v^* = -1 \dots -3\ m_0$ (heavier) holes

Density of states

↳ central notion

DOS in k-space:

(1D) $\Delta k = \frac{2\pi}{L}$; $\boxed{D(k) = \frac{L}{2\pi}}$ dimensions of m density for $\frac{1}{m}$ (?)

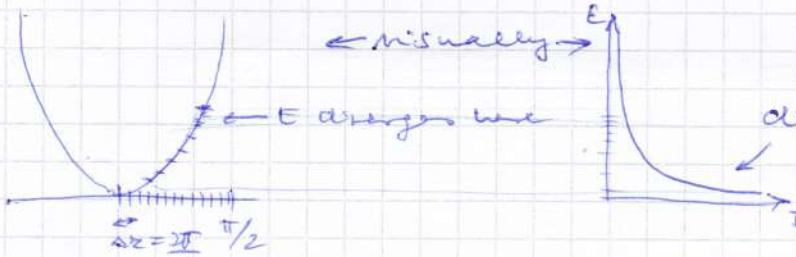
Number of atoms: $N = \int_{-\pi/a}^{\pi/a} D(k)\,dk = \int_{-\pi/a}^{\pi/a} \frac{L}{2\pi}\,dk = \frac{L}{2\pi}\frac{2\pi}{a} = \frac{L}{a} = N$ //

// Statistics

density function
e.g. Gauss $\langle x \rangle = \int_{-\infty}^{\infty} x\, f(x)\,dx$ //

(2D) $\boxed{D(k) = \frac{A}{(2\pi)^2}}$ (area) (3D) $\boxed{D(k) = \frac{V}{(2\pi)^3}}$ plus: ×2 for spin degeneracy

Go from $D(k) \rightarrow D(E)$ (better to work in E-space)



(1D) $\boxed{D(k)\,dk = D(E)\,dE}$ ← preservation of states or def. of $D(E)$

$D(k)\frac{dk}{dE} = D(E) = D(k)\left(\frac{dE}{dk}\right)^{-1}$

// $\frac{dk}{dE} = \left(\frac{dE}{dk}\right)^{-1}$; derivative of inverse function

$y = f(x) \quad [f^{-1}(x)]' = \frac{1}{f'(f^{-1}(x))}$

e.g.: $y = x^2 \rightarrow x = \sqrt{y} \quad \frac{dy}{dx} = 2x \quad (\sqrt{y})' = \frac{1}{2\sqrt{y}} = \frac{1}{2x}$ //

(1D) $\boxed{\left(\frac{dE}{dk}\right) = \frac{\hbar^2 k}{m^*}}$

$D(E) = \frac{L}{2\pi} \cdot \underbrace{2}_{\text{spin}} \cdot \frac{m^*}{\hbar^2 k} \overset{(3)}{=} \frac{L\sqrt{m^*}}{\pi\sqrt{2}\hbar\sqrt{E}}$

$E = \frac{\hbar^2 k^2}{2m^*} \quad k = \sqrt{\frac{E 2m^*}{\hbar}}$

$k \rightarrow E \quad = \frac{1}{\pi}\frac{m^*}{\hbar^2}\sqrt{\frac{\hbar^2}{E 2m^*}} = \frac{L\sqrt{m^*}}{\pi\sqrt{2E}\hbar}$

$\boxed{D(E) = \frac{L}{\pi}\frac{\sqrt{m^*}}{\sqrt{2E}\hbar}}$

(2D) $\boxed{D(k)\,2\pi k\,dk = D(E)\,dE}$

elementary surface in 2D

$\frac{2A}{4\pi^2} \cdot 2\pi \cdot \frac{m^*}{\hbar^2} = \boxed{D(E) = \frac{Am^*}{\pi\hbar^2}}$ constant, no singularity!

~~Spin?~~ spin ?

(3D) $D(k)\,4\pi k^2\,dk = D(E)\,dE$

$$D(E) = \frac{V}{4\pi^3}\cdot 4\pi k^2 \frac{m^*}{\hbar^2 k} = \frac{V}{2\pi^2}\sqrt{E}\left(\frac{2m^*}{\hbar^2}\right)^{3/2}$$

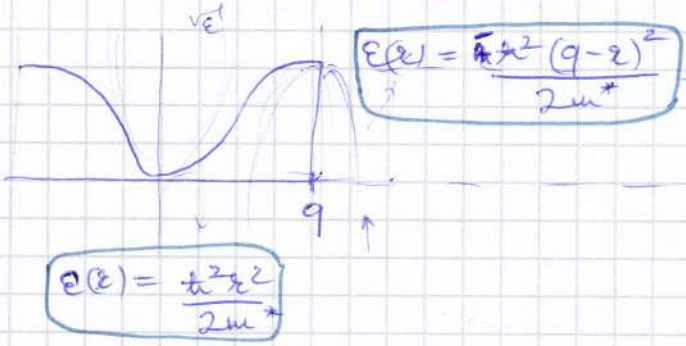
linear in k

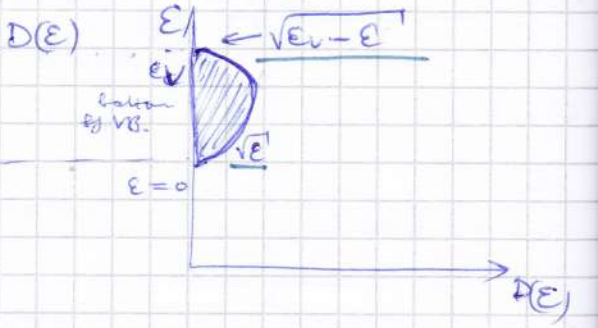
$D(k) \sim k \sim \sqrt{E}$

$D(E)$:

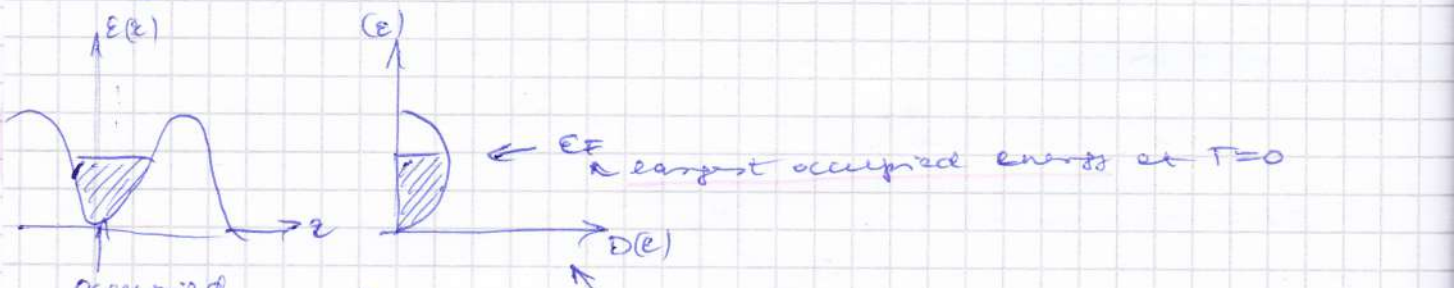
(1D) $\sim \frac{1}{\sqrt{E}}$ (2D) const (3D) $\sim \sqrt{E}$ ← important!

Occupation of bands in 3D

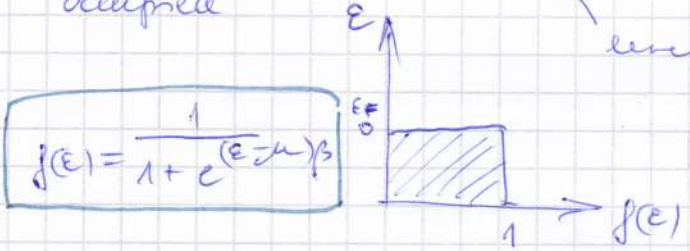




T = 0



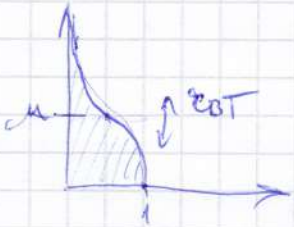
level of e^- is sharp : consequence of $f(E)$



$f(E) = \frac{1}{1 + e^{(E-\mu)\beta}}$

at T=0 $\mu = E_F$

T > 0



E_F only makes sense at T=0

$\lim_{T\to 0} \mu(T) = E_F$

Number of CCs

$$N_c = nV = \int D(E) f(E)\,dE$$

$D(E) \to \frac{V}{2\pi^2}\cdot$

n: density of e^-s

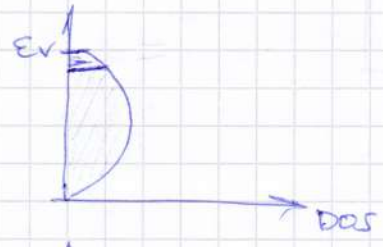
N_c: number of CCs in CB

We like this equation it describes everything

$f(E)$: quantum statistics of e^-s

$D(E)$: contains the system specific information

Top of the VB

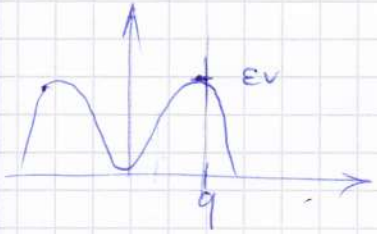


$$D_v(\varepsilon) = \frac{V}{(2\pi)^3}\left(\frac{2m_v^*}{\hbar^2}\right)^{3/2}(\varepsilon_v - \varepsilon)^{1/2}$$

↓ valence

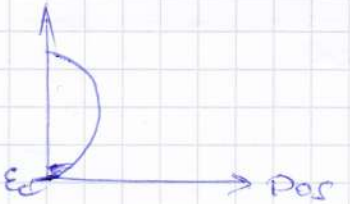
$$\varepsilon(k) = \varepsilon_v - \frac{\hbar^2(q-k)^2}{2m_v^*}$$

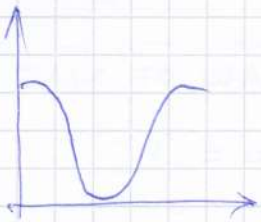
measure ε back from ε_v



(1ppm doping makes Sc semi-metal)

Bottom of CB

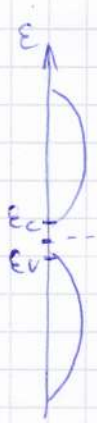




$\varepsilon_c + (?)\ \frac{\hbar^2(q+k^2)}{2m_c^*}\ (?)$

$$\varepsilon(k) = \frac{\hbar^2 k^2}{2m_c^*}$$

$$D_c(\varepsilon) = \frac{V}{2\pi^2}\left(\frac{2m_c^*}{\hbar^2}\right)^{3/2}(\varepsilon - \varepsilon_c)^{1/2}$$



$$f(\varepsilon) = \frac{1}{1+e^{\beta(\varepsilon-\mu)}} \underset{\varepsilon \gg \mu}{\simeq} e^{-(\varepsilon-\mu)\beta}$$

$\mu \simeq \frac{\varepsilon_c+\varepsilon_v}{2}$; $|\varepsilon - \mu| \simeq \frac{1}{2}\varepsilon_g$

average ε of vehicles / chem. pot.

$\varepsilon_c - \varepsilon_v = \varepsilon_g$ and $\varepsilon_g \gg k_B T$

ε_g: 1–2 eV ; $k_B T$: 26 meV

density of e^-: $n = \frac{1}{V}\int_{\varepsilon_c}^{\infty} D_c(\varepsilon) f(\varepsilon)\, d\varepsilon$

density of h^+: $p = \frac{1}{V}\int_{-\infty}^{\varepsilon_v} D_v(\varepsilon) f(\varepsilon)\, d\varepsilon$

$$n \sim \int_{\varepsilon_c}^{\infty} \sqrt{\varepsilon_{\text{kin}} - \varepsilon_c}\; e^{-(\varepsilon-\mu)\beta}\, d\varepsilon = \int_0^{\infty} \sqrt{x}\, e^{-x}\, \beta^{-3/2}\, e^{-(\varepsilon_c-\mu)\beta}\, dx \Rightarrow$$

$(\varepsilon - \varepsilon_c)\beta = x$; $\int_0^{\infty} \sqrt{x}\, e^{-x} dx = \frac{\sqrt{\pi}}{2}$; $e^{-(\varepsilon_c-\mu)\beta}$: const

$$n = 2\left(\frac{m_c^* k_B T}{2\pi\hbar^2}\right)^{3/2} e^{-\frac{(\varepsilon_c-\mu)}{k_B T}}$$

$$p = 2\left(\frac{m_v^* k_B T}{2\pi\hbar^2}\right)^{3/2} e^{-\frac{(\mu-\varepsilon_v)}{k_B T}}$$

if I know $\mu \to n, p$ can be given

Conservation of charge (μ is unknown)

$n = p$ Solve the equations for μ:

$$\mu = \underbrace{\frac{1}{2}(\varepsilon_c + \varepsilon_v)}_{\mu \text{ is in the middle of the gap}} + \underbrace{\frac{3}{4} k_B T \ln\left(\frac{m_v^*}{m_c^*}\right)}_{\text{small / slow function}}$$

n in Na: $n \sim 10^{21} - 10^{22} \frac{1}{cm^3}$

calculate n in a SC

$$n_c = 2\left(\frac{k_B T}{2\pi\hbar^2}\right)^{3/2} (m_c^* m_v^*)^{3/4} e^{-\frac{\varepsilon_g}{2k_B T}} = \frac{1}{10}\left(\frac{10^{-20}}{10^{-68}}\right)^{3/2} (10^{-21})^3 \cdot \underbrace{e^{-40}}_{10^{-20}} \approx 10^{+10}$$

↑ substituting back μ

intrinsic SC → n_c

Charge carriers in intrinsic SCs

DOS: $D(E)$

$n = \frac{1}{V}\int D(E) f(E)\, dE$ in 3D $D(E) \sim \sqrt{E}$

$$n = p = 2\left(\frac{k_B T}{2\pi\hbar^2}\right)^{3/2} (m_c^* m_v^*)^{3/4} e^{-\frac{\varepsilon_g}{2k_B T}} \qquad n \sim e^{-\frac{\varepsilon_g}{2k_B T}}$$

$\boxed{n \cdot p = n_i^2(T)}$ → T dependent constant always valid not only for intrinsic also for extrinsic : LAW OF MASS ACTION

$$c = \frac{[OH^-][H^+]}{[H_2O]} = 10^{-14} \qquad pH = \frac{[H^+]}{[H_2O]}$$

recombination can happen

Mobility vs. charge concentration

Drude: $ma = -eE - kv \rightarrow a = -\frac{eE}{m} - \frac{k}{m}v$ ($\frac{k}{m} = \frac{1}{\tau}$)

stationary: $0 = -eE - kv$

e: elementary charge >

$j = \sigma E$; $j = nev_D$; $\boxed{\frac{|v_D|}{E} = \mu}$: mobility

~~Electrons~~	Electrons	Holes te charge
Drift velocity	$v_{De} = -\frac{eE\tau_e}{m_e^*}$	$v_{Dh} = \frac{eE\tau_h}{m_h^*}$
~~current density conductivity~~	$j_e = nev_{De} = -nev_{De}$	$j_h = pev_{Dh}$
Conductivity $j = \sigma E$ positive for both	$\sigma_e = \frac{ne^2\tau_e}{m_e^*}$	$\sigma_h = \frac{pe^2\tau_h}{m_h^*}$
Mobility	$\mu_e = \frac{v_{De}}{E} = \frac{e\tau_e}{m_e^*}$	$\mu_h = \frac{v_{Dh}}{E} = \frac{e\tau_h}{m_h^*}$
Full conductivity: e^- + h^+ current are additive $\sigma = ne\mu_e + pe\mu_h$		

Dr $0 = eE$

p: ei is n or h pos $j_e =$

HEMT: high e⁻-mobility transistor
↳ revolutionized analogue electronics

$[\mu] = \frac{m}{s}\frac{m}{V} = \frac{m^2}{Vs} \Rightarrow \frac{cm^2}{Vs}$ are used in literature /why(?)/

typical values: ("the larger the better")

Si $\mu_e = 1000 \frac{cm^2}{Vs}$ ← e⁻-phonon scattering

$\mu_h = 100 \frac{cm^2}{Vs}$ ← intra-band scattering → U_c, U_v, h

GaAs $\mu_e = 30\,000 \frac{cm^2}{Vs}$ (better barrier, than Si)

