F1

$m = 1\,kg$

$A = 0{,}1\,m$

$t = 0{,}5\,s$

Egyik szélsőhelyzetből a másikba ér:

$$T = 2t = 1\,s = \frac{2\pi}{\omega}$$

$$v_{max} = A\cdot\omega = \frac{2\pi A}{T} = 0{,}6\,m/s$$

F2

$m = 50\,kg$

$D = 35\,N/m$

$A = 4{,}0\,cm$

$y = 1{,}0\,cm$

$$v(t) = A\omega\cdot\cos(\omega t)$$

$$y(t) = A\sin(\omega t)$$

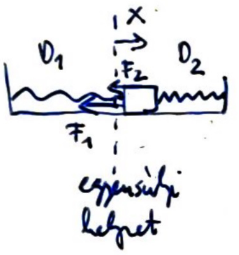
$$\omega = \sqrt{\frac{D}{m}} = 0{,}84\,\frac{1}{s}$$

$$\sin(\omega t) = \frac{y}{A} = 0{,}25$$

$$\cos(\omega t) = \sqrt{1-\sin^2(\omega t)} = \frac{\sqrt{15}}{4}$$

$$v(t) = A\cdot\omega\cdot\frac{\sqrt{15}}{4} = 3{,}3\,cm/s$$

F3

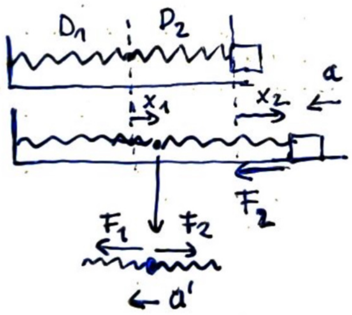


$$F_1 + F_2 = D_1 x + D_2 x = m\cdot a$$

$$a = \underbrace{\frac{D_1+D_2}{m}}_{\omega^2}\cdot x$$

a és x ellentétes

$$T = 2\pi\sqrt{\frac{m}{D_1+D_2}}$$



$$F_2 = D_2(x_2 - x_1) = m\cdot a$$

$$F_1 - F_2 = 0\cdot a' \Rightarrow F_1 = F_2 = D_1 x_1 = D_2(x_2 - x_1) \rightarrow x_1 = \frac{D_2}{D_1+D_2}x_2$$

$$D_2\cdot x_2\left(1 - \frac{D_2}{D_1+D_2}\right) = ma \rightarrow \frac{D_1 D_2}{D_1+D_2}\cdot x_2 = ma \rightarrow \omega^2 = \frac{1}{m}\cdot\frac{D_1 D_2}{D_1+D_2}$$

$$\rightarrow T = 2\pi\sqrt{\frac{m(D_1+D_2)}{D_1 D_2}}$$

F4

$m = 20\,kg$

$f_1 = 2\,Hz$

$f_2 = 1\,Hz$

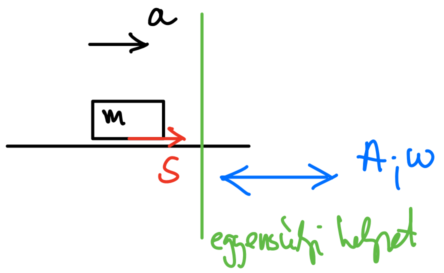
$$\omega_1 = \sqrt{\frac{D}{m}} = 2\pi f_1$$

$$\omega_2 = \sqrt{\frac{D}{m+M}} = 2\pi f_2$$

$$\frac{f_2}{f_1} = \sqrt{\frac{m}{m+M}}$$

$$\left(\frac{f_2}{f_1}\right)^2 = \frac{m}{m+M} \Rightarrow M = m \cdot \frac{1-\left(\frac{f_2}{f_1}\right)^2}{\left(\frac{f_2}{f_1}\right)^2} = 3m = 60\,kg$$

F5



Talaj rendszerében a testet a tapadási súrlódási erő gyorsítja:

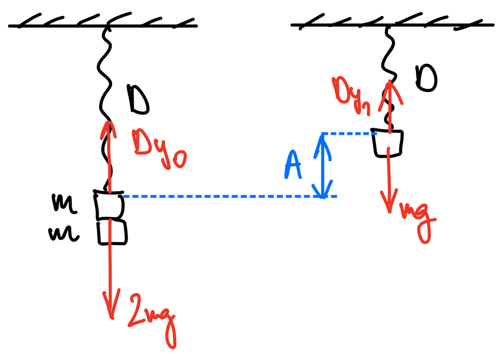
$$S = ma = mA\omega^2 \sin(\omega t) \leq \mu_0 N = \mu_0 mg$$

Amikor a test gyorsulása maximális (szélsőhelyzet), akkor a legnagyobb a tapadási súrlódási erő, tehát nem csúszik meg a test sehol, ha

$$A\omega^2 \leq \mu_0 g \quad \Rightarrow \quad A \leq \frac{\mu_0 g}{\omega^2} = \frac{\mu_0 g}{\left(\frac{2\pi}{T}\right)^2} = \frac{\mu_0 g T^2}{4\pi^2} \approx 0{,}03\,m$$

F6

egyensúlyi helyzetek:



$Dy_0 = 2mg$ $\qquad$ $Dy_1 = mg$

$y_0 = \frac{2mg}{D}$ $\qquad$ $y_1 = \frac{mg}{D}$

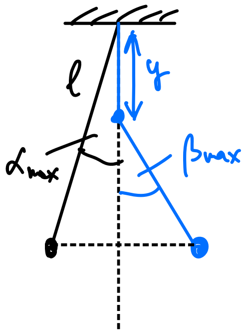
A rezgés amplitúdója:

$$A = y_0 - y_1 = \frac{mg}{D} = \frac{y_0}{2} = 1\,cm$$

A rezgés frekvenciája:

$$\omega = \sqrt{\frac{D}{m}} = \sqrt{\frac{2g}{y_0}} \simeq 30\,\frac{1}{s} \Rightarrow T = \frac{2\pi}{\omega} \approx 0{,}2\,s$$

F7



Az inga kis kitérésű lengéseket végez: $\alpha_{max} \to 0$

$$\ell \cdot \underbrace{\cos\alpha_{max}}_{\approx 1} = y + (\ell - y)\cdot\cos\beta_{max}$$

$\approx 1 \;\to\; \cos\beta_{max} \approx 1 \,:\, \beta_{max} \to 0$

Fél periódus alatt eltelt idők:

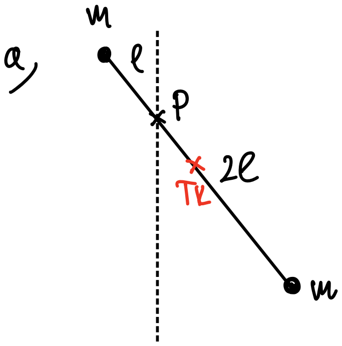
$$t_1 = \frac{T_1}{2} = \frac{1}{2}\cdot 2\pi\sqrt{\frac{\ell}{g}}$$

$$t_2 = \frac{T_2}{2} = \frac{1}{2}\cdot 2\pi\cdot\sqrt{\frac{\ell - y}{g}}$$

$$T = t_1 + t_2 = \pi\cdot\frac{\sqrt{\ell}+\sqrt{\ell-y}}{\sqrt{g}} \approx 1{,}9\,s$$

F8

a)



$$\Theta_p = m\ell^2 + m\cdot(2\ell)^2 = 5m\ell^2$$

TK helye a felfüggesztési ponttól:

$$s = \frac{3\ell}{2} - \ell = \frac{\ell}{2}$$

$$T_a = 2\pi\sqrt{\frac{\Theta_p}{2mgs}} = 2\pi\cdot\sqrt{\frac{5m\ell^2}{2mg\cdot\frac{\ell}{2}}} = 2\pi\cdot\sqrt{\frac{5\ell}{g}} \approx 4{,}4\,s$$

b)

$$\Theta_p = 5m\ell^2 + \frac{1}{12}m\cdot(3\ell)^2 + m\left(\frac{\ell}{2}\right)^2 = 6m\ell^2$$

