Inducing a magnetic monopole with topological surface states

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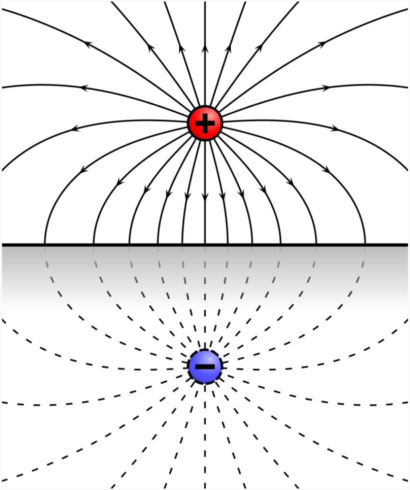
Main idea

The method of images



(source: https://www.imp.kiev.ua/kord/wiki/methodofimages.html)

- magnetic monopole

- topological insulator

- topological magneto-electric effect

- quantum gas carrying fractional statistics

Topological insulators

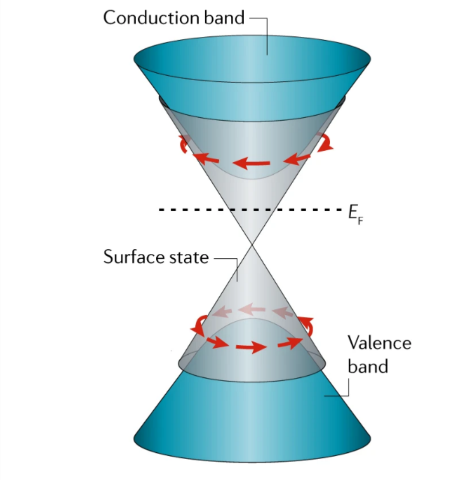
- interior - insulator $\longleftrightarrow$ surface - conducting states

- symmetry-protected surface states

- bulk energy gap

- gapless excitations

Topological insulators



(source: https://www.nature.com/articles/s42254-018-0011-5

Theoretical background

- physical origin of the topological magneto-electric effect (TME):

Hall-current on the surface $\longleftrightarrow$ magnetic polarization

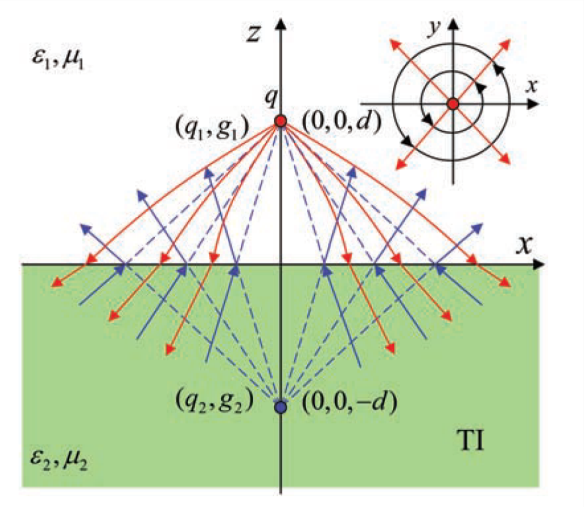
- quantized Hall-conductance:

$$\sigma_{xy} = \left(n + \frac{1}{2}\right)\frac{e^2}{h}$$

- interpretation:

a magnetic monopole charge is induced as a mirror charge of an electric charge

Theoretical background



(source: X-L. Qi, R. LI, J. Zang, S-C. Zhang, Science, 323-5918, 2009)

Theoretical background

- complete boundary-value problem
- method of images
- the constituent equations:

$$\mathbf{D} = \mathbf{E} + 4\pi\mathbf{P} - 2\alpha P_3 \mathbf{B}$$
$$\mathbf{H} = \mathbf{B} - 4\pi\mathbf{M} + 2\alpha P_3 \mathbf{E}$$

- magneto-electric polarization: $P_3 = \pm 1/2$
- fine structure constant: $\alpha = \frac{e^2}{\hbar c}$

Theoretical background

- at $z = 0$, the standard boundary condition gives:

$$q_1 = q_2 \qquad g_1 = -g_2$$

$\rightarrow$ for an electric charge both an image magnetic monopole and image magnetic charge will be induced

- the physical origin of the image magnetic charge:

$$\nabla \times \mathsf{B} = 2\alpha P_3 \delta(z) \mathsf{n} \times \mathsf{E}$$