$\mu_h = 1000 \frac{cm^2}{Vs}$

poly-Si: in solar cells $\mu_e \simeq 1 \frac{cm^2}{Vs}$

Cu $\mu = 0{,}1 \frac{cm^2}{Vs}$

graphene $\mu \simeq 2$ million $\frac{cm^2}{Vs}$

hoped to revolutionize modern electronics
quantum reasons
/$\frac{1}{2}mv_D^2$ = thermal large lattice vibrations → falls apart/

Doping

Replacing some atoms by heteroatoms

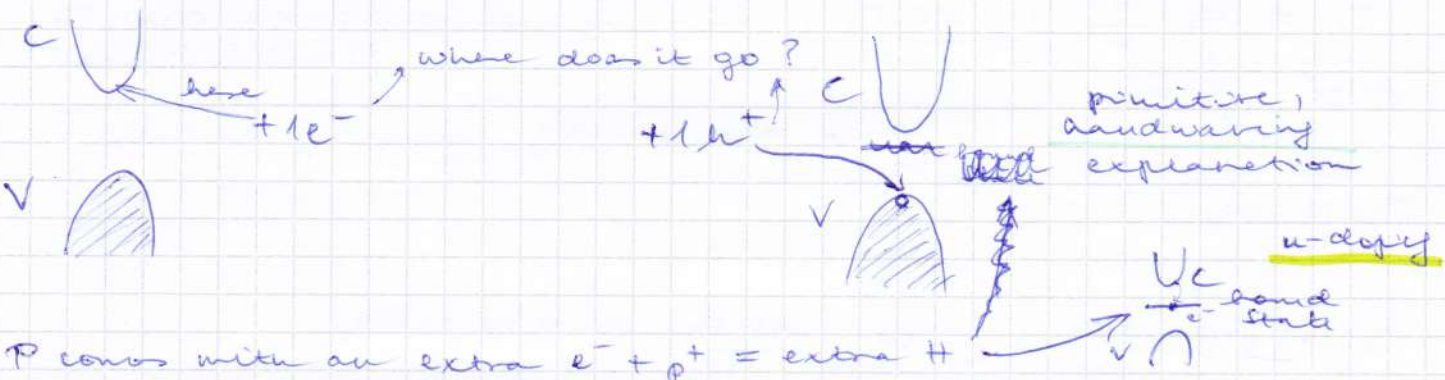
e.g.: 

III B: Si or Al: Si p-dopant

V P: Si As: Si n-dopant

handwaving explanation
like in high-school
($\pm e^-$)

Band picture



Schrödinger-eq. for Si

$$\left[\underbrace{-\frac{\hbar^2}{2m}\Delta}_{\text{kinetic}} + \underbrace{U(r)}_{\text{atomic potentials}}\right]\psi(r) = E\psi(r)$$

We have the solution! (running)

+1 Si replaced by P → same Sch.-eq. ± Sch.-eq. for ±

H-atom, Bohr-model : good for H ground and excited states, L not

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} \quad \& \quad L = mvr = n\hbar \quad \Rightarrow \quad r = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{mv^2}$$

$$mv\hbar = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{v} \Rightarrow \boxed{v_n = \frac{e^2}{4\pi\varepsilon_0}\frac{1}{n\hbar}}$$

introduce: $\alpha = \frac{1}{137} \approx \frac{e^2}{\hbar c 4\pi\varepsilon_0}$ fine structure constant

n: principle Q. number

$$\boxed{v = \alpha\frac{c}{n}} \qquad r_n = \frac{1}{4\pi\varepsilon_0}\frac{e^2 n^2}{m\alpha^2 c^2} = \underbrace{\frac{e^2}{4\pi\varepsilon_0 \hbar c}}_{\alpha}\frac{\hbar}{mc}\frac{n^2}{\alpha^2} = \frac{n^2\hbar}{mc\alpha}$$

$$\boxed{r_n = a_B n^2}$$

$\frac{4\pi m^2 s}{s^2 4\pi m} = [m]$

Introduce: Bohr-radius $a_B = \frac{\hbar}{mc}\frac{1}{\alpha} \approx 0{,}5\,\text{Å} = 5\cdot 10^{-11}\,m$

Binding energy:

$$\boxed{E_{bn} = \frac{1}{2}mv_n^2 - \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n} = \frac{1}{2}m\alpha^2c^2\frac{1}{n^2} - \frac{\alpha\hbar c}{a_B n^2} = \underbrace{-\frac{1}{2}mc^2\alpha^2\frac{1}{n^2}}_{0{,}5\,MeV}}$$

$\frac{\alpha c^2 m}{n^2}$ // 10^{-4}

$$\boxed{E_1 = -13{,}6\,eV} \quad (H: \text{first } E \text{ level})$$

In a SC:

$m \longrightarrow m_e^*$ $\quad m_e^* = 0{,}1 \ldots 1\,m_e$ (carriers the qualities of the lattice)

$\varepsilon_0 \rightarrow \varepsilon_0\varepsilon_r$ extra $e^- + p^+$ Coulomb potential is screened

$\varepsilon_r(Si) = 11{,}9$ (from other e^-) $\quad \alpha \longrightarrow \frac{\alpha}{\varepsilon_r}$

Bohr radius of a dopant

$$a_B^* = \frac{m}{m_e^*}\varepsilon_r a_B \approx 100\,a_B \; ; \; \underline{a_B^* \gg a_B}$$

loosely like H but much larger radius

$$E_1^* = \underbrace{E_1}_{\text{binding energy}} \cdot \frac{m^*}{m}\frac{1}{\varepsilon_r^2} \ll E_1 \; ; \text{ typically } E_1^* = 0{,}0001 \ldots 0{,}001\,E_1$$

$\uparrow -13{,}6\,eV$ $\qquad$ 10 meV .. 100 meV

these are mostly accurate values (correct)

Typically:

$E_c \rightarrow$ donor level energy: E_d $\quad E_d - E_c$: binding energy

$E_v \rightarrow$ acceptor level energy: E_a

Donor density: n_D

$$\boxed{d = \frac{1}{\sqrt[3]{n_D}}}$$ average distance between donors

$D(E)$

if $d \gg a_B^*$ $\quad$ (1 ppm doping)

← discrete states

($d = 30\,nm$)

$D(E)$

$a_B \approx d$ (?)

if $d \ll a_B^*$ $\quad$ (1 ... 1000 ppm doping)

← donor band

(from degenerate state

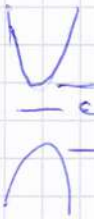
$d \ll a_B^*(R)$

$d \approx a_B^*$

← degenerate Sc (acts like a metal)
/can make an Sc fully conducting/

typically occurs at a few % doping
>1000 ppm doping

Doping vs charge carriers concentration



intrinsic Sc $\mu \approx \frac{1}{2}(E_c + E_v)$

extrinsic Sc
eg. n-doped $\mu \approx \frac{1}{2}(E_c + E_d)$ ← derivation is complicated (Subject II. hoar)

$\langle n \rangle$: expectation value of e^- occupation (in general)

$$\langle n \rangle = \frac{\sum_i n e^{-(\varepsilon_n - \mu n)\beta}}{\Xi} \qquad \Xi = \sum_i e^{-\beta(\varepsilon_n - \mu n)} \qquad \beta = \frac{1}{k_B T}$$

grand canonical ensemble (N can change)

partition function

due to coulomb repulsion (not allowed)

for donor levels: $n = 0, 1, \cancel{2}$

↑↓ double (spin) multiplicity

probability of single occupation:

$$f_d(\varepsilon_d) = \frac{2e^{-\beta(\varepsilon_0 + \varepsilon_d - \mu)}}{e^{-\beta(\varepsilon_0 + 0)} + 2e^{-\beta(\varepsilon_0 + \varepsilon_d - \mu)}} = *$$

ε_0: ground state E
$\varepsilon_0 + \varepsilon_d$: ground + donor is occupied

donor statistics

$$* = \frac{1}{1 + \frac{1}{2} e^{\beta(\varepsilon_d - \mu)}} = f_d(\varepsilon_d) \iff \text{c.f.} \quad f(\varepsilon) = \frac{1}{1 + e^{\beta(\varepsilon - \mu)}}$$

compare to Fermi func.

↑ consequence of spin degeneracy

probability of ionizing the donor state: $(n = 0)$

$$1 - f_d(\varepsilon_d) = \frac{1 + \frac{1}{2} e^{\beta(\varepsilon_d - \mu)}}{1 + \frac{1}{2} e^{\beta(\varepsilon_d - \mu)}} - \frac{1}{1 + \frac{1}{2} e^{\beta(\varepsilon_d - \mu)}} = \frac{1}{1 + \frac{1}{2} e^{\beta(\mu - \varepsilon_d)}}$$

$\frac{1}{1} = 1$

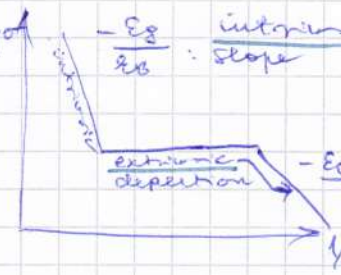
charge carrier concentration in the conduction band

$$n = n_d \cdot \frac{1}{1 + \frac{1}{2} e^{\beta(\mu - \varepsilon_d)}} \quad ; \quad \mu - \varepsilon_d = \underbrace{\frac{1}{2}(\varepsilon_c - \varepsilon_d)}_{\text{donor binding E}} \qquad /p: \frac{1}{2}(\varepsilon_a - \varepsilon_v)/$$

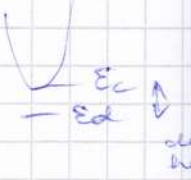
(1.) $T \to 0$ $\quad n = 0$

(2.) $k_B T \approx \varepsilon_c - \varepsilon_d$ donors are ionized $\quad n \approx n_d$; depletion or extrinsic regime

(3.) $k_B T \gg \varepsilon_c - \varepsilon_d$ & $k_B T \approx \varepsilon_g$ intrinsic; band-band excitation



depletion is the ... relevant in today's technology devices work in ... and in het temper... → can control n → stable : σ = const



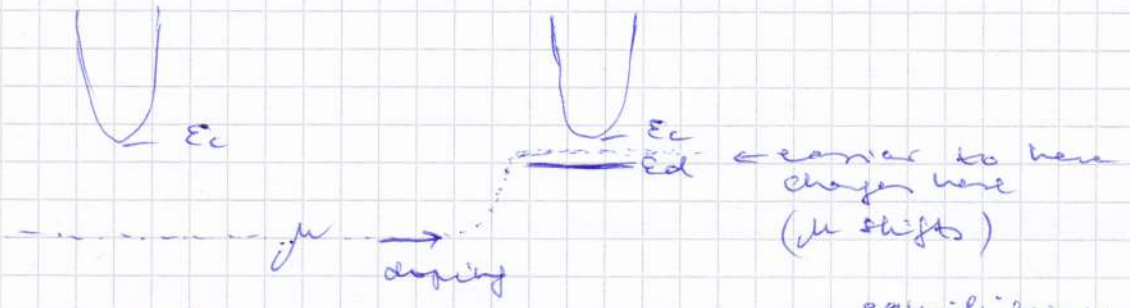
Law of mass action

intrinsic Scs $n = p = \sqrt{n_i p_i} = n_i^2(T) = n_p = n_c n_v e^{-\frac{\varepsilon_g}{2k_B T}}$

$$n = n_c e^{-(\varepsilon_c - \mu)\beta}$$
$$p = n_v e^{-(\mu - \varepsilon_v)\beta}$$

valid also for doped

Handwaving proof of law of mass action



equilibrium proces...

intrinsic

$\mu \approx \frac{\varepsilon_c + \varepsilon_v}{2}$

$n = n_c e^{-(\varepsilon_c - \mu)\beta}$

$p = n_v e^{-(\mu - \varepsilon_v)\beta}$

extrinsic

$n = n_d = n_d e^{-(\varepsilon_c - \mu)\beta}$ → goes up

$p = n_v e^{-(\mu - \varepsilon_v)\beta}$ → goes down

⇒ their pr... remains same

Band structure calculations in Scs

what determines the band structure ?

Importance: understand the conduction properties (m_i^*, ε_g)
optical properties (ε_g)
intelligent design (band-engineering) (making predictions (designs))

Band structure calculation methods (from scratch to advanced)

- empty lattice modell (handwaving model)
  gives k-space quantisation
  ↓ QM: partical in a box
- quasi-free electron approximation ($\frac{\hbar^2 k^2}{2m} + U(\vec{r})$ + periodic reference from atom)
- semi-empirical methods (tight-binding, TB; pseudopotential method)
  → fit to experimental (properties) (m^*, ε_g) (predicting sop unknown)
- ab initio (first principles) DFT
- interacting e⁻ models GW

(we don't talk about this)

$H\psi = E\psi$ simple, but the details are complicated

$$H = -\frac{\hbar^2}{2m}\Delta + U(\underline{r}) \quad ; \quad U(\underline{r}) = \sum_{\text{atom-ion}} H_{at} + \sum_{\text{for all } e^-} H_{e-e}$$

e^- interaction — e^--e^- interaction

most complications come from here

Simplification: use 1 e^- ψ_s (wavefunction)

ions fixed

$$\left[-\frac{\hbar^2}{2m}\Delta + U(\underline{r})\right]\psi(\underline{r}) = E\psi(\underline{r})$$

↑ 1 e^- wavefunction

In solids: $U(\underline{r}) = U(\underline{r}+\underline{R})$ ← lattice periodic ; $\underline{R}$: lattice vector

Bloch-theorem: $\psi_{\underline{k}}(\underline{r}) = e^{i\underline{k}\underline{r}} \cdot u_{\underline{k}}(\underline{r})$ → plain wave

phase factor: does not change the system

where $u_{\underline{k}}(\underline{r}) = u_{\underline{k}}(\underline{r}+\underline{R})$: lattice periodic function

$\underline{k}$ are given by the Born-Karman periodic boundary conditions

eg. 1D ..... ↔ Na, N points

$$\underline{k} = \frac{2\pi}{Na}\cdot n \qquad n = -\frac{N}{2} \dots \frac{N}{2}$$

$\underline{k}$: crystal wavenumber, crystal momentum

$\hbar\underline{k}$ ← Why? → next lecture

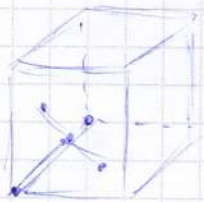
when $\underline{k} = 0 \rightarrow \psi_{\underline{k}=0}(\underline{r}) = u_{\underline{k}}(\underline{r})$ same for each lattice site

in 1D when $\underline{k} = \pm \pi/a$; then + u(r) − u(r) + − + − ...

wavefunction alternates ↔ a

Structure and $\underline{k}$-space

Real space:



$\underline{k}$-space

First Brillouin-zone (W-S-cells)

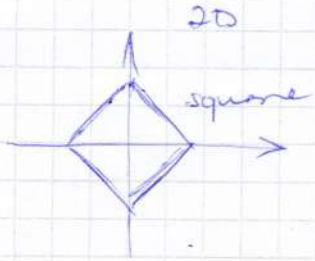
of course on all sides

diamond structure

FCC + 2 atom basis

at (0,0,0) ; $(\frac{1}{4},\frac{1}{4},\frac{1}{4})a$

2D



for 3D we'd need 4D to draw

Distinguished points in $\underline{k}$-space:

notations held for diamond lt. (changes for diff. struct.)

Γ: Brillouin zone center (0,0,0)
L: middle of the hexagon
X: middle of the square
K: middlepoint of 2 touching hexagon

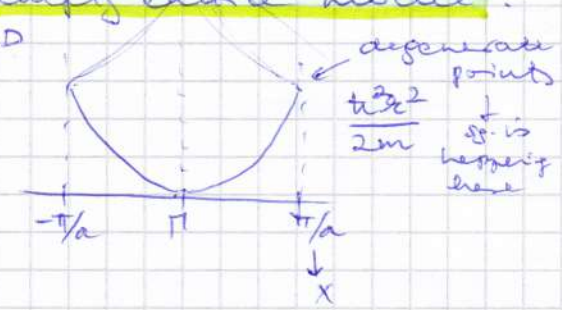
Distinguished directions:

100 $\Gamma \xrightarrow{\Delta} X$: Δ line
111 $\Gamma \xrightarrow{\Lambda} L$: Λ line
110 $\Gamma \xrightarrow{\Sigma} K$: Σ line

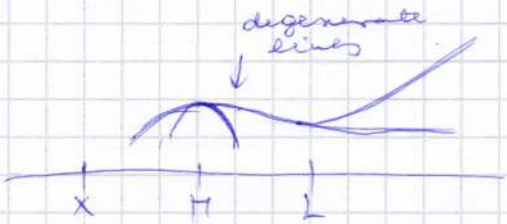
Spaghetti diagrams

Empty lattice model:

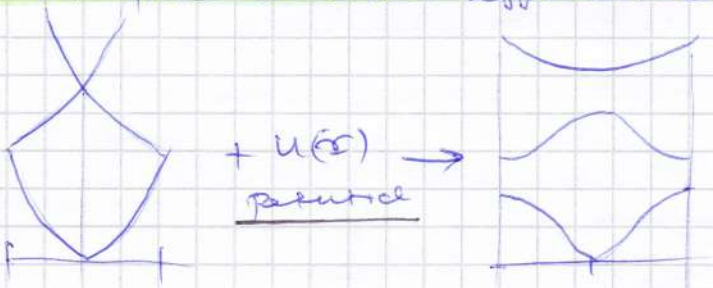
1D



2 or 3D



Quasi-free electron approximation



degeneracy lifts with (perturbation?)

