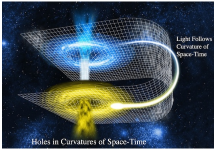
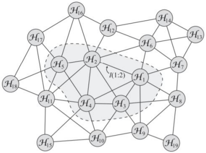
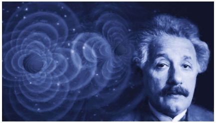




Space from Hilbert-space

Budai Ákos



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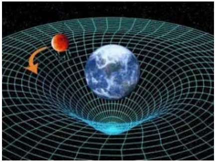


MŰEGYETEM 1782

Quantum Theory of Gravity

- respects the principles of GR and QM
- same predictions as QFT at low energies
- reproduces GR at cosmological scales
- difficulty: quantum gravity only appears at very high energies

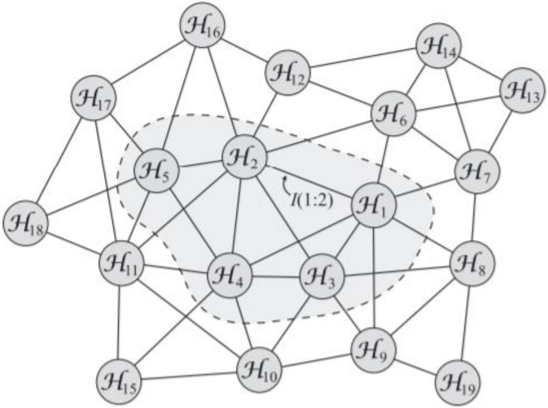
Attempts so far

- "like an attempt to breathe in empty space" (Einstein)
- e.g. Loop Quantum Gravity, string theory
- procedure: start with a classical theory and impose the laws of QM!

Starting from Quantum

Idea: the world is quantum mechanical from the get go

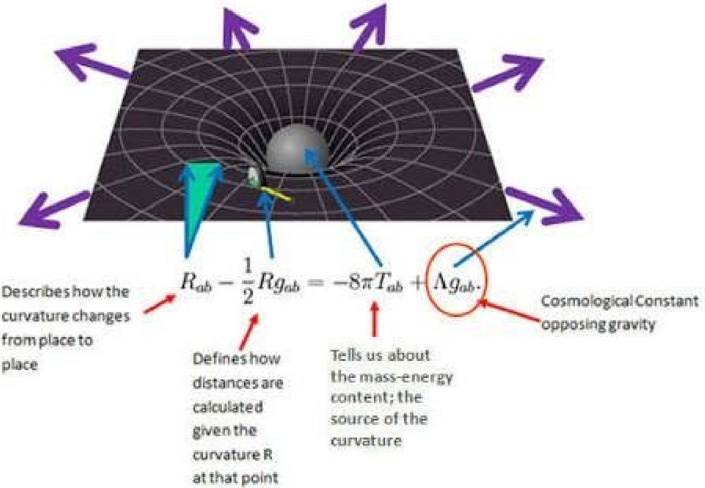
- no need to start from a classical theory
- instead:
  - take a quantum system (Hamiltonian + wave function)
  - find the emergent classical world inside



Emergent spacetime

- existence of particles not assumed
- existence of spacetime itself not assumed!

Aim: find the structure of spacetime encoded in the wave function



Emergent spacetime

1. divide Hilbert-space into "pieces" ($\sim$ particles)

$$\mathcal{H} = \bigotimes_i \mathcal{H}_i$$

$$\downarrow$$

$$|\Psi\rangle = \sum \bigotimes_i |\Psi_i\rangle = \sum |\Psi_1\rangle |\Psi_2\rangle |\Psi_3\rangle |\Psi_4\rangle \dots$$

Emergent spacetime

2. entropy ($\sim$ information content of the state):

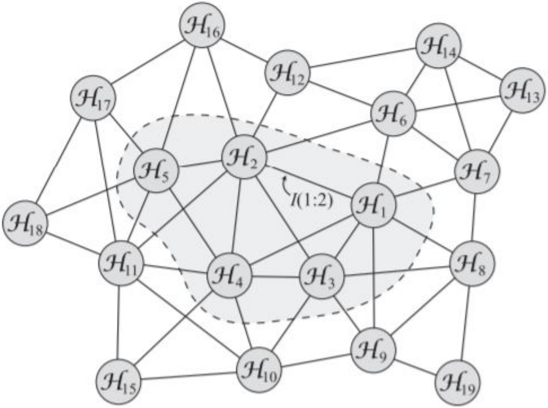
$$S(A) = -\,\mathrm{Tr}\{\rho_A \log \rho_A\}$$

