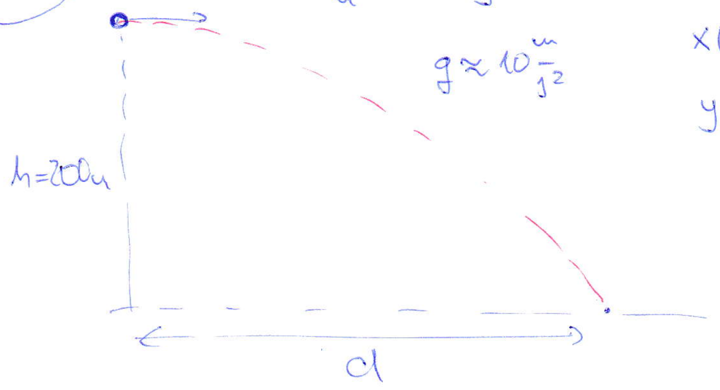
4. feladatok megoldása

F1 $v_0 = 180 \frac{km}{h} = 50 \frac{m}{s}$



$x(t) = v_0 \cdot t$

$y(t) = h - \frac{g}{2} \cdot t^2$

$t_{be}: \quad y = 0$

$\Downarrow$

$h - \frac{g}{2} t_{be}^2 = 0$

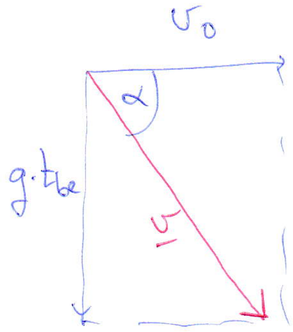
$t_{be} = \sqrt{\frac{2h}{g}} = 6{,}38\ s$

dobás távolsága x irányban:

$d = x(t_{be}) = v_0 \cdot t_{be} = v_0 \cdot \sqrt{\frac{2h}{g}} \approx 320\,m$

Sebességvektor: $v_x = v_0$

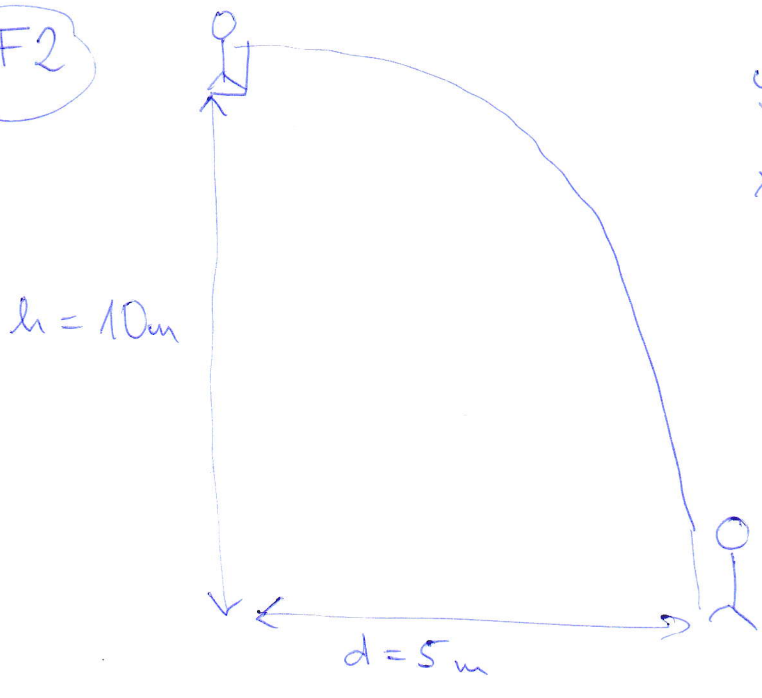
$v_y = -g \cdot t_{be}$



$tg\,\alpha = \frac{g \cdot t_{be}}{v_0} = 1.26$

$\alpha = 0.90\,(rad) \approx 52^\circ$

F2



$y = h - \frac{g}{2} \cdot t^2 \Rightarrow t_{be} = \sqrt{\frac{2h}{g}} = 1.4\ s$