$U(x) = U(x + n\cdot a)$

$\psi_k(r) = \frac{1}{\sqrt{V}} e^{ikr}$ → solves $H = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + U(x)$ ← solves E_k

$E(k) = \frac{\hbar^2 k^2}{2m}$ $\quad H\psi_k = E\psi_k$ $\quad E_k = \varepsilon_k + \langle \psi_k | U(x) | \psi_k \rangle$

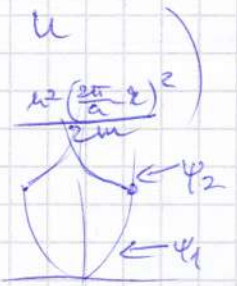
good formula from degenerate points

non-degenerate part. calc.

Degenerate part. calc.:

$H = \begin{pmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{pmatrix} = \begin{pmatrix} \frac{\hbar^2 k^2}{2m} & U^* \\ U & \frac{\hbar^2}{2m}\left(\frac{2\pi}{a} - k\right)^2 \end{pmatrix}$

Hamiltonian mx.



where $U = \int e^{-iGr} U(G) e^{i(k-G)r}$ $\quad = U(G)$ ≈ first terms

$U_{11} = \langle \psi_{1k} | U | \psi_{1k} \rangle$

$U_{12} = \langle \psi_{1k} | U | \psi_{2k} \rangle$

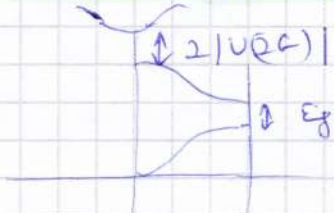
$\psi_1 = \frac{1}{\sqrt{V}} e^{ikr}$

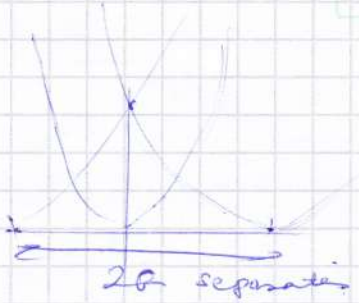
$\psi_2 = \frac{1}{\sqrt{V}} e^{i(k+G)r}$ 1D:

$\varepsilon^0(k) = \frac{\hbar^2 (k+G)^2}{2m}$ non

$U(G)$: Fourier ∫ of U

Solutions at degeneracy





Tight-binding approximation

1D, 1 atom lattice $\quad H\psi = E\psi$; $H = -\frac{\hbar^2}{2m}\Delta + U(r)$

$U(r) = \sum_R V_{atom}(r - R)$

sum of atomic potentials

assuming atomic solution is known $H_{at}\psi_a = \varepsilon_a \psi_a$

(neglect e-e⁻ interactions)

$H_{at} = -\frac{\hbar^2}{2m}\Delta + V_{atom}$

Ansatz ("trial"):

$\psi_k(r) = \frac{1}{\sqrt{N}} \sum_n e^{ikna} \psi_a(r - na)$ ← it satisfies the Bloch thm.

shorthand: $\psi_k(r) = \sum_n e^{ikna} \underbrace{|n\rangle}_{\varphi_a(r-na)}$

substitute into Sch.-eq.:

$$H\psi = \sum_n e^{ikna}\left[-\frac{\hbar^2}{2m}\Delta + \sum_{n'} V_{atom}(r-n'a)\right]|n\rangle = E\sum_n e^{ikna}|n\rangle$$

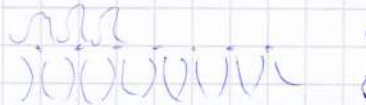
project, multiply $\langle n''|$ left $\rightarrow \langle n''|n\rangle = \delta_{n,n''}$ (wavefunctions are orthogonal)

$$\sum_{n\neq n'} V_{atom}(r-n'a) + \underbrace{V(r-na)}_{\text{on site potential}}$$

$$\sum_n e^{ikna}\left[\varepsilon_a\delta_{n'n} + \sum_{n\neq n'}\langle n''|V_{atom}(r-n'a)|n\rangle\right] = E(k)\sum_n e^{ikna}\underbrace{\langle n''|n\rangle}_{\delta_{n''n}}$$

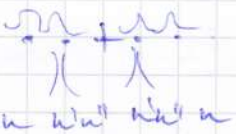
Result: $$E(k) = \varepsilon_a + \sum_{n'\neq n} e^{ik(n-n')a}\langle n''|V_{atom}(r-n'a)|n\rangle$$

seeing graphically



orbitals potentials

from the sum: nearest neighbor tight-binding



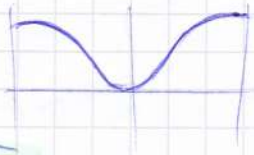
Let's denote:

$\langle n-1|V_{atom}|n\rangle = -|t| = \langle n+1|V_{atom}|n\rangle$

negative $V<0$ (constant interaction)

$$E(k) = \varepsilon_a + \underbrace{\left(e^{ika} + e^{-ika}\right)}_{2\cos(ka)}\cdot(-|t|)$$

$$E(k) = \varepsilon_a - 2|t|\cos(ka)$$



Band Structure Calculations II

Revision: $H\psi = E\psi$ $\quad H: -\frac{\hbar^2}{2m}\Delta + U(r)$ $\quad U(r) = \sum V_{atom}(r-na)$

$H_{atom}\varphi_a = \varepsilon_a\varphi_a$ $\quad H_{atom} = -\frac{\hbar^2}{2m}\Delta + V_{atom}(r)$

Ansatz:

$$\psi_k(r) = \frac{1}{\sqrt{N}}\sum_n e^{ikna}\varphi_a(r-na) \quad \text{(satisfies Bloch-theorem)}$$

new notation ⇕

$$\psi_k(r) = \sum_n e^{ikna}|n\rangle; \quad \langle n'|n\rangle = \delta_{nn'}$$

$\int d^3r\, \varphi_a^*(r-n'a)\varphi_a(r-na)$

plug into $H\psi = E\psi$

$$H\psi = \sum_n e^{ikna}\left[-\frac{\hbar^2}{2m}\Delta + \sum_n V_{atom}(r-na)\right]|n\rangle = E(k)\sum_n e^{ikna}|n\rangle \quad / \cdot\langle n''| \text{ project}$$

$$\sum_{n'} V_{atom}(r-n'a) = \underbrace{V_{atom}(r-na)}_{\text{on-site pot.}} + \sum_{n'\neq n} V_{atom}(r-n'a)$$

$$\sum_n e^{ikna}\left[\varepsilon_a \delta_{nn'} + \sum_{n\neq n'} \langle n''|V_{atom}(r-n'a)|n\rangle\right] = E(k)\sum_n e^{ikna}\delta_{nn''}$$

e^{ikna}

$$E(k) = \varepsilon_a + \sum_{n\neq n'} e^{ik(n''-n')a}\langle n''|V_{atom}(r-n'a)|n\rangle \quad (2)$$

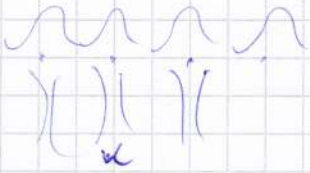
exact ma
← equation

Approximations

(1) Nearest neighbor tight-binding – NNTB

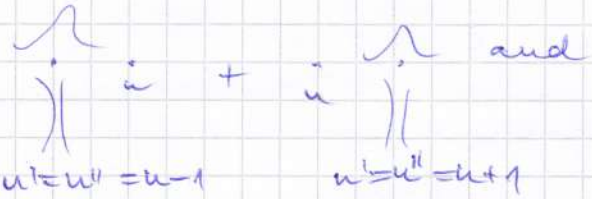
$\varphi_a(r-n'a)$ s-type
$V_{atom}(r-na)$ Coulomb-pot.

NNTB:



n

$\Rightarrow$ Approx.



$n'=n''=n-1$ $\qquad$ $n'=n''=n+1$

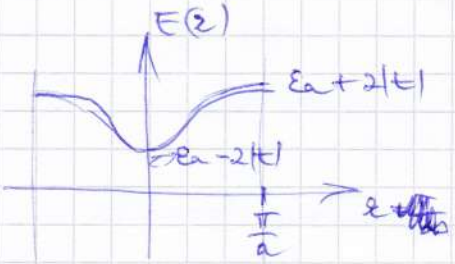
keeping where $n'' = n\pm 1$

$$\langle n-1|V_{atom}(r-(n-1)a)|n\rangle = \langle n+1|V_{atom}(r-(n+1)a)|n\rangle$$
$$= \int \varphi_a^*(r-(n-1)a)\,V(r-(n-1)a)\,\varphi_a(r-na)\,d^3r$$
$$= -|t| \;;\quad |t|>0$$

t: overlap integral

NNTB: $E(k) = \varepsilon_a + \underbrace{\left(e^{ika}+e^{-ika}\right)}_{2\cos(ka)}\cdot(-1)\cdot|t|$

$n''-n = \pm 1$



(0?)

$u^+ > 1$ → price curve

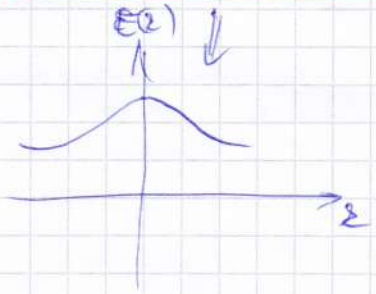
$\Leftrightarrow$ ssσ type overlap

$\oplus\cdot\oplus\cdot\ominus = \ominus$

π-type orbital: ppσ

orbitals have a sign
sign of overlap is positive
$\ominus\cdot\oplus\cdot\ominus = \oplus$ (?)

$$E(k) = \varepsilon_a + 2|t|\cos(ka)$$



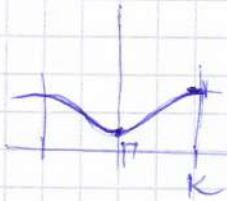
hole type band: $u^* < 0$
(at zero the curve is $\ominus$)

ppπ tight binding

graphene (is like this)

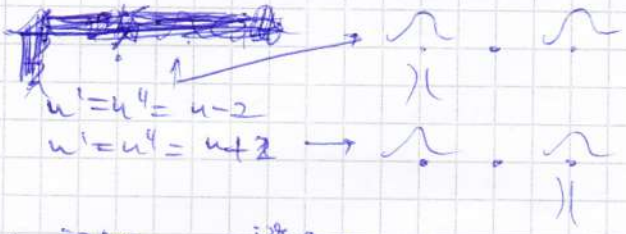
overlap is ⊖

⊕⊕⊖ = ⊖



in graphene interesting things happen here
Tight-Binding Approx. describes its properties well (quite)

2nd nearest neighbor TB (2nd NNTB)



$(e^{ikx a} + e^{-ikx a})\,|t|$ 2nd NNTB

$$E(k) = E_a - 2|t_{nn}|\cos(k_x a) - 2|t_{2nn}|\cos(2k_x a)$$

double periodicity → double wiggles

how many TB terms is enough to describe the system?
(I can consider many terms, as much as I want)
further I go, the more wiggles I have

⇒ TB is semi-empirical:

- DFT band-structure (gives numbers) + fitting with TB parameters (|t|) (gives function) (wiggles: where do they come from)
- fit |t|-s from experiment like E_g or m^*

example: effective mass $(m^*)^{-1} = \frac{1}{\hbar^2}\frac{\partial^2 E(k)}{\partial k^2}$

$$E(k) = E_a - 2|t|\cos(k_x a) = E_a - 2|t|\left[1 - \frac{k_x^2 a^2}{2}\right] \rightarrow (m^*)^{-1} = \frac{2|t|a^2}{\hbar^2}$$

$\cos x a \cong 1 - \frac{x^2 a^2}{2}$ (Taylor series)

dim: $[t] = J$ (energy), $[(m^*)^{-1}] = \frac{J m^2}{J^2 s^2} = \frac{m^2}{J s^2} = \frac{1}{kg}$ (indeed)

t changes exponentially with a
constricting a → cc.s get lighter (→ faster, hotter)

2D TB Approximation

$$E(k) = E_a - 2|t_A|\cos(k_x a) - 2|t_B|\cos(k_y b)$$

$E(k_x, k_y)$

"k·p model" (new topic, new for everyone)

- good semi-empirical model near band extreme ⇔ ideal for ħ (relevant fields in Sci. important)
- gives clear results for transport, optics and magnetic properties

$H\psi = E\psi$ we want to solve this still
Solution always the Bloch-term!

$$\psi_{n,k}(r) = e^{ikr}\, u_{nk}(r)$$

n: band index, u_{nk}: lattice periodic function

$\underline{k}$: crystal momentum ; generally $\hbar\dot{\underline{k}} = \underline{F}_{ext}$ ← external forces

(separation of ext and int. forces is possible)

$\underline{p} = -i\hbar\underline{\nabla}$ $\left(= \frac{\hbar}{i}\underline{\nabla}\right)$ canonical momentum

$\hbar\underline{k} \neq \underline{p}$ except for plane waves

$H = \frac{-\hbar^2}{2m}\Delta + V(\underline{r})$ ↓ periodic part.

substitute : $\psi_{n\underline{k}}(\underline{r})$ into $H\psi$: $u_{n\underline{k}}(\underline{r})$

$\Delta\psi_{n\underline{k}}(\underline{r}) = (i\underline{k})^2 u_{n\underline{k}}(\underline{r}) + 2i\underline{k}\, e^{i\underline{k}\cdot\underline{r}}\underline{\nabla} u_{n\underline{k}}(\underline{r}) + e^{i\underline{k}\cdot\underline{r}}\Delta u_{n\underline{k}}(\underline{r})$

$\Delta\psi_{n\underline{k}} = (i\underline{k})^2 u_{n\underline{k}}(\underline{r}) + 2i\underline{k}\, e^{i\underline{k}\cdot\underline{r}}\underline{\nabla} u_{n\underline{k}}(\underline{r}) + e^{i\underline{k}\cdot\underline{r}}\Delta u_{n\underline{k}}(\underline{r})$ → into $H\psi$

$$\left[\frac{\hbar^2\underline{k}^2}{2m} - \frac{i\hbar^2\underline{k}}{m}\cdot\underline{\nabla} - \frac{\hbar^2}{2m}\Delta + V(\underline{r})\right] u_{n\underline{k}}(\underline{r}) = E_{n\underline{k}}\, u_{n\underline{k}}(\underline{r})$$

$\underline{p} = -i\hbar\underline{\nabla}$ put into $H\psi$ gives :

$$\boxed{\left[\frac{\hbar^2\underline{k}^2}{2m} + \frac{\hbar\underline{k}\cdot\underline{p}}{m} + \frac{p^2}{2m} + V(\underline{r})\right] u_{n\underline{k}}(\underline{r}) = E_{n\underline{k}}\, u_{n\underline{k}}(\underline{r})}$$

So far exact !

$\underline{k}\cdot\underline{p}$ approx. : consider solution of Sch. near band – ed where $\underline{k} = 0$ (like Γ)

then $\boxed{\left[\frac{p^2}{2m} + V(\underline{r})\right] u_{n0}(\underline{r}) = E_{n0}\, u_{n0}(\underline{r})}$: Sch. eq. at the center

+ consider $\frac{\hbar^2\underline{k}^2}{2m} + \frac{\hbar\underline{k}\cdot\underline{p}}{m}$ as perturbation ! works if $\underline{k}$ small

2nd order perturbation theory

suppose : $H|u\rangle = \varepsilon_u|u\rangle$ is solved $|u\rangle$ is a set of orthonorm. wavefunctions

$\langle u'|u\rangle = \delta_{uu'}$

$(H+V)\psi = E\psi$ what is E and ψ ?