3. mutual information between "pieces":

$$I(A:B) = S(A) + S(B) - S(AB)$$

Emergent spacetime

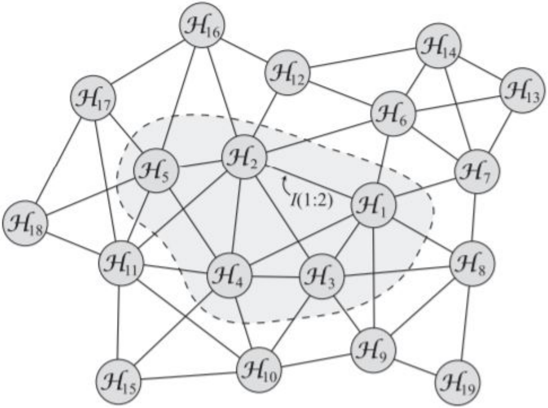
4. Define a graph
   - nodes: pieces
   - edge weights: mutual information



Emergent spacetime

5. Redundancy-constrained states:

$$S(R) \propto \sum_{a \in R, b \in \bar{R}} I(a:b)$$



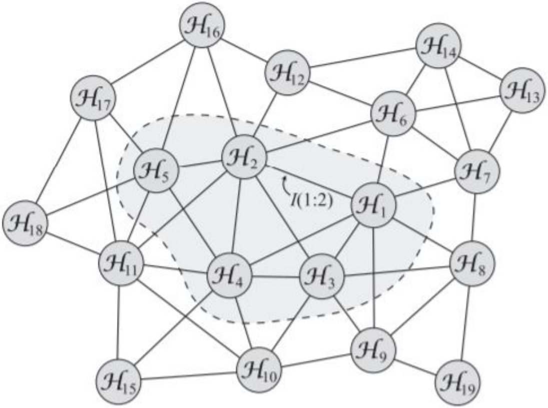
Emergent spacetime

Idea: pieces closer together should have more mutual information

6. Distance from mutual information:

$$d_{AB} = \ell_0 \cdot \Phi(\frac{I(A:B)}{I_0})$$

Spatial metric!



Emergent spacetime

Recover the dimension:

1. consider an r-ball centered around some point p
2. calculate its total entropy
3. get the dimension from the scaling:

$$S(r, p) \sim r^{D_f}$$

4. $D = D_f + 1$

Recovering a smooth geometry is also possible:

- Multidimensional scaling
- Regge calculus

Examples

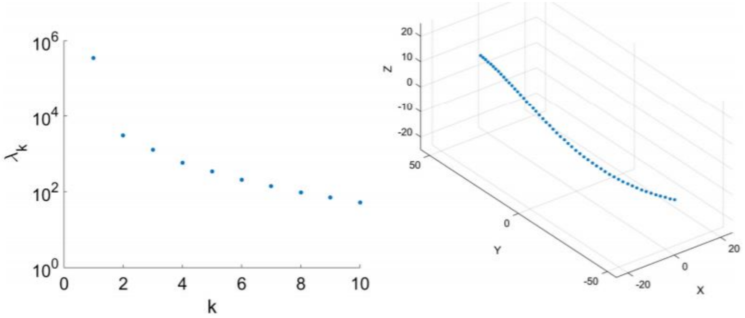


Figure: Ground state of 1D antiferromagnetic Heisenberg spin chain

Examples

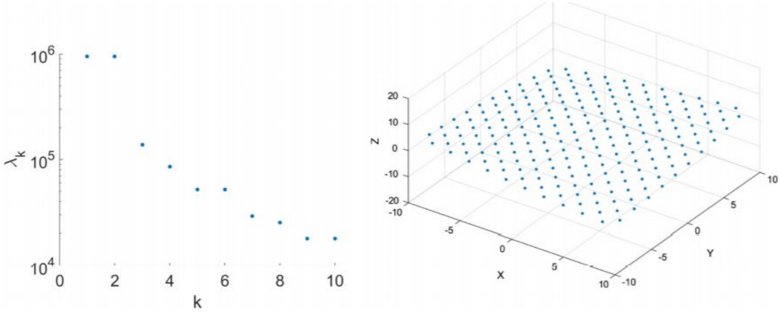
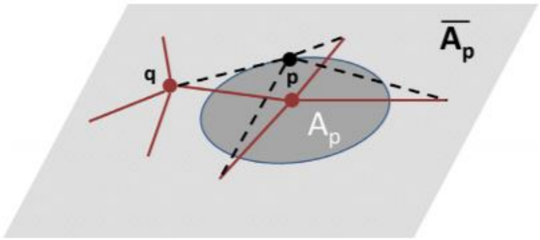