$x = v_0 \cdot t$

$v_0 \cdot t_{be} = d \Rightarrow v_0 = \frac{d}{t_{be}} = d\sqrt{\frac{g}{2h}}$

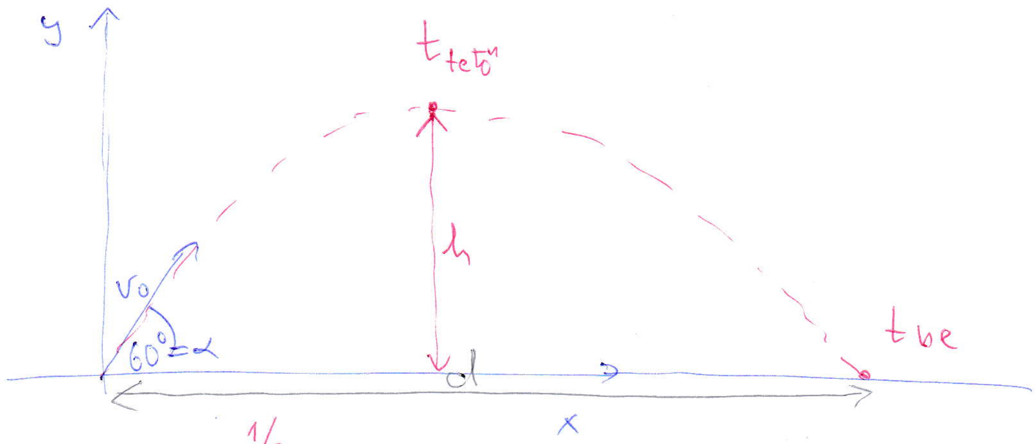
$v_0 = 3.5\ \frac{m}{s}$

$v_x = v_0 = 3.54\ \frac{m}{s}$

$v_y = -g \cdot t_{be} = 14{,}1\ \frac{m}{s}$

$|\underline{v}| = \sqrt{v_x^2 + v_y^2} \approx 15\ \frac{m}{s}$

F3



$$v_x = v_0 \cdot \overbrace{\cos\alpha}^{1/2} = \frac{v_0}{2} = 12{,}5\ \frac{m}{s}$$

$$v_y = \underbrace{v_0 \cdot \sin\alpha}_{v_0^y = 21{,}6\ \frac{m}{s}} - g \cdot t$$

a.) tetőponton: $v_y = 0$

$$\Rightarrow \quad v_0 \cdot \sin\alpha - g \cdot t_{tető} = 0$$

$$\boxed{t_{tető} = \frac{v_0 \cdot \sin\alpha}{g}} = 2.165\ s \approx 2{,}2\ s$$

$$\Rightarrow \quad x(t) = v_0 \cdot \cos\alpha \cdot t$$

$$y(t) = v_0 \sin\alpha \cdot t - \frac{g}{2} \cdot t^2$$

b.) $h = y(t_{tető}) = v_0 \cdot \sin\alpha \cdot t_{tető} - \frac{g}{2} \cdot t_{tető}^2 = 23\ m$

c.) t_{be}: $y = 0$ $\quad v_0 \cdot \sin\alpha \cdot t_{be} - \frac{g}{2} \cdot t_{be}^2 = 0$

$\Rightarrow$ $t_{be} = 0$ (kilövés)

$$t_{be} = \frac{2 \cdot v_0 \sin\alpha}{g} \approx 4{,}3\ s$$

észrevétel: $\boxed{t_{be} = 2 \cdot t_{tető}}$

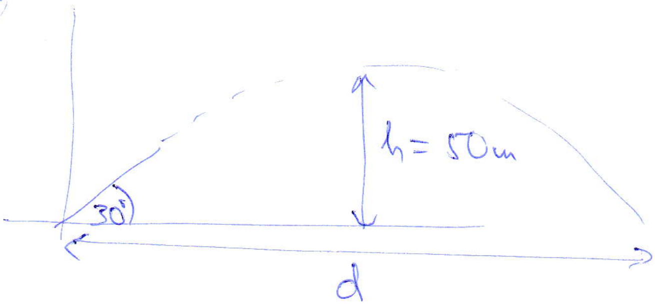
(szimmetria)

$\Rightarrow$ $d = x(t_{be}) = 54\ m$

d.) ötlet: $x = v_0 \cdot \cos\alpha\, t \Rightarrow t = \frac{x}{v_0 \cdot \cos\alpha}$

$$\Rightarrow \boxed{y = v_0 \sin\alpha \frac{x}{v_0 \cos\alpha} - \frac{g}{2} \frac{x^2}{v_0^2 \cos^2\alpha} = x \cdot tg\alpha - \frac{g}{2 v_0^2 \cos^2\alpha} \cdot x^2}$$

F4.



$$t_{tető} = \frac{v_0^y}{g} = \frac{v_0 \cdot \sin\alpha}{g}$$

$$h = v_0 \cdot \sin\alpha \cdot t_{tető} - \frac{g}{2} \cdot t_{tető}^2 = \frac{v_0^2 \sin^2\alpha}{g} - \frac{g}{2}\,\frac{v_0^2 \sin^2\alpha}{g^2} = \boxed{\frac{v_0^2 \sin^2\alpha}{2g}}$$

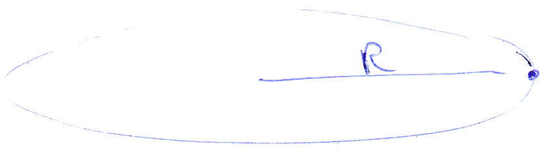
$$\Downarrow$$

$$v_0 = \sqrt{\frac{2gh}{\sin^2\alpha}} \approx \boxed{63\,\frac{m}{s}}$$

$$d = v_0 \cdot \cos\alpha \cdot \underbrace{2t_{tető}}_{t_{be}} = 2 v_0 \cos\alpha \cdot \frac{v_0 \sin\alpha}{g} = \frac{2v_0^2 \sin\alpha \cdot \cos\alpha}{g} = 346{,}5\,m$$

$$\approx 350\,m$$

F5.