Ansatz : $E_u = \varepsilon_u + \underbrace{\langle u|V|u\rangle}_{\text{1st order}} + \sum_{u'\neq u} \frac{|\langle u'|V|u\rangle|^2}{\varepsilon_{u'} - \varepsilon_u}$ (no summ.)

$\psi_u = |u\rangle + \sum_{u'\neq u} \frac{\langle u'|V|u\rangle}{\varepsilon_{u'} - \varepsilon_u}|u'\rangle$

orig basis : $u_{n\phi}(\underline{r})$;

result : $u_{n\underline{k}}(\underline{r}) = u_{n\phi}(\underline{r}) + \sum_{n'\neq n} \frac{\hbar}{m} \frac{\langle u_{n'\phi}|\underline{k}\cdot\underline{p}|u_{n\phi}\rangle}{\varepsilon_{n,\phi} - \varepsilon_{n',\phi}}|u_{n'\phi}\rangle$

$\langle u_{n\phi}|\left[\frac{\hbar^2\underline{k}^2}{2m} + \frac{\hbar\underline{k}\cdot\underline{p}}{m}\right]|u_{n\phi}\rangle$

1st term : $\frac{\hbar^2\underline{k}^2}{2m}\langle u_{n\phi}|u_{n\phi}\rangle = \delta_{nn'}$ but $n\neq n'$: it's nothing but

So: $u_{nk}(r) = u_{n0}(r) + \frac{\hbar}{m}\sum_{n\neq n'} \frac{\langle u_{n'0} | k\cdot p | u_{n0}\rangle}{E_{n'0} - E_{n0}} |u_{n'0}\rangle$

$$E_{nk} = E_{n0} + \frac{\hbar^2 k^2}{2m} + \frac{\hbar^2}{m^2}\sum_{n'\neq n} \frac{|\langle u_{n'0} | k\cdot p | u_{n0}\rangle|^2}{E_{n'} - E_n}$$

$\langle u_{n0} | \frac{\hbar^2 k^2}{2m} + \frac{\hbar}{m} k\cdot p | u_{n0}\rangle$ gives: $\frac{\hbar^2 k^2}{2m} + \phi$ → disappears (?)

↑ linear term in k disappears near the band-edge

$\langle u_{n0} | \frac{\hbar^2 k^2}{2m} | u_{n0}\rangle = \delta_{nn}$

↑ it's a number not an operator

effective mass approximation:

$E_{nk} = E_{n0} + \frac{\hbar^2 k^2}{2m^*}$ → semi-empirical model

↑ band edge extreme energy

upper / calculation (e^- mass is retained)

Relation between the two is: $\frac{1}{m^*} = \frac{1}{m} + \frac{2}{mk^2}\sum_{n'\neq n} \frac{|\langle u_{n'0} | k\cdot p | u_{n0}\rangle|^2}{E_{n0} - E_{n'0}}$

the mx. element is measurable:

$P^2 = \frac{1}{k^2} |\langle u_{n'0} | k\cdot p | u_{n0}\rangle|^2$

↳ experimentally determined: $\frac{2P^2}{m} \approx 200\ eV$ $(20\ eV)^3$ → for most group IV and III–V Sc.

this gives: $\frac{1}{m^*} = \frac{1}{m} + \frac{2P^2}{mE_g}$ this describes m^* well for III–V Sc.s and IV

↑ gap energy

Envelope function method (funrolegh. moldener)

- works well in SC with slowly varying external fields (magnetic, electric fields, doping, heterostructures ($Ga_xAl_{1-x}As$) bandgap engineering)

slowly: characteristic lengthscale of V is longer than lattice constant

$H_0\psi(r) = E\psi(r)$

Assume: solution without fields or doping is known.

↑ original H includes $U(r)$ (periodic pot.)

↑ E_{nk}: n band index, k crystal mom. index

$\psi_{nk}(r) = e^{ikr} u_{nk}(r)$ ← most generic solution

$u_{nk}(r) = u_{nk}(r+R)$ /periodic/

External field is a perturbation:

$H = H_0 + V(r)$

↑ slowly varying function

FT: $V(k) = \int e^{ikr} V(r)\, d^3r \simeq \delta_k V_0$ ← FT of slowly varying function

↑ strongly localized

FT of constant → δ

$H\Phi = E\Phi$; **Ansatz**: $\Phi(r) = \sum_{nk} F_n(k)\psi_k(r)$ (total function)

linear combination of atomic orbitals (LCAO)

(intuitive result)

$F_n(k)$: multiplying coeff for fixed k → Envelope function

Sch. eq.: substitute $\Phi(r)$ in here

(*) $\sum_{n k} \psi_{nk}\left[E_{nk}(r) - E + V(r)\right] F_n(k) = 0$ — no longer a differential

$\psi_{n,k}(r)$ forms an orthonormal set of functions

$\int \psi^*_{n'k'}(r)\psi_{nk}(r)\, d^3r = \delta_{nn',kk'} = \delta_{nn'}\delta_{kk'}$

we $\int \psi_{n'k'}(r)$ the (*) equation / $\langle \psi_{nk}(r)|$ — $\langle \psi_{n'k'}|\psi_{nk}\rangle = \delta$

$\Rightarrow$ (**) $\sum_{nk}\left[(E_{nk}(r) - E)\delta_{nn'}\delta_{kk'} + \langle \psi_{n'k'}|V(r)|\psi_{nk}\rangle\right] F_n(k) = 0$ — Since Most...

← set of linear ... for $F_n(k)$

$\langle \psi_{n'k'}|V(r)|\psi_{nk}\rangle = \int u^*_{n'k'}(r)\, u_{nk}(r) \cdot e^{i(k-k')r} \cdot V(r)\, d^3r$ ← difficult, it's approximat...

fast varying (rapidly) ↑ slowly varying

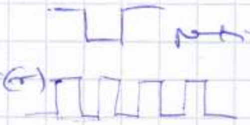
EFA is separation: if functions are different in property

$\langle \psi_{n'k'}|V(r)|\psi_{nk}\rangle \approx \underbrace{\int u^*_{n'k'}(r)\, u_{nk}(r)\, d^3r}_{\text{Bloch's theorem } \delta_{nn'}\delta_{kk'}} \cdot \underbrace{\int e^{i(k-k')r} V(r)\, d^3r}_{\delta(k k')\cdot V(k-k')\ \ (FT)}$

Substitute back to (**) eq.

$\sum_k \left[(E_{n k}(r) - E)\delta_{kk'} + \underset{\delta_{kk'}(r)}{V(k-k')}\right] F_{n'}(k) = 0$

This is a Sch.-eq. for $F_{n'}(k)$ ↑ — particle in a box



Example:

planewave solution + effective mass approximation

$E_n(k) = E_c + \frac{\hbar^2 k^2}{2m_c^*}$ with this we get the EFA equation as: (known solution)

$\frac{\hbar^2 k^2}{2m_c^*} F_c(k) + \sum_k V(k-k') F_c(k) = (E - E_c) F_c(k)$ ← set of linear equations

↑ n: band index; EF. for conduction band

We approximate canonical impulse with:
$-i\hbar\nabla \equiv k$ (is valid for planewaves)

the Schrödinger equation for $F_c(k)$:

$\left[\frac{-\hbar^2\Delta}{2m_c^*} + E_c + V(r)\right] F_c(r) = E F_c(r)$ — Sch. eq. for $F_c(r)$ function

FT. both sides: $F_c(r) \to F_c(k)$

$FT(f'(r)) = ik \cdot f(k)$; $FT(f'(r)) = \int e^{ikr} f'(r)\, d^3r = \underbrace{e^{ikr} f(r)}_{0} - \int ik\ldots$

($\int uv' = uv - \int u'v$) partial.

$FT(V(r)F_c(r)) = \int V(k-k')F_c(k)\, d^3r$ ← convolution ∫

Substituting it back gives:

$\frac{\hbar^2 k^2}{2m_c^*} F_c(k') + \sum_k V(k-k') F_c(k) = (E - E_c(k)) F_c(k)$

Exam: Sch. for ... (EF: ψ) potential, EF is ...

Semiempirical method as it needs E_c and m_e^* as input

Example: extra charge (doping)

charge density: $n_e(r) = \sum_i |\psi(E_i)|^2 f(E_i)$ ← probabilistic interpretation
(Born) ↑ Fermi fkt.

EFA ↓

$$n(r) \overset{EFA}{=} \sum_i |\psi(r)|^2 f(E_i) \overset{EFA}{=} n_{eff} \cdot \sum_i F_i(r) f(E_i)$$

original fkt.

e.g. n doped Si: P: Si

(EFA)
everything is self-consistent
works well if $a_B^* \gg d$
(Bohr) (atomic distance)

Transport phenomena in Scr

current: j
thermal current: j_Q
magnetoresistance:
frequency dependent conductivity
Seebeck-effect, Peltier-effect

size (length) scales

- cc. diffusion length — charge is created, annihilates
typically: 10 µm – cm (solar cells)

Diffusion: 

↓ propagation in material is diffusive

mean-free-path:

$$l = v_F \tau$$

v_F: Fermi velocity; τ: momentum scattering time = relaxation time

$\tau = 10^{-13} \ldots 10^{-15}$

τ_c: cc. lifetime: $\tau_c(Si) = 10\,\mu s \ldots 10\,ms$

$l < \delta_\varphi < \delta_s < \delta_c$

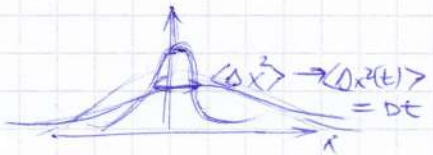
Fick's law: $\underline{j} = -D \nabla n$

$\underline{j}$: diffusion current; D: diffusion constant; $D \approx 1/3 \cdot v_F^2 \tau \approx 1/3\, v_F \cdot l$

charge carrier diffusion length: $\delta_c = \sqrt{D \tau_c}$; solution of diffusion eq.

Spin diffusion length; τ_s: spin lifetime

$\tau_s = 1\,ns \ldots 1\,\mu s$ $\quad \delta_s = \sqrt{D \tau_s}$ typical values: 1 … 100 µm



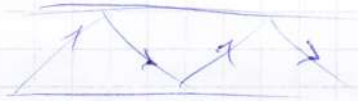
phase coherence length: 100 nm … 1 µm

wave scatters coherently on an impurity

e^- propagates as a planewave

important for: weak localization
ballistic transport

optical analogy of ballistic transport: optical fibre



no attenuation
always phase coherent

Momentum relaxation time: $10^{-13} \ldots 10^{-15}$

mean-free path 1 nm … 100 nm $l = v_F \tau$

↑ ohm's law (diffusion + drift)

→ mesoscopic physics 100 nm – 1 µm Landauer-Büttiker formalism: (ohm's law may not work below a given length scale)

Transport in SCs

Diffusion → ohm's law

	Free electrons (Sommerfeld model)	Bloch-electrons
quantum No.	$\underline{k}$ ε periodic b.c. (boundary condition)	$\underline{k}$ ε periodic b.c. + n: band index
velocity	$\underline{v} = \frac{1}{\hbar}\frac{\partial \varepsilon}{\partial \underline{k}} = \frac{\hbar \underline{k}}{m}$	$\underline{v}_n(\underline{k}) = \frac{1}{\hbar}\frac{\partial \varepsilon_n}{\partial \underline{k}} = \frac{1}{\hbar}\nabla_k \varepsilon(\underline{k})$
Energy dispersion	$\varepsilon(\underline{k}) = \frac{\hbar^2 k^2}{2m}$	$\varepsilon(\underline{k}) = \varepsilon_n(\underline{k}+\underline{G})$; $\underline{G} \in$ reciprocal lattice
wave-fn (ψ)	$\psi_k(x) = \frac{1}{\sqrt{V}} e^{ikr}$	$\psi_{nk} = e^{ikr} u_{nk}(x)$; where $u_{nk}(x) = u_{nk}(x+R)$
effective mass	$m^* = m$	$(\underline{\underline{m}}^*)^{-1} = \frac{1}{\hbar}\frac{\partial^2 \varepsilon}{\partial \underline{k}^2}$; $\underline{\underline{m}}^*$ tensor: $\underline{\underline{m}} = \begin{pmatrix} \frac{\partial^2 \varepsilon}{\partial k_x^2} & \frac{\partial^2 \varepsilon}{\partial k_y \partial k_x} & \frac{\partial^2 \varepsilon}{\partial k_x \partial k_z} \\ & \ddots & \\ & & \frac{\partial^2 \varepsilon}{\partial k_z^2}\end{pmatrix}$

Remarks:

Drude-model: $ma = -eE + kv$
↑ damping, viscous force constant

$m\dot{v} = -eE - \frac{m}{\tau}v \rightarrow \sigma = \frac{ne^2\tau}{m}$

resistivity is due to collisions with ions, viscous environment

Bloch-model: motion of e⁻ in a perfect lattice (ions at equilibrium position)

$\sigma = \infty$
$\tau = \infty$

velocity of e⁻ : $\underline{v}(\underline{k}) = $ const.

impurities, lattice vibrations : phonons
↳ ions motion

Canonical vs Crystal momentum (impulse)

conservation of momentum → result of translational invariance

In a crystal we have a discrete translational invariance

$\underline{r} \rightarrow \underline{r} + \underline{R}$ ← lattice vector → crystal momentum is conserved: $\hbar \underline{k}$

Forces: $\boxed{F_{ext} = \hbar \dot{\underline{k}}}$ ← periodic potential is transformed out from the problem! (eg. $-eE$, Lorentz...)

Price paid: $u_{nk}(r)$ are not nice
canonical impulse ≠ crystal momentum → $p \neq \hbar k$

canonical impulse : $p = \frac{\hbar}{i}\nabla$, $p\psi_{nk}(r) = \frac{\hbar}{i}\nabla(e^{ikr}u_{nk}(r)) = \hbar k\, \psi_{nk}(r) +$

Bloch - fns are no eigenstates of canonical impulse $p \neq \hbar k$

contribution of a closed e^- band to conduction

$$j = -e \int \underbrace{\frac{d^3k}{4\pi^3}}_{D(k)} \cdot \underbrace{\frac{1}{\hbar}\frac{\partial \varepsilon}{\partial k}}_{v(k)} \cdot 1 \qquad j = -e \int D(k) \, f(k) \, v(k) \cdot d^3k$$

number; c.c.; (DOS); Fermi-fn. (occupation fn.)

at $T=0$ $\quad f(k)=1$

$$f(k) = \frac{1}{e^{\frac{\varepsilon - \mu}{k_B T}} + 1} \qquad \varepsilon = \varepsilon(k)$$

j for a closed band: $= 0$ ← strong statement (Aschroft-Mermin beer)

$$\boxed{j = -e \int_{\text{closed band}} \frac{d^3k}{4\pi^3} \frac{1}{\hbar}\frac{\partial \varepsilon}{\partial k} = 0}$$

(no conduction in closed bands) / handwaving arguments / (sponge tilting demonstration)

$$\int \underbrace{\frac{d^3k}{4\pi^3}}_{\text{symmetric domain}} \frac{\partial \varepsilon}{\partial k} = 0 \quad \leftarrow \text{odd fn.}$$

Concept of holes:

$$0 = \int_{\text{closed band}} \frac{d^3k}{4\pi^3} v(k) = \int_{\text{states under } \varepsilon} \frac{d^3k}{4\pi^3} v(k) + \int_{\text{states above } \varepsilon} \frac{d^3k}{4\pi^3} v(k)$$

separate

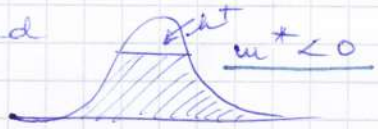
$$-e \int_{\text{states below } \varepsilon} \frac{d^3k}{4\pi^3} v(k) = +e \int_{\text{states above } \varepsilon} \frac{d^3k}{4\pi^3} v(k)$$

electron conduction

hole conduction

→ if the cc.-s were present

hole is just a concept



$m^* a = \mp$

Boltzmann-equation

We describe the system with particle distribution function.