- surface current density $\rightarrow$ quantized Hall-current

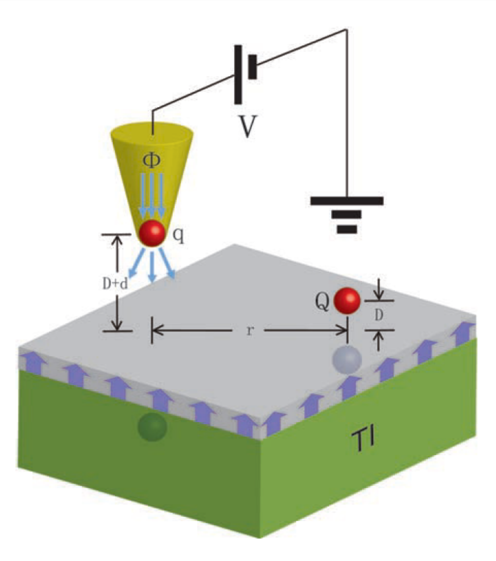
Experimental approach

- fractionalization or de-confinement phenomena in condensed matter physics

- magnetic force microscope (MFM)

- topological insulator surface with a localized charged impurity

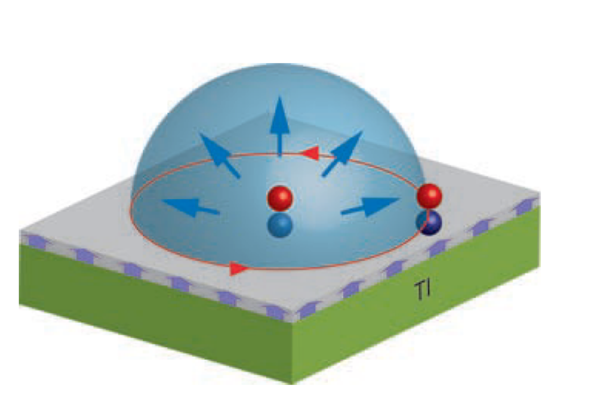
Experimental approach



(source: X-L. Qi, R. LI, J. Zang, S-C. Zhang, Science, 323-5918, 2009)

Experimental approach

- the bound state of an electron and its image monopole: dyon



(source: X-L. Qi, R. LI, J. Zang, S-C. Zhang, Science, 323-5918, 2009)

Experimental approach

- with q dyon charge and g monopole flux, the statistical angle:

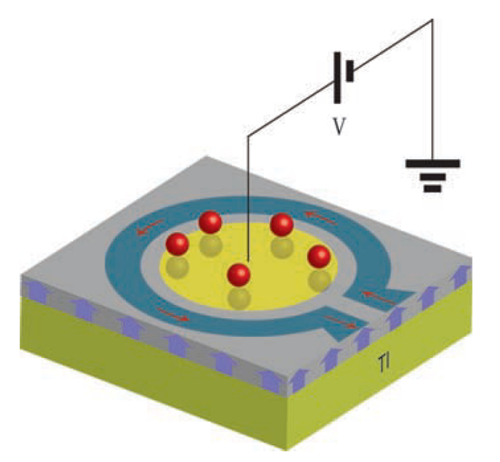
$$\varphi = \frac{gq}{2\hbar c}$$

- for $P_3 = 1/2$ and $\varepsilon_1, \mu_1, \mu_2 \sim 1$, $\varepsilon_2 \sim 100$:

$$\boxed{\varphi \approx 2.6 \cdot 10^{-7}\ rad}$$

- small, but physically observable

Experimental approach



(source: X-L. Qi, R. LI, J. Zang, S-C. Zhang, Science, 323-5918, 2009)

Experimental approach

- flux-change $\rightarrow$ supercurrent
- with typical electron-density and island size:
  - net magnetic flux:

$$\boxed{n\varphi\pi R^2 \frac{\hbar c}{e} \approx 2.6 \cdot 10^{-4}\ \frac{hc}{2e}}$$

  - magnetic field:

$$\boxed{B = n\varphi \frac{\hbar c}{e} \approx 1.7 \cdot 10^{-3}\ G}$$

Conclusion

- the surface of a topological insulator images an electron as a magnetic monopole

- direct manifestation of the TME effect

- electron gas becomes a dyon gas with fraction statistics

- experimental approach to observe the field and the statistical angle

Thank you!