$R = 10\,km$

$v = 810\,km/h = 225\,\frac{m}{s}$

a.) $\omega = \frac{v}{R} = \frac{225}{10\,000}\,\frac{1}{s} = \boxed{0.0225\,\frac{1}{s}} \approx 0{,}02\,\frac{1}{s}$

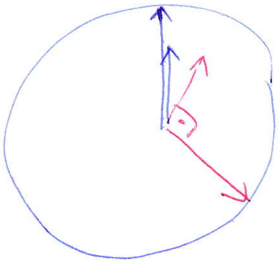
b.) $t_{félkör}$: $\omega \cdot t_{félkör} = \pi$

$t_{félkör} = \frac{\pi}{\omega} = 140\,s$

c.) a : egyenletes körmozgás

$\Rightarrow a_{cp} = \frac{v^2}{R} = R \cdot \omega^2 \approx 5{,}1\,\frac{m}{s^2}$

F6



$$\omega_k = \frac{2\pi}{12\,h} = 0.524\,\frac{1}{h}$$

$$\omega_n = \frac{2\pi}{1\,h} = 6.28\,\frac{1}{h}$$

$\Downarrow$

$$\varphi_k = \omega_k \cdot t$$

$$\varphi_n = \omega_n \cdot t$$

$$(\varphi_n - \varphi_k) = (\omega_n - \omega_k)\cdot t =$$

$$= 2\pi \cdot \left(1 - \frac{1}{12}\right)\frac{1}{h} \cdot t$$

$$= \frac{11}{12}\cdot 2\pi\,\frac{1}{h}\cdot t$$

kérdés: $\varphi_n - \varphi_k = \frac{\pi}{2}$

$$\frac{\cancel{\pi}}{2} = \frac{11}{6}\cancel{\pi}\cdot t \qquad \boxed{t = \frac{3}{11}\,h}$$

F7

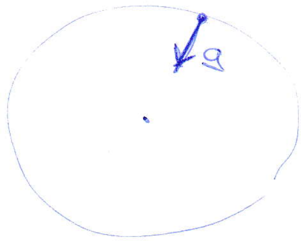
$$\boxed{v = \frac{5\,m}{2\,s} = 2.5\,\frac{m}{s}}$$

$R = ?$ 5 m: félkör

$$R\cdot\pi = 5\,m$$

$\hookrightarrow R = 1.59\,m \approx 1{,}6\,m$

$$\omega = \frac{v}{R} = 1.57\,1/s \approx 1{,}6\,\frac{1}{s}$$

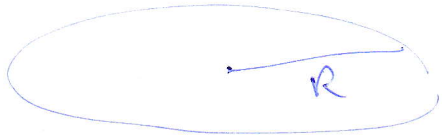


b.) egyenletes körmozgás

$$a_{cp} = R\cdot\omega^2 = 3.9\,\frac{m}{s^2}$$ sugár irányú

c.) $$t_{100\,kör} = \frac{100\cdot 2\pi}{\omega} = 400\,s$$ (4 s: 1 kör)

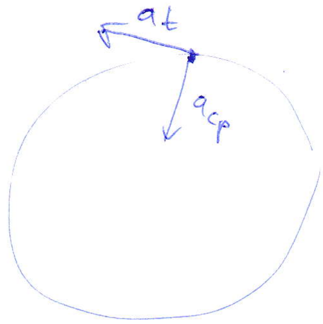
F8.



$R = 50\,cm = 0.5\,m$

$a_t = 0.2\,\frac{m}{s^2}$

$\llcorner$ tangenciális (érintő irányú)



$v = a_t \cdot t$

$a_{cp} = \frac{v^2}{R} = \frac{a_t^2 \cdot t^2}{R}$

? $a_{cp} = a_t$ mikor?

$a_t = \frac{a_t^2 \cdot t^2}{R} \rightsquigarrow \boxed{t = \sqrt{\frac{R}{a_t}} = \underline{\underline{1{,}6\,s}}}$

$s = \frac{a_t}{2} \cdot t^2 = \frac{a_t}{2} \cdot \frac{R}{a_t} = \boxed{\frac{R}{2}}$

F9.

$1400\,RPM \Rightarrow \omega = \frac{1400 \cdot 2\pi}{60\,s} \approx \underline{\underline{150\,\frac{1}{s}}}$

$d = 50\,cm$

$R = 25\,cm = 0.25\,m$

$\hookrightarrow \boxed{a_{cp} = R \cdot \omega^2 \approx 5400\,\frac{m}{s^2}} \sim \boxed{540g}$