$f(k, r, t)$ distribution function simplest form:

$$f_0(\varepsilon(k), k, r, t) = \frac{1}{e^{\frac{\varepsilon(k) - \mu}{k_B T}} + 1}$$

(for fermions)

stationary → no (t, r)

measureable quantities:

$$j = -e \int \tau(k) \, f(\varepsilon(k)) \, v(k) \, d^3k$$

↑ external forces affect the distribution

$\frac{\hbar}{i} e^{\frac{ikr}{}} \nabla$ kinetic

Boltzmann:

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \dot{\underline{x}}\,\nabla_{\underline{x}} f + \dot{\underline{v}}\,\nabla_{\underline{v}} f = \left.\frac{\partial f}{\partial t}\right|_{\text{collision term}}$$ (no general solution)

collision term ↔ (analogous) Drude-model $m\dot{\underline{v}} = -e\underline{E} - \frac{m\underline{v}}{\tau}$ (collision)

↑ introduced heuristically

$f(\underline{E}(\underline{x}), \underline{x}, \underline{v}, t)$; $\frac{df}{dt}$ (scalar) $\overset{\text{chain's rule}}{=}$ $\frac{\partial f}{\partial t}$ (4.) + $\frac{\partial f}{\partial \underline{x}}\dot{\underline{x}}$ (2. $\nabla_{\underline{x}} f$) + $\frac{\partial f}{\partial \underline{v}}\dot{\underline{v}}$ (3. $\nabla_{\underline{v}} f$) + $\frac{\partial f}{\partial \underline{E}(\underline{x})}\frac{\partial \underline{E}(\underline{x})}{\partial \underline{x}}\dot{\underline{x}}$ (1.)

$\left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$

$f^0 = \frac{1}{e^{(\varepsilon-\mu)\beta}+1} = \frac{1}{e^x+1}$ additional terms: $\frac{\partial f}{\partial x}\frac{\partial x}{\partial \mu}\frac{\partial \mu}{\partial \underline{r}}\dot{\underline{r}} + \frac{\partial f}{\partial x}\frac{\partial x}{\partial T}\frac{\partial T}{\partial \underline{r}}\dot{\underline{r}}$

- $\dot{\underline{x}}\nabla_{\underline{x}} f$: force term $m\dot{\underline{x}} = \underline{F}_{ext} = -e\underline{E} = -e\nabla\times\underline{B}$
- $\dot{\underline{r}}\nabla_{\underline{r}} f$: diffusion term (we neglect it)

Assume force term only

$$\frac{\partial f}{\partial t} + \frac{1}{m}\underline{F}_{ext}\underbrace{\frac{\partial f}{\partial \underline{v}}}_{\nabla_{\underline{v}} f} = \left.\frac{\partial f}{\partial t}\right|_{\text{collision term}} = \frac{-g(\underline{r},t)}{\tau}$$ relaxation time approximation

$$f(\underline{r},t) = \underbrace{f^0(\underline{r},t)}_{\text{eq. distribution}} + \underbrace{g(\underline{r},t)}_{\text{difference in equilibrium}}$$

τ - relaxation time

scattering probability per unit time from $\underline{v} \to \underline{v}'$; usually: $\tau = \tau(\underline{v})$

τ is the same as τ in the Drude-model

Assume: $\underline{F}_{ext} = 0$ relaxation approximation (τ = const)

$$\frac{\partial f}{\partial t} = \underbrace{\frac{\partial f^0}{\partial t}}_{0} + \frac{\partial g(\underline{r},t)}{\partial t} = \frac{-g(\underline{r},t)}{\tau} \rightarrow g(t) = g(t=0)e^{-t/\tau}$$

origin of the name: relaxation time

f: distribution fu. $f^0(\underline{v}) = \frac{1}{e^{\frac{\varepsilon(\underline{v})-\mu}{k_B T}}+1}$

$f(\underline{r}, \underline{v}, t)$

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial t} + \underbrace{\dot{\underline{x}}\nabla_{\underline{x}} f}_{\text{force}} + \underbrace{\dot{\underline{r}}\nabla_{\underline{r}} f}_{\text{diffusion}} = \left.\frac{\partial f}{\partial t}\right|_{\text{coll.}}$$

Stationary case: $\frac{\partial f}{\partial t} = 0$ plus DC electric field $\underline{F}_{ext} = -e\underline{E}$

Neglect diffusion term: $\nabla_{\underline{r}} f = 0$

force term: $\nabla_{\underline{x}} f = \frac{\partial f}{\partial x} = \frac{\partial f}{\partial \varepsilon}\frac{\partial \varepsilon(\underline{x})}{\partial \underline{x}}$ $\quad \underline{v}(\underline{x}) = \frac{1}{\hbar}\frac{\partial \varepsilon(\underline{x})}{\partial \underline{x}}$

$\Rightarrow \nabla_{\underline{k}} f = \frac{\partial f}{\partial \varepsilon} \hbar \underline{v}(\underline{k})$

$\dot{\underline{k}} = \frac{1}{\hbar} \underline{F}_{ext}$

Substitute back to Boltzmann-eq.:

$$-\frac{e\underline{E}}{\hbar} \cdot \frac{\partial f}{\partial \varepsilon} \hbar \underline{v}(\underline{k}) = -\frac{g(\underline{k},t)}{\tau}$$

Solution for g: $g(\underline{k},t) = \frac{\partial f}{\partial \varepsilon} \cdot e\underline{E} \cdot \underline{v}(\underline{k}) \cdot \tau$

Final result: $\boxed{f(\underline{k},t) = f^0(\underline{k}) + \frac{\partial f}{\partial \varepsilon} \cdot e\underline{E} \cdot \underline{v}(\underline{k}) \tau}$

leads to a linearization

approx: $\frac{\partial f}{\partial \varepsilon} \approx \frac{\partial f^0}{\partial \varepsilon}$

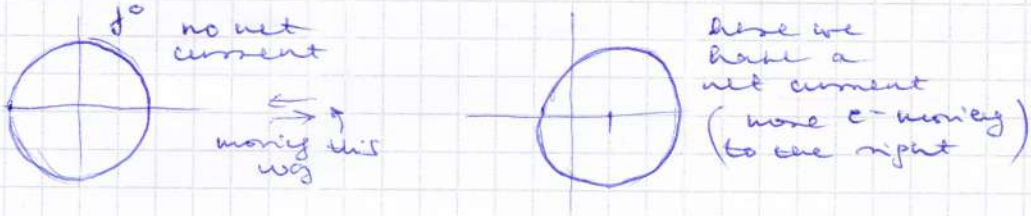
consider:

$$f^0\left(\underline{k} + \frac{e\underline{E}\tau}{\hbar}\right) \approx f^0(\underline{k}) + \frac{e\underline{E}\tau}{\hbar} \cdot \frac{\partial f^0}{\partial \varepsilon} \cdot \underbrace{\frac{\partial \varepsilon}{\partial \underline{k}}}_{\hbar \underline{v}(\underline{k})} = f^0(\underline{k}) + e\underline{E}\tau \frac{\partial f^0}{\partial \varepsilon} \underline{v}(\underline{k})$$

$$f(x+dx) = f(x) + dx \frac{\partial f}{\partial x} + O(dx^2)$$

perturbed distribution function looks like as if it was shifted in $\underline{k}$-space by $\Delta\underline{k} = -\frac{e\underline{E}\tau}{\hbar}$ (parallel to $\underline{E}$)

net current:



net drift velocity from Boltzmann-eq.

$$v_d = \frac{\hbar \Delta k}{m^*}$$

$$\boxed{v_d = -\frac{e\underline{E}\tau}{m^*}}$$ ← Boltzmann eq. => same as Drude

Drude:

$m^* \dot{v} = eE - kv$

$\dot{v} = \frac{eE}{m^*} - \frac{v}{\tau}$ => stationary: $\boxed{v_d = -\frac{eE\tau}{m^*}}$

Drude — conceptual difference — **Bloch (Boltzmann eq.)**

lot of small moving e^- | a few e^- only those on the Fermi surface, but they move fast with v_F

$v = v_d \ll v_F = \left(10^5 - 10^6 \frac{m}{s}\right)$ (Fermi-velocity)

$\boxed{j = nev_d}$ (net current)

$\boxed{j = e v_F \frac{n v_d}{v_F}}$ → number of charge carriers

same result, but conceptually different! QM is right!

Transport in SCs II

τ is a principal quantity

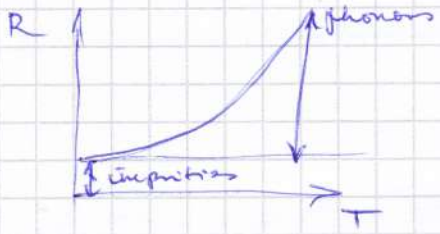
origins: phonons, impurities, defects
↳ everything that breaks the perfect lattice (the symmetry)

low T: impurity
high T: ionized dopants

Matthiessen's rule:

$$\frac{1}{\tau} = \frac{1}{\tau_{e\text{-}e}} + \frac{1}{\tau_{e\text{-}phonon}} + \frac{1}{\tau_{imp}} + \dots$$

shortest timescale dominates

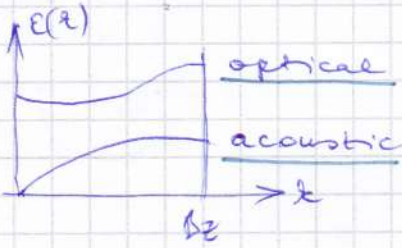


R: resistivity

examples:

$$\frac{1}{\tau_{e\text{-}e}} \sim T^2 \; ; \quad \frac{1}{\tau_{e\text{-}ph}} = \begin{cases} \sim T & T \gg T_{Debye} \\ \sim T^5 & T \ll T_{Debye} \end{cases} \qquad (k_B T_D = \hbar\omega_D)$$

phonon - dispersion



optical: only excited with optical phonons ($E \gtrsim k$)

acoustic: looks like an acoustic ~~wave~~: $E \sim k$

Debye-model: (acoustic)

Debye - Einstein model: acoustic + optic

Ω: phonon dispersion frequency

Eliashberg - formalism: (phonon only)

$$\frac{\hbar}{\tau} = 2\pi k_B T \lambda$$

λ: e⁻-phonon coupling constant typ.: 0,1-1

$\frac{\hbar}{\tau}$: scattering rate

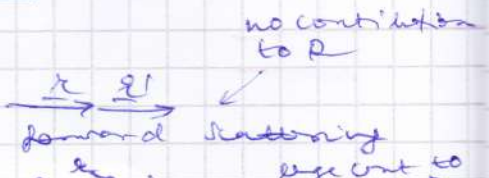
τ is always around $10^{-13} - 10^{-15}$ second

only valid if T is large

$$\lambda = 2\int \frac{d\Omega}{\Omega} \alpha^2 F(\Omega); \quad \underbrace{\alpha^2 F(\Omega)}_{\downarrow \text{DOS}}: \text{notation: how strong } e^- \text{ are coupled to phonons}$$

if T is small

$\alpha^2 F(\Omega)$ contains the so-called $1-\cos\vartheta$ term:



no contribution to R

large contribution to R

R: resistivity

$\vartheta \neq 0$ $\xrightarrow{k \; k'}$ $\vartheta = 0 \rightarrow 1 - \cos\vartheta = 0$ (does nothing)

$\vartheta \neq \pi$ $\xrightarrow{k}$ k' $\vartheta \neq \pi \rightarrow 1 - \cos\vartheta = 2$

Bloch-Grüneisen-fn : (acoustic, isotropic phonons)

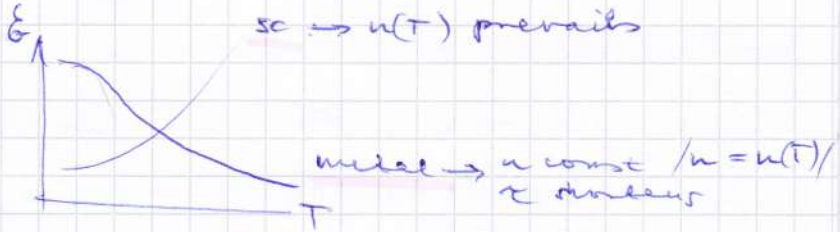
Debye model with a single T_D — explicit, analytic solution

$$\frac{1}{\tau} = 2\pi\lambda k_B T \int_0^{\Omega_D} \left(\frac{d\Omega}{\Omega_D}\right) \left[\frac{\hbar\Omega/k_B T}{\sinh\left(\frac{\hbar\Omega}{2k_B T}\right)}\right]^2$$

high T constant

$\begin{cases} 1 \text{ if } T \to \infty \\ \sim 10^4 \text{ if } T \to 0 \end{cases}$

in SCs : $\sigma = \frac{ne^2\tau}{m^*} = \xi$



Magnetotransport

Hall-effect :

applying Drude-model $\quad \underline{F} = q\underline{v} \times \underline{B}$ ↓ Lorentz force

in stationary conditions : internal field builds up to compensate for it

$V_H = dE_{tr}$ (voltage)

$$v_d = \frac{eE_{long}\tau}{m^*}$$

$$eE_{tr} = ev_d \cdot B \Rightarrow \quad E_{tr} = v_d B = \frac{eE_{long}\tau}{m^*} B$$

Hall voltage : $V_H = d v_d \cdot B = \frac{de E_{long} B \tau}{m^*}$

define : R_H : Hall resistivity

$j = \xi E$, $j\rho = E$

$$R_H = \frac{E_{tr}}{j_{long}} = \frac{n_D B}{j_{long}} = \frac{v_d B}{n e v_d} = \frac{B}{ne}$$

$$[R_H] = \frac{Vs}{m^2}\frac{m^3}{As} = \frac{V}{A} m = \Omega m \; \checkmark$$

Hall constant : $R \triangleq \frac{R_H}{B} = \frac{1}{ne}$ cc. sign sensitive ! (e^-: q^+, or $n <$ for e^-, $n >$ for h^+) ($n = -10^{23} \frac{1}{cm^3}$)

large for SC / metype eg /

Q. Hall-effect (quantum) : QHE

Zeeman Energy

$\mu_B B \gg \frac{\hbar}{2\tau}$

$\mu_B B$; $\frac{\hbar}{\tau}$: broadening parameter

Cyclotron effect

$v = \omega r$

$m\omega^2 r = qvB$

$m\omega^2 r = q\omega r B$

$$\boxed{\omega_c = \frac{qB}{m}}$$ cyclotron frequency

$\hbar\omega_c$ = level spacing $\underbrace{\frac{q\hbar}{2m}}_{\mu_B = \frac{e\hbar}{2m}} B$

QHE appears when:

$\hbar\omega_c > \frac{\hbar}{\tau}$ or $\omega_c \tau > 1$

$\Rightarrow \omega_c > \frac{1}{\tau}$

physical meaning:

e^- can close a cyclotron orbit before scattering

More precisely:

$$\begin{pmatrix} j_x \\ j_y \end{pmatrix} = \underbrace{\begin{pmatrix} \sigma_{xx} & -\sigma_{xy} \\ \sigma_{xy} & \sigma_{xx} \end{pmatrix}}_{\text{isotropic material}} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

no simple inversion: $\rho \neq \frac{1}{\sigma}$

E_y: builds up (Hall effect)

E_x: applied

$$\rho_{xx} = \frac{\sigma_{xx}}{\sigma_{xy}^2 + \sigma_{xx}^2} = \frac{m^*}{ne^2\tau} = \frac{1}{\sigma_{DC}}$$

without derivation

$\rho_{xx} \propto \frac{1}{\sigma_{xx}}$ if σ_{xx} is small

$$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_{xy}^2 + \sigma_{xx}^2} = \frac{B}{|e|n} \Leftrightarrow \text{Hall resistivity}$$

In practice: $n = \frac{1}{|e|\left(\frac{\partial \rho_{xy}}{\partial B}\right)\Big|_{B=0}}$; $\mu = \frac{\left(\frac{\partial \rho_{xy}}{\partial B}\right)\Big|_{B=0}}{\rho_{xx}}$

sign — carrier mobility

Magnetothermal effects

Boltzmann eq.