Figure: Ground state of 2D toric code

Curvature

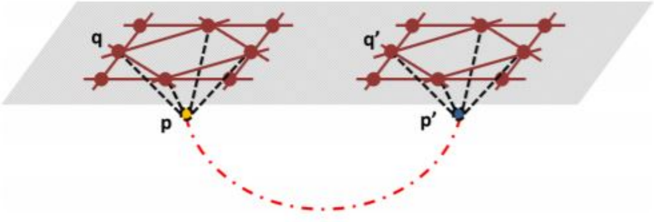
Local entropy perturbation:
curvature at the point



$$\mathcal{R}_p \propto -\delta S_p$$

Curvature

Nonlocal entropy perturbation:
wormhole-like structure

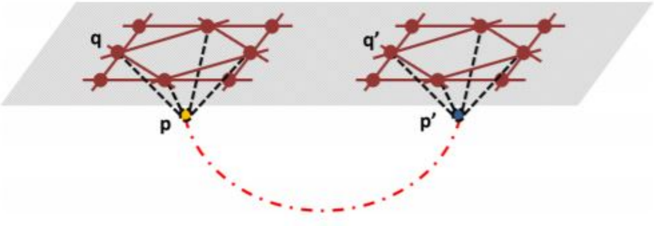


$$\mathcal{R}_p \propto -F'_\lambda(p,p')\delta S_p$$

always positive!

Curvature

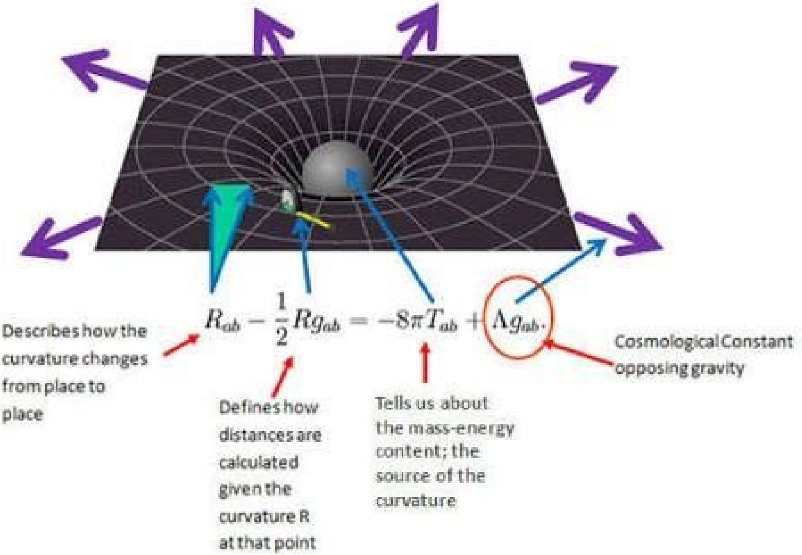
Quantum proto-wormhole



increasing $\delta S_p \sim$ formation of a classical wormhole

$$ER = EPR$$

Einstein's equation



$$G_{\mu\nu} = C \cdot T_{\mu\nu}$$

Einstein's equation

- modular Hamiltonian: $K = -\log \rho$
- Entanglement first law:

$$\delta S = \delta \langle K \rangle$$

- modular energy density: $\epsilon_p = -\delta \langle K \rangle$
- CFT: connection to stress-energy tensor:

$$\epsilon_p^{CFT} = C \cdot \delta \langle T_{00}^{CFT} \rangle$$

Einstein's equation

- assume framework adapted for time evolution
- If no external curvature:

$$\mathcal{R}_p = 2G_{00}(p)$$

$$\downarrow$$

$$G_{00} = C \cdot \delta\langle T_{00}^{CFT} \rangle$$

- Apply same reasoning for all spatial slices:

$$G_{\mu\nu} = C \cdot \delta\langle T_{\mu\nu}^{CFT} \rangle$$

Einstein's equation holds for local perturbations of a smooth manifold

Future research directions

- Time evolution
- Emergent Lorentz invariance

References

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