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \dot{x}\nabla_x f + \dot{r}\nabla_r f = -\frac{g}{\tau}$$

$f(x, r, t)$ Experiment shows $\nabla T \rightarrow j$ (Seebeck effect)

$j \rightarrow jQ$ (Peltier effect)

heat current

$\mu(r)$; $T(r)$; $E(r)$ ← spatial variations

neglect diffusion: $\dot{r}\nabla_r f = 0$

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \dot{x}\nabla_x f + \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial T}\cdot\underbrace{\frac{\partial T}{\partial r}}_{\nabla T}\dot{r} + \frac{\partial f}{\partial x}\underbrace{\frac{\partial x}{\partial \mu}}_{\nabla\mu}\frac{\partial \mu}{\partial r}\underbrace{\frac{\partial r}{\partial t}}_{\dot{r}} + \frac{\partial f}{\partial x}\frac{\partial x}{\partial \epsilon}\nabla E\cdot\dot{r}$$

$$f^0 = \frac{1}{e^{\beta(\epsilon-\mu)}+1} = \frac{1}{e^x+1}$$

$$\frac{\partial f}{\partial x}\frac{\partial x}{\partial T}\nabla T\,\dot{r} + \frac{\partial f}{\partial x}\frac{\partial x}{\partial(\epsilon-\mu)}\nabla(\epsilon-\mu)\,\dot{r} \quad ; \quad \frac{\partial x}{\partial T} = \frac{-x}{T}$$

Collect all terms:

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial \varepsilon} \cdot \underline{v}(\underline{k}) \left[\underline{F}_{ext} - \underline{\nabla}\mu + \frac{\varepsilon(\underline{k}) - \mu}{T} \left(-\underline{\nabla} T\right) \right]$$

(don't have to remember this ↑)

Force term: $\underline{\nabla}_{\underline{k}} f \cdot \dot{\underline{k}}$

$$\dot{\underline{k}} = \frac{\underline{F}_{ext}}{\hbar} \qquad \underline{\nabla}_{\underline{k}} f = \frac{\partial f}{\partial \varepsilon} \frac{\partial \varepsilon}{\partial \underline{k}} = \frac{\partial f}{\partial \varepsilon} \hbar \underline{v}(\underline{k})$$

$$\varepsilon = \varepsilon(\underline{k}), \quad \underline{v}(\underline{k}) = \frac{1}{\hbar} \frac{\partial \varepsilon}{\partial \underline{k}}$$

reminder: we had: $\frac{df}{dt} = \frac{\partial f}{\partial \varepsilon} \underline{v}(\underline{k}) \cdot (-e\underline{E}) = -\frac{f}{\tau}$

Solution: $f = f^0 + e E \tau \underline{v}(\underline{k}) \cdot \frac{\partial f}{\partial \varepsilon}$ stationary solution

why don't we use this above?

$-\underline{\nabla}\mu$ and $-\underline{\nabla} T$ act as forces

Particle current

$$\underline{j}_n = \int \frac{d^3k}{4\pi^3} \underline{v}(\underline{k}) f(\underline{k}) \qquad \underline{j} = -e \int \frac{d^3k}{4\pi^3} \underline{v}(\underline{k}) f(\underline{k}) \leftarrow \text{electric current}$$

~~heat~~ current (?)

First law of thermodynamics: $dU = \delta Q + \delta W \rightarrow dU = T dS + \mu dn$

(δQ: heat transfer)

$T dS = dU - \mu dn$: $dU \leftrightarrow$ energy current

$$\underline{j}_E = \int \frac{d^3k}{4\pi^3} \underline{v}(\underline{k}) f(\underline{k}) \cdot \varepsilon(\underline{k})$$

~~thermal~~ current: $T dS \rightarrow \underline{j}_Q = \int \frac{d^3k}{4\pi^3} \underline{v}(\underline{k}) f(\underline{k}) (\varepsilon - \mu)$

without derivation:

$$\underline{j} = K_0 \left(\underline{E} + \frac{\underline{\nabla}\mu}{e} \right) - K_1 \left(- \frac{\underline{\nabla} T}{T} \right)$$

$$\underline{j}_Q = -K_1 \left(\underline{E} + \frac{\underline{\nabla}\mu}{e} \right) + K_2 \left(- \frac{\underline{\nabla} T}{T} \right)$$

Ks are integrals of $\underline{v}(\underline{k}) f(\underline{k})$

two K_1's are the same cross-coefficients

Onsager-relation (III law of thermodynamics / principle of detailed balance)

examples:

(1) Heat conductivity: $\underline{j}_Q = -K \underline{\nabla} T$ Fourier-law

$\underline{j} = 0$

$$K_0 \left(\underline{E} + \frac{\underline{\nabla}\mu}{e} \right) = K_1 \left(- \frac{\underline{\nabla} T}{T} \right)$$

substitute into second

$$K = \frac{K_2}{T} - \frac{K_1^2}{K_0 T}$$

(2) Seebeck effect $\underline{\nabla} T \neq 0$, $\underline{E} = 0 \Rightarrow \underline{j} \neq 0$ or open circuit $\underline{j} = 0$, $\underline{E} \neq 0$, $\underline{\nabla} T \neq 0$

definition of Seebeck-const: $\frac{\underline{E} + \underline{\nabla}\mu}{e} \triangleq S \underline{\nabla} T$ (S: Seebeck-coeff.)

gives: $S = -\frac{1}{T} \cdot \frac{K_1}{K_0}$

(3) Peltier-effect : $j \neq 0 \rightarrow j_Q \neq 0$ ($\nabla T = 0$)

definition of Peltier-coef: $j_Q \triangleq = \Pi j$

$$\Pi = -\frac{K_1}{K_0}$$

relation between Π and S → $\Pi = ST$ Kelvin-relation
special case of Onsager

(4) Wiedemann-Franz law:

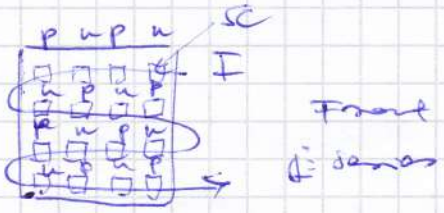
$$\frac{K}{\sigma} = LT \; ; \; L = \frac{\pi^2}{3}\left(\frac{k_B}{e}\right)^2$$

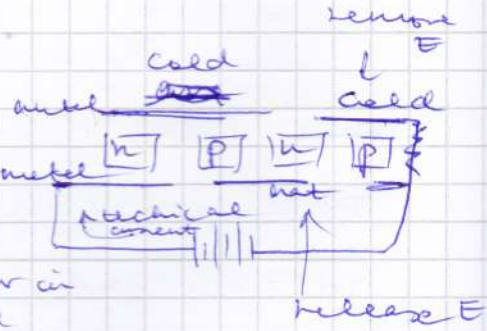
↑ for free e⁻
Lorenz number

what is good electric conductor usually good heat conductor
↑ (counter example: diamond)
not lattice induced

$$\frac{K}{\sigma} = \frac{K_2}{K_0 T} \; ; \text{ gives } L = \frac{K_2}{K_0 T^2}$$

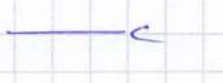
Thermoelectric-cooler (Peltier)

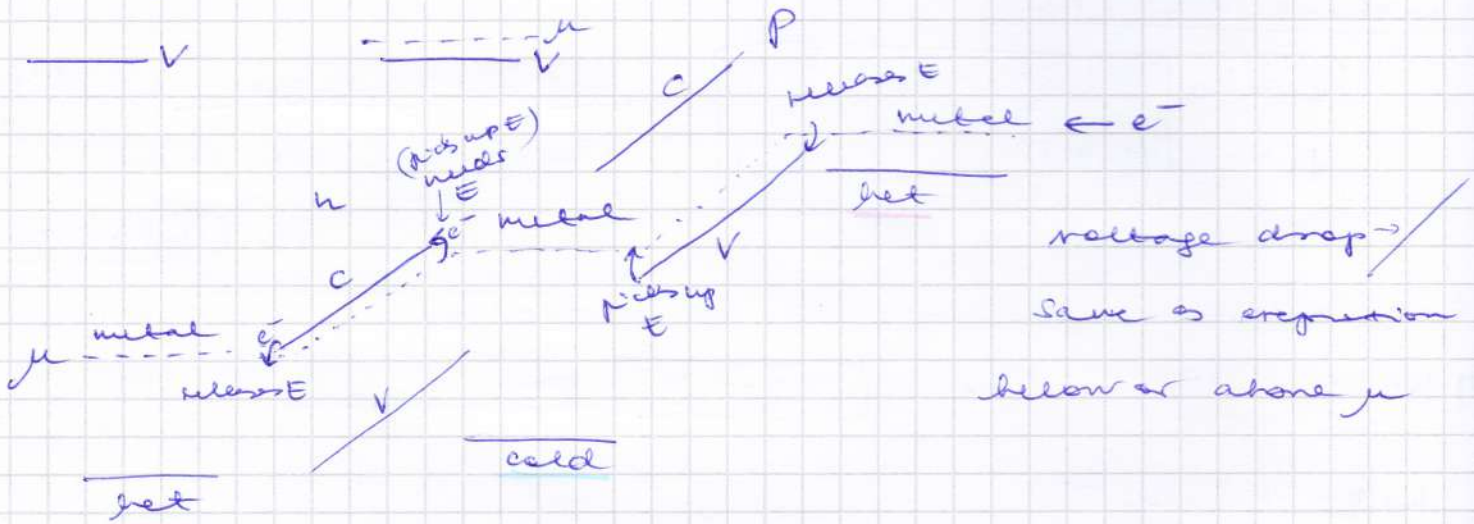




n-type

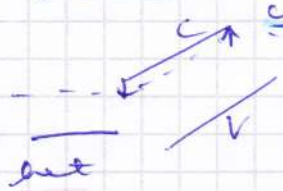
p-type





voltage drop →
same as evaporation
below or above μ

Single n is a Peltier-cooler, but not practical



↓ this wouldn't work

each electrode is cold and hot at the same time

Diffusion effects in SCs

non-uniform charge distribution → strip two SCs n+p
→ doping homogeneously

Intrinsic SCs

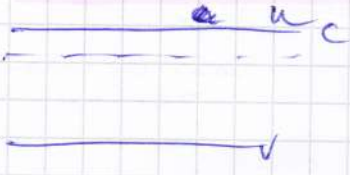
$p = n = n_i(T) = 2\left(\frac{k_B T}{2\pi\hbar^2}\right)^{3/2} (m_n^* m_p^*)^{3/4} e^{-\frac{E_g}{2k_B T}}$ ← don't have to remember this (?)

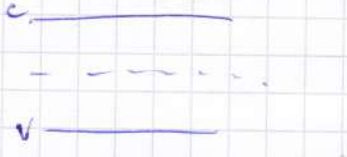
depends on T

Law of mass action: (like water pH: dissociates)
(ett par dissociate, recombine)

$n_p = n_i^2(T)$

Extrinsic SCs





doped: majority c.c.
minority

n-type $n \gg p$ n: maj. p: min

p-type $p \gg n$ p: maj. n: min

important: $n_p = n_i^2(T)$ is still valid

e.g. why is $n_p = n_i^2(T)$ retained

n doping

int	ext
μ_1	μ_2

n increases by a $e^{-\beta(\mu_1-\mu_2)}$

p decrease by $e^{-\beta(\mu_2-\mu_1)}$

product stays the same

$n \sim e^{-\beta(E_c-\mu_1)}$ $n \sim e^{-\beta(E_c-\mu_2)}$

$p \sim e^{-\beta(E_v-\mu_1)}$ $p \sim e^{-\beta(E_v-\mu_2)}$

a non-equilibrium change concentration

excitation (by light, injection, thermal excitation)

equilibrium concentration n_0, p_0

non-equilibrium n, p

Net recombination rate $U = R - G_{thermal} = c(np - n_0 p_0)$

recombination rate $R = p n$

thermal excitation $G n_0 p_0 = R^0$

$G n_0 p_0 = G_{thermal}$

(c: constant)

$$U = c\left[\underbrace{(n-n_0)(p-p_0)}_{\text{small, second order neglect}} + n_0(p-p_0) + p_0(n-n_0)\right]$$

$$U \cong c\left[n_0(p-p_0) + p_0(n-n_0)\right]$$

n-type $n_0 \gg p_0 \rightarrow U \approx c n_0 (p-p_0)$

p-type $p_0 \gg n_0 \rightarrow U \approx c p_0 (n-n_0)$

} recombination is given by minority c.c. concentration

E.g. $\dot{n} = -c(n\ell - n_0)$; $\boxed{\dot{n} = -\frac{n-n_0}{\tau_n}}$ $\boxed{\dot{p} = -\frac{p-p_0}{\tau_p}}$

τ_n, τ_p : cc lifetime
↓ how long it can survive
cc recombination time

Revision :

n-type	$n \gg p$	$n \approx n_d$ (donors)
p-type	$p \gg n$	$p \approx n_a$ (acceptors)

in equilibrium $\boxed{n_0 p_0 = n_i^2(T)}$ always ! / Law of mass action /

recombination is given by the minority cc.

n-type : $\frac{\partial n}{\partial t} = -\frac{p-p_0}{\tau_p}$ $\frac{\partial p}{\partial t} = -\frac{p-p_0}{\tau_p}$ if $p_0 \ll n_0$ / $p_0 \ll p_0$

if $p > p_0$ → annihilation

τ_n, τ_p : cc. lifetime $\tau_n, \tau_p \gg \tau$ up to 1 ms

Continuity equation :

in SC : change in balance due to injection

$$\boxed{\frac{\partial \rho}{\partial t} + \nabla \vec{j} = \sigma}$$

ρ: current, σ: source of the liquid

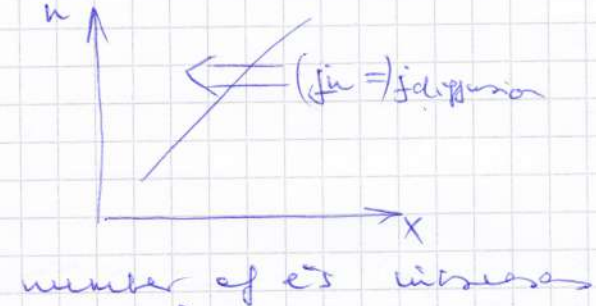
For our case :

e charge

n-type
$$\begin{cases} \frac{\partial n}{\partial t} - \frac{1}{e}\nabla \vec{j}_n = -\frac{p-p_0}{\tau_p} \\ \frac{\partial p}{\partial t} + \frac{1}{e}\nabla \vec{j}_p = -\frac{p-p_0}{\tau_p} \end{cases}$$

p-type
$$\begin{cases} \frac{\partial n}{\partial t} - \frac{1}{e}\nabla \vec{j}_n = -\frac{n-n_0}{\tau_n} \\ \frac{\partial p}{\partial t} + \frac{1}{e}\nabla \vec{j}_p = -\frac{n-n_0}{\tau_n} \end{cases}$$

Case of spatial inhomogenity (stationary)



number of e's increases → diffusion

diffusion current (particle current)

$\boxed{j_{diff} = -D\nabla n}$ Fick - law

D : diffusion constant

$\boxed{D \approx v^2 \tau}$ in the mean-free path approximation

τ: momentum scattering time

v usually v_F (Fermi velocity)

In the presence of an external field ⇒ drift current

$$\underline{j} = n e \underline{v}_{drift} = n e \mu \underline{E} = -ne\mu \underline{\nabla} V \qquad \underline{E} = -\underline{\nabla} V$$

(mobility; potential)

particle current → $\underline{j}_{diff} + \underline{j}_{drift} = 0$ ← electric current

Stationary case:

$$(*) \quad +eD\underline{\nabla} n - ne\mu \underline{\nabla} V = 0$$

n (density of e^-)

$$f(E,T) = \frac{1}{2\frac{E-\mu}{2k_BT} + 1}$$

$$n \approx f(\mathcal{E} - eV, T) = \frac{1}{e^{\frac{\mathcal{E}-eV-\mu}{k_BT}} + 1}$$

($\mathcal{E}$: energy of electron at a given potential)

few meVs

if $2k_BT \ll \mathcal{E} - eV - \mu$

$$\approx e^{-\frac{(\mathcal{E}-eV-\mu)}{2k_BT}}$$

with this: $\underline{\nabla} n = \underline{\nabla} f = n \frac{e}{2k_BT} \cdot \underline{\nabla} V$ (important)

if $V = V(x)$

Write it back to (*):

$$\left(\frac{e^2 D n}{2k_BT} \frac{1}{n} - ne\mu \right) \underline{\nabla} V = 0 \Rightarrow$$

Einstein-Relation: $\mu = \frac{e}{2k_BT} D$ (mobility)

special case of fluctuation dissipation theory

Charge inhomogenity + stationary condition

example: charge injection

n-type Si, inject holes in one line in 1D

$$\begin{cases} \frac{1}{e}\frac{\partial j_p}{\partial x} = -\frac{p - p_0}{\tau_p} & \text{(1) eq. of continuity} \\ j_p = -eD\frac{\partial p}{\partial x} & \text{in 1D} \end{cases}$$

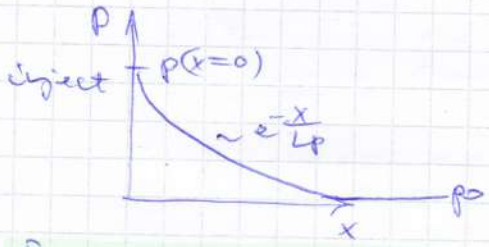
$$\frac{\partial^2 p}{\partial x^2} = \frac{p - p_0}{\tau_p} \cdot \frac{1}{D}$$ diff. eq.

Solution: $p(x) = p_0 + [p(x=0) - p_0] e^{-\frac{x}{L_p}}$

$\frac{\partial^2 f}{\partial x^2} = a \rightarrow e^{-\frac{x \cdot \sqrt{a}}{1}}$

where $L_p = \sqrt{D\tau_p}$: charge diffusion length (length just with diffusion)

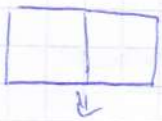
since $D = v_F^2 \tau$: $L_p = v_F \sqrt{\tau \tau_p}$ → can be up to 1 cm



Semiconductor devices (basics)

p-type n-type

p-n junction



diffusion

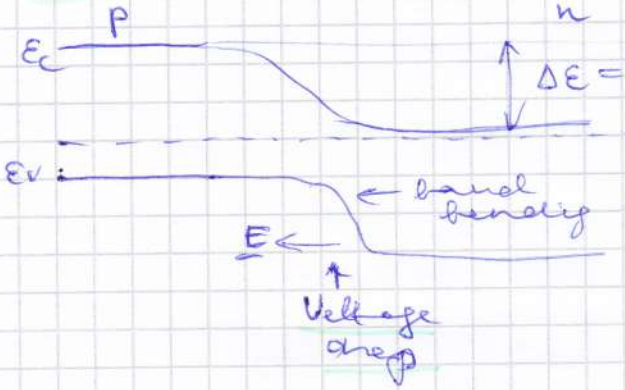
When does the diffusion stop ?
(V stops it)

depletion region ; no free cc.s
also called : diffusion or contact region

built in electric field

⇒ electric shielding

band structure after contact



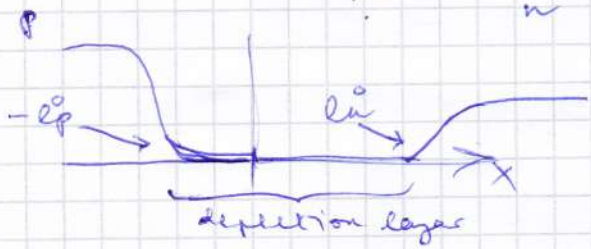
driving force of diffusion is

$j_{drift} + j_{diff} = 0$

$$n e \mu E + eD \nabla n = 0$$

built-in e-field

Schottky-approximation



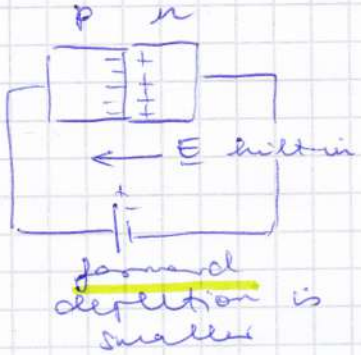
charge neutrality: $p \cdot l_p^0 = n \cdot l_n^0$

$\rightleftharpoons = 0$ (?)

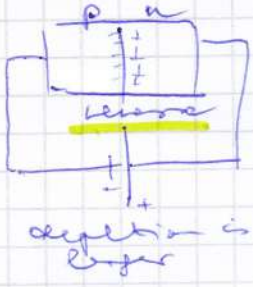
$$n_A l_p^0 = n_D l_n^0$$

heavily doped → small depl. layer
lightly doped → wide depl. layer

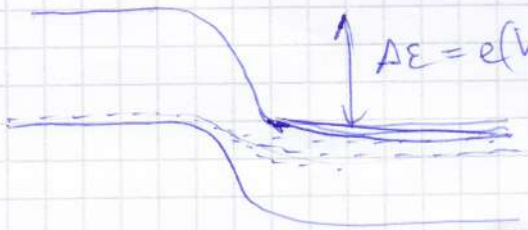
p-n junction + bias



if $E_{ext} \parallel E_{built-in}$ ⇒ reversed
if $E_{ext} \perp\!\!\!\perp E_{built-in}$ (?) ⇒ forward



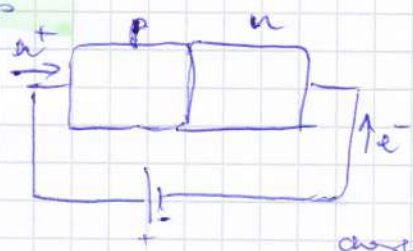
(pn open, np closed)



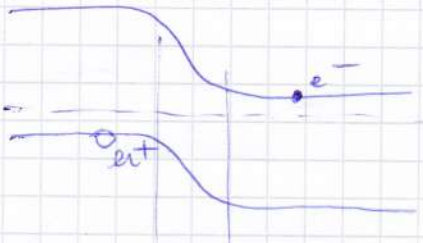
Schockley-formula: $I(V) = I_c \left(e^{\frac{eV}{\lambda k T}} - 1 \right)$

↑ reverse current

Diode characteristics



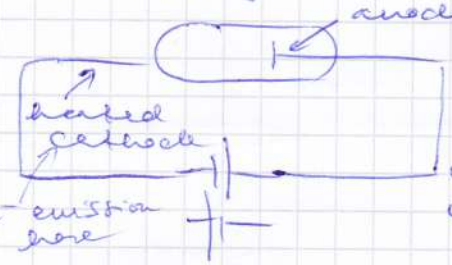
charge carrier: happy when majority
sad when minority
↳ recombination



LED: light is emitted

depl. layer → charge annihilation is happening

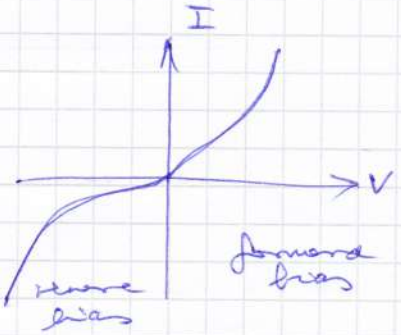
old vacuumtubes
→ rectifying device



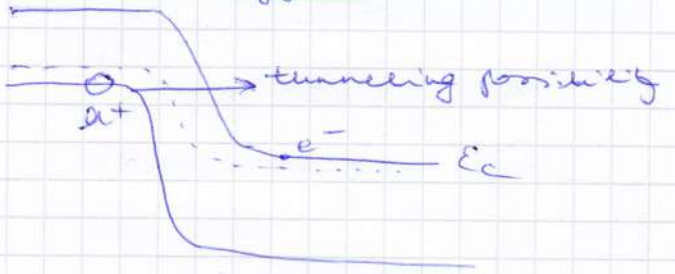
origin of name: diode

avalanche breakdown:

- reverse bias
- minority ccs are accelerated
  ⇓
  excite additional minority ccs
  ⇓
  ...
- don't like it: destroys the device
- occurs in weakly doped SCs.



Zener-effect

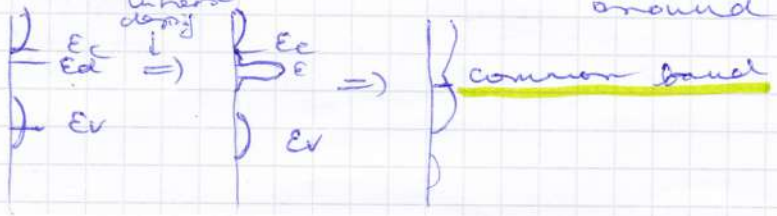


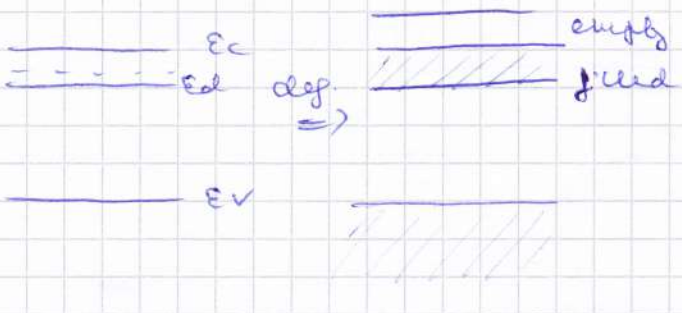
(cannot go there otherwise)

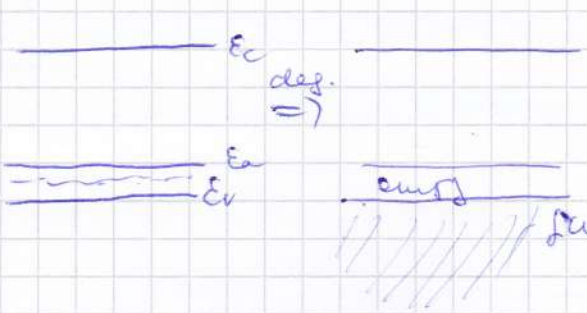
- thin depletion layer because doped
- reversible: doesn't destroy the device
- similar physics as tunnel-diode

Tunnel diode (Esaki)

degenerate doping: it occurs around 1% doping



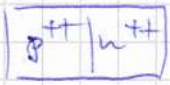




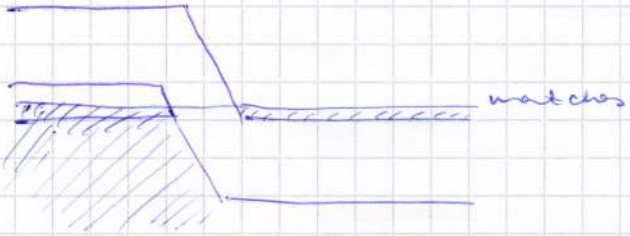
n-type

p-type

Esaki diode : heavily doped pieces



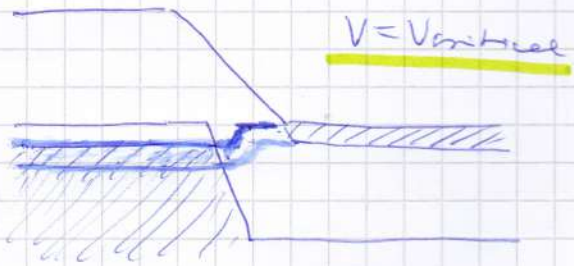
I



V=0
(0 bias)
tunnel current

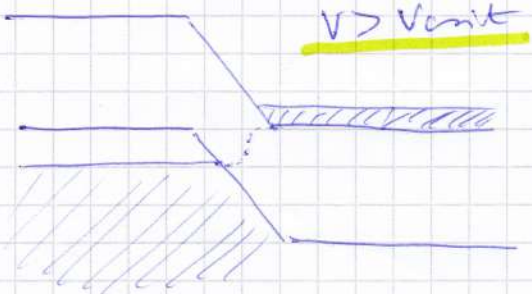
$V > 0$

II.



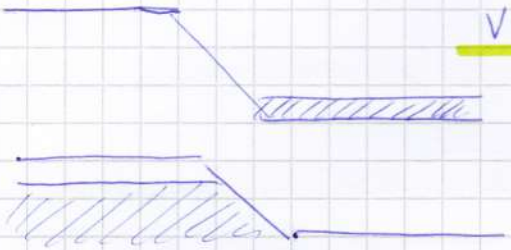
(forward bias)
tunnel current increases

III.

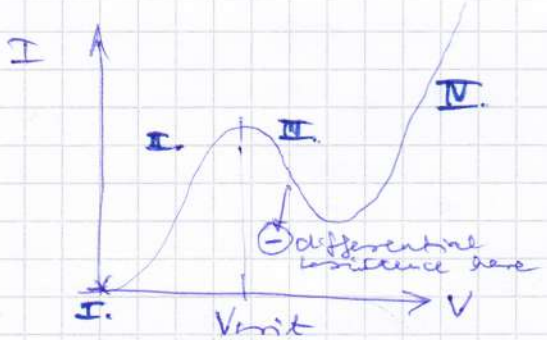


tunnel current drops
$V > V_{max}$

IV.



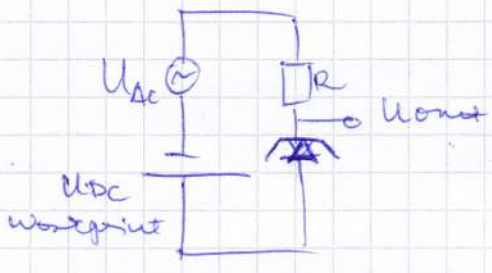
again current ~~drops~~ flows just like in a normal diode



circuit notation :

diode

Esaki diode



$U_{out} = U_{AC} \cdot \frac{-r}{R-r} = U_{AC} \frac{r}{r-R} > U_{AC}$ if $R < r$

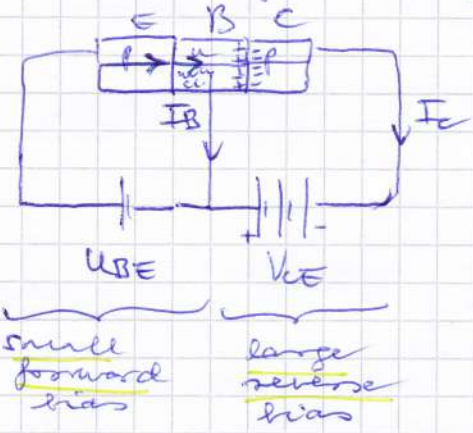
$-r = \left(\frac{\partial I}{\partial V}\right)^{-1}\Big|_{\text{Esaki at working point}}$

analogue amplifier

SC devices

dec 18th 9:00 - 12:00 unofficial exam date (mine)

BJT: bipolar junction transistor



Emitter, Base, Collector

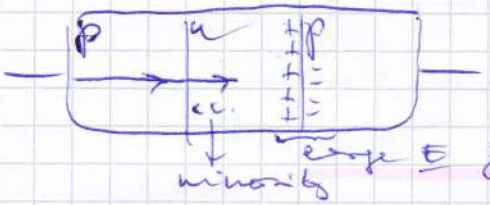
E B C

npn or pnp type

(later for mobility)

BJT ≠ np + pn diode

base is thin: $l_B \ll L_{diff.}$ smaller than the diffusion length



BJT: minority carrier device

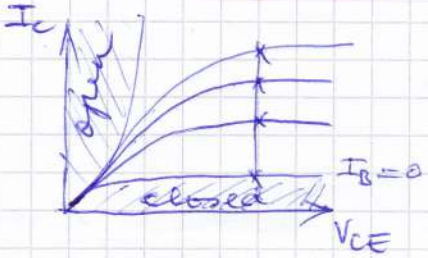
I_C depends on the injected charges (V_{BE}) and is proportional to I_B (not explaining it)

→ I_B controls I_C; construction is such that

$$\frac{I_C}{I_B} = \beta = 10 \dots 100$$

current amplification factor

$I_C = I_E - I_B$ – recombination



Working ways:

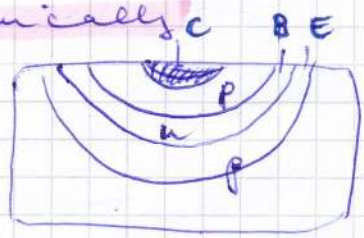
(1) digital device (TTL, TTL) → disappeared (old computers)
ON-OFF: open-closed

control it with I_B

(2) analogue amplification

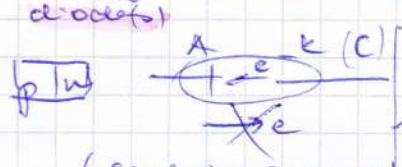
$I_C = \beta I_B$ (this is still used)

Technically

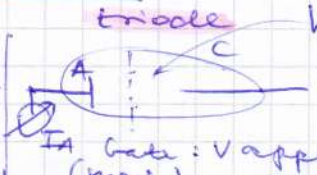


Analogy: vacuum tubes

diode



triode

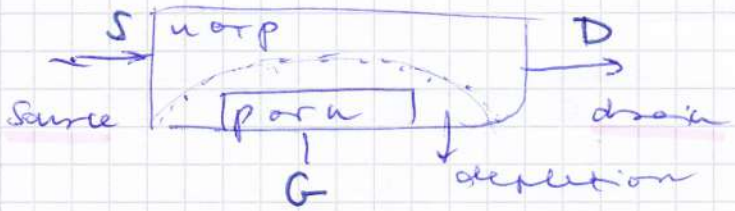


Grid: V applied (mesh) → stop or let e^- go)

I_A controlled by V_{CC}

($I_B = 0$ → voltage controlled)

JFET: junction field effect transistor



gate-source is forward
→ large I_{SD}

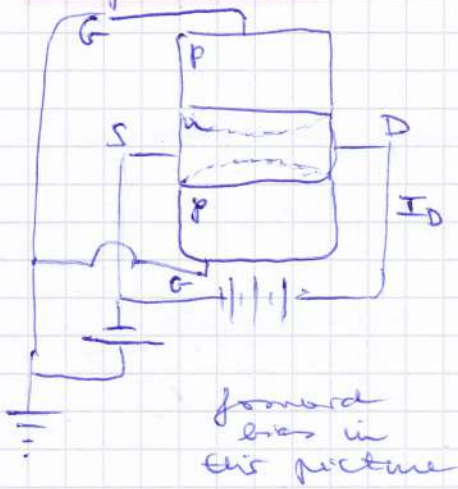
gate-source reversed
→ small I_{SD}
(large depletion)

Majority CC. device
↓ in high speed device

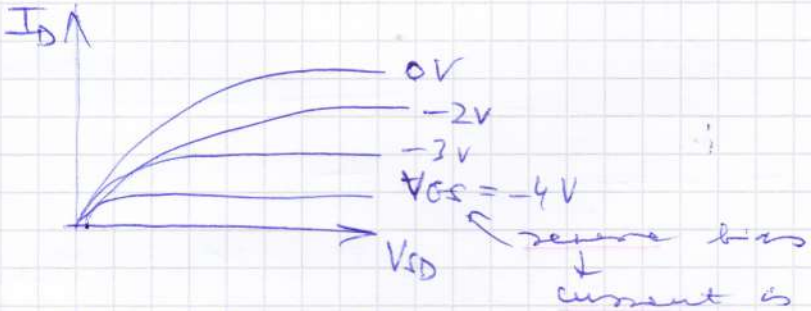
I_{SD} is controlled by V_{GS} and $I_G = 0$

(eg. oscilloscope input high imp)

in practice:



forward bias in this picture

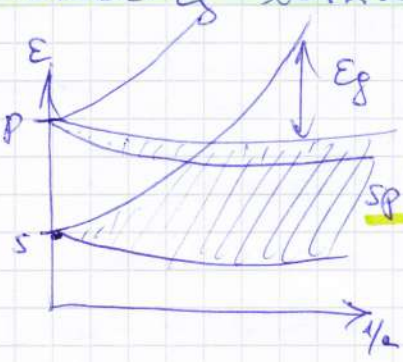


SD: drives V
GS: controls

JFET vs. BJT

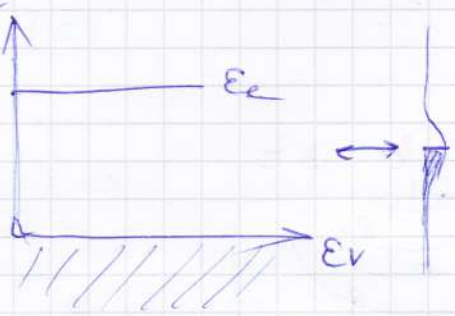
JFET	BJT
large control impedance ↓ (1–10 MΩ) ($I_G = 0$)	finite control impedance (I_B finite)
better for digital and low freq. analogue amplification	better for high freq. amplification

Schottky barrier and surface states



bulk material

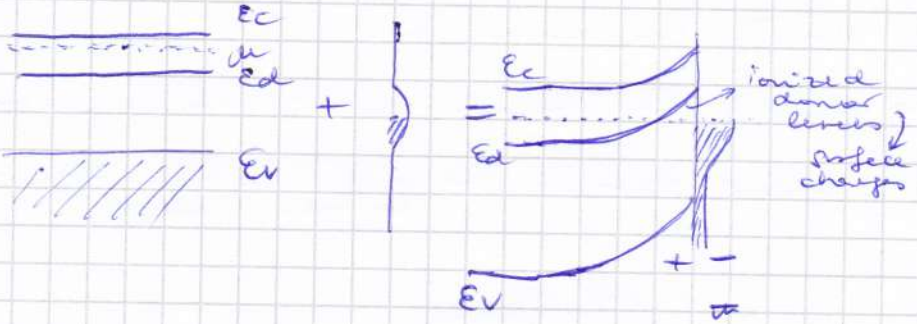
⇒ sp^3 hybrid states are in the middle of the gap (dangling bonds).

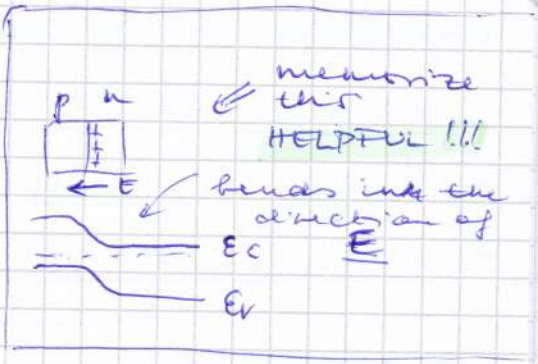


bulk — surface

Schottky requires surface states, understanding it

Surface of an n-type SC





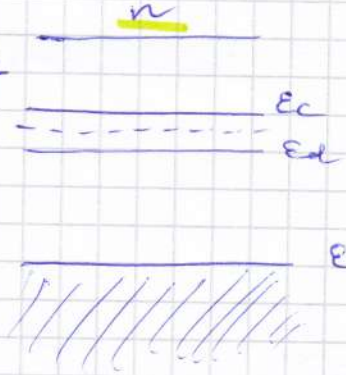
Schottky barrier : metal + SC | M | n/p |

1 type of c.c. only

M :

E=0 vacuum

+



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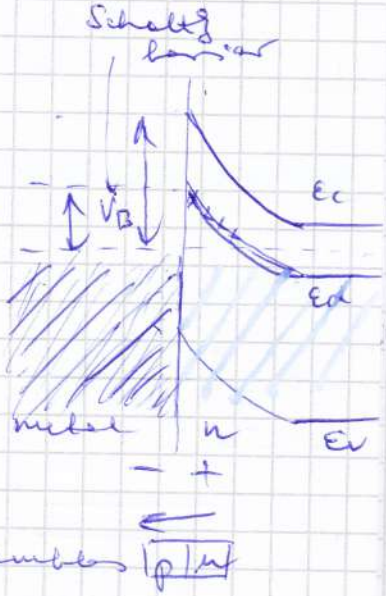
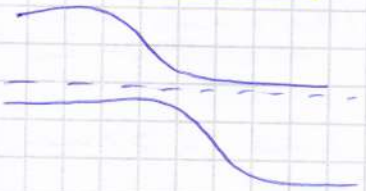
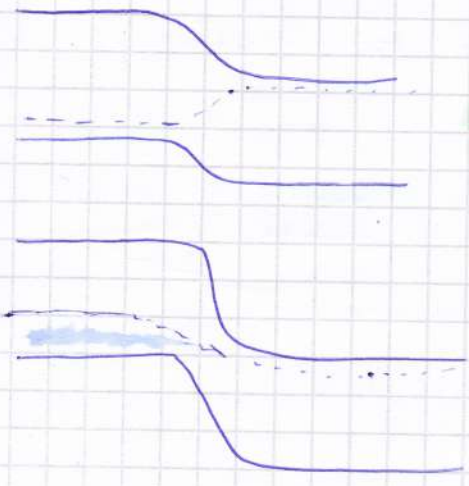


remember | p | M |

reminder :



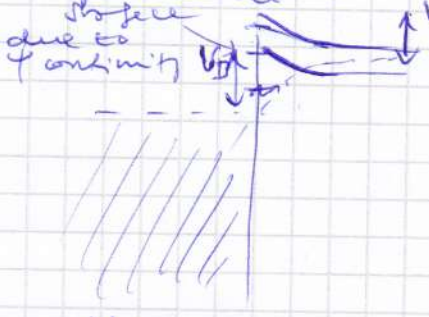
V ≠ 0



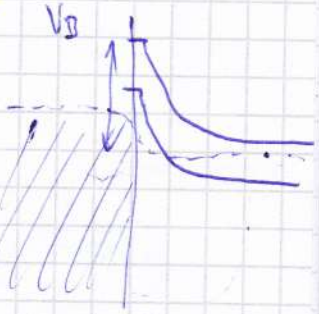
V > 0
forward bias
(up)

V < 0
reversed bias
(down)

Schottky diode under bias



VB reduced
forward

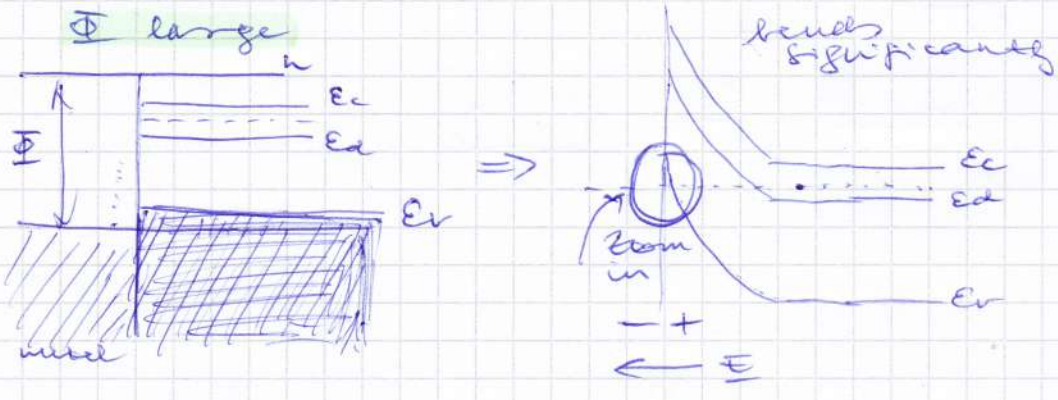


VB is larger
reverse

Advantages of Schottky devices

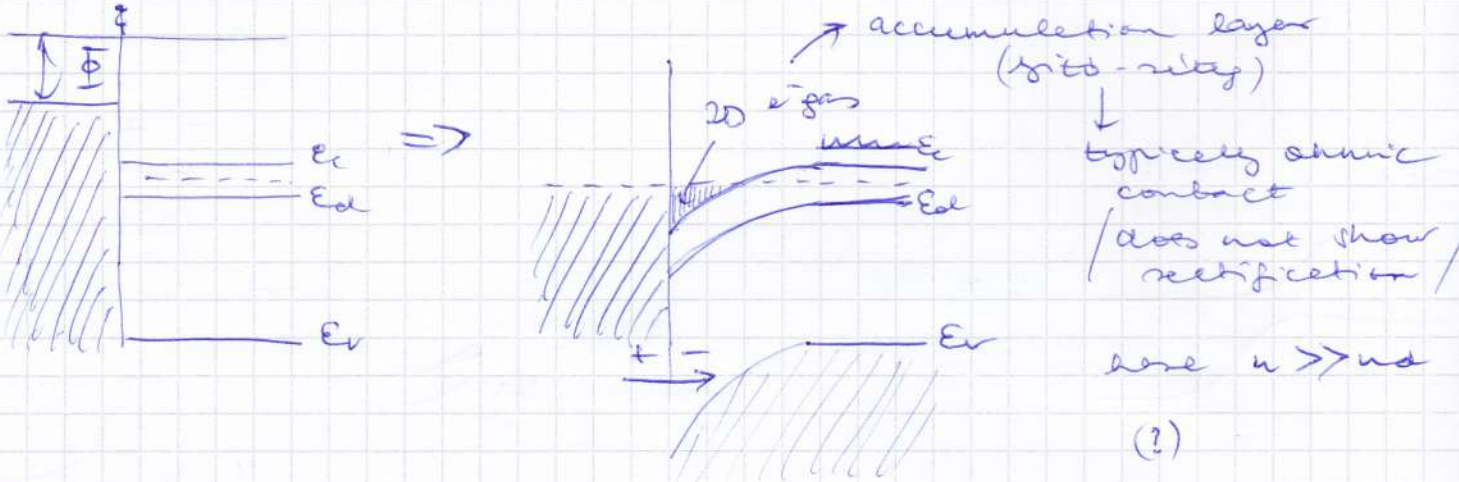
- tuneable barrier due to Φ (different metal)
- V_{th} can be small → device has small power consumption
- no minority cc.s → faster devices (no recombination effect) ↳ less time

Φ large



Inversion layer: 2D hole gas; p-type cc.s in an n-type

Φ small



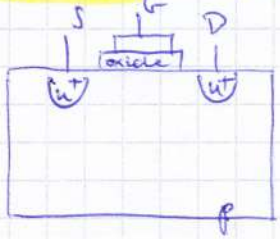
↓
typically ohmic contact
/does not show rectification/

here $n \gg n_d$

(?)

MOSFET: metal-oxide-SC field effect transistor

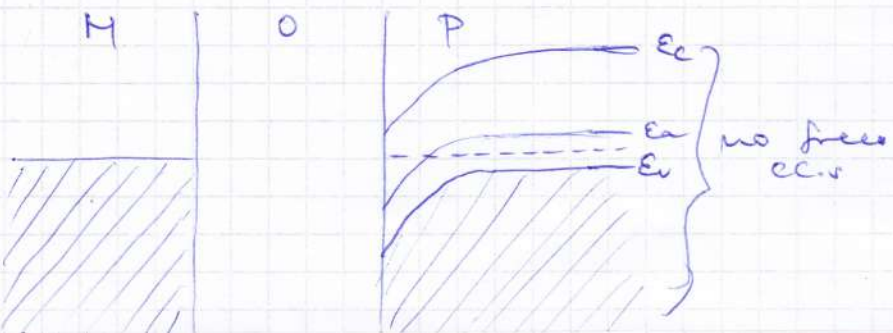
n^+: strong



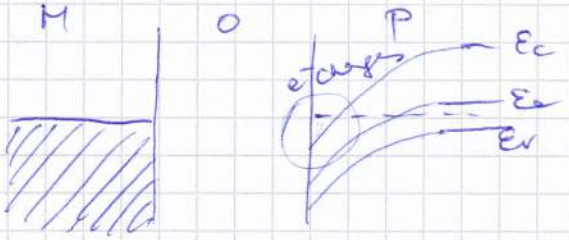
$V_G > 0$ → n channel forms
$(E > 0)$ S-D current is finite (I_{SD})

$V_G < 0$ → p channel forms
$(E < 0)$ I_{SD} is zero

band picture

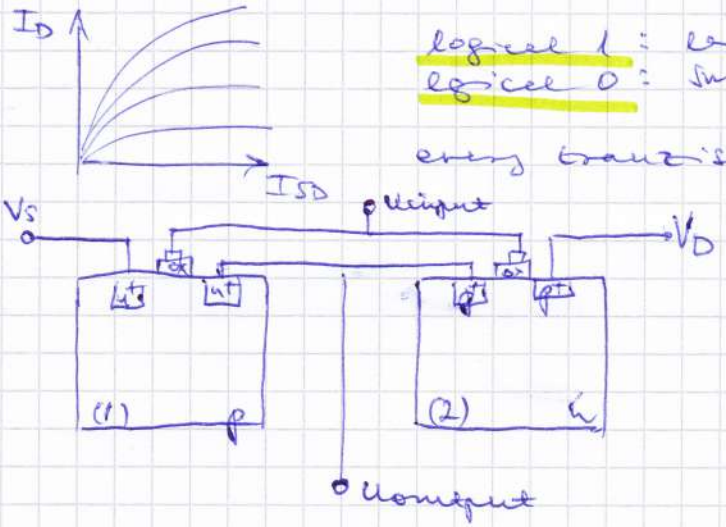


if $V_{GS} > 0$



like the ~~accumulation~~ inversion layer
(e^-s in a p-type)

CMOS: Complementary metal-oxide-Sc ~~FET~~



logical 1: large I_D (usually 0 is large)
logical 0: small I_D

every transistor has 1 mA

if $U_{input} > 0$

(1) open (2) closed (n-channel) → $U_{output} = V_S$

if $U_{input} < 0$

(1) closed (2) open (p-channel) → $U_{output} = V_D$

always $I_{SD} = 0$ except for switching
↳ this gives consumption

if V_S = low U_{input}
if V_D = high U_{input}

} logical NOT gate
you can build up everything from NOT and OR, XOR, ~~AND~~
→ to be double operation
(make three in pins in tech.)

Bring lecture notes!
→ later more if will :)