A rendszer TK-ja ugyanott van, így $s = \frac{\ell}{2}$

$$T_b = 2\pi\cdot\sqrt{\frac{6m\ell^2}{3mg\cdot\frac{\ell}{2}}} = 2\pi\cdot\sqrt{\frac{4\ell}{g}} \approx 4{,}0\,s$$

F9

Csillapított rezgés:

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0 \;\Rightarrow\; x(t) = \underbrace{Ae^{-\beta t}}_{A(t)}\cdot\sin(\omega t + \varphi)\,;\; \omega = \sqrt{\omega_0^2 - \beta^2}$$

$$A(t + 3T) = \frac{A(t)}{10}$$

$$e^{-\beta t}e^{-3\beta T} = \frac{1}{10}\cdot e^{-\beta t} \;\Rightarrow\; e^{-3\beta T} = \frac{1}{10} \;\Rightarrow\; T = -\frac{1}{3\beta}\cdot\ln\frac{1}{10}$$

$$\frac{2\pi}{\omega} = -\frac{1}{3\beta}\cdot\ln\frac{1}{10}$$

$$\frac{2\pi}{\sqrt{\omega_0^2-\beta^2}} = -\frac{1}{3\beta}\cdot \ln\frac{1}{10}$$

$$3\beta\cdot\frac{2\pi}{-\ln\frac{1}{10}} = \sqrt{\omega_0^2-\beta^2}$$

$$9\beta^2\cdot\left(\frac{2\pi}{\ln\frac{1}{10}}\right)^2 = \omega_0^2-\beta^2 \Rightarrow \beta^2 = \frac{\omega_0^2}{1+\frac{9\cdot(2\pi)^2}{\left(\ln\frac{1}{10}\right)^2}} \approx 0{,}03\,\frac{1}{s^2}$$

$$\omega_0^2 = \frac{D}{m} = 2\,\frac{1}{s^2}$$

$$\underline{T} = \frac{2\pi}{\sqrt{\omega_0^2-\beta^2}} = \frac{2\pi}{\omega_0\cdot\sqrt{1-\frac{1}{1+\frac{9(2\pi)^2}{\left(\ln\frac{1}{10}\right)^2}}}} \approx \underline{4{,}5\,s}$$

F10

Gerjesztett rezgés:
$$\ddot{x} + \underbrace{2\beta\dot{x}}_{\approx 0} + \omega_0^2 x = \frac{F_0}{m}\cdot\cos(\omega t)$$

Fázisábra:

$$\left|A\omega_0^2 - A\omega^2\right| = \frac{F_0}{m} \Rightarrow A(\omega) = \frac{F_0/m}{\left|\omega_0^2-\omega^2\right|}$$

$A_1 = 9\,mm$; $f_1 = \frac{\omega_1}{2\pi} = 400\,Hz$ $\left(\omega_1 \approx 2513\,\frac{1}{s}\right)$

$A_2 = 5\,mm$; $f_2 = \frac{\omega_2}{2\pi} = 405\,Hz$ $\left(\omega_2 \approx 2545\,\frac{1}{s}\right)$

$$A_1\cdot\left|\omega_0^2-\omega_1^2\right| = A_2\cdot\left|\omega_0^2-\omega_2^2\right|$$

1, ha $\omega_0 > \omega_1$ és $\omega_0 > \omega_2$:

$$A_1\omega_0^2 - A_1\omega_1^2 = A_2\omega_0^2 - A_2\omega_2^2$$

$$\omega_0^2 = \frac{A_1\omega_1^2 - A_2\omega_2^2}{A_1 - A_2} \Rightarrow f_0 \approx 394\,Hz \text{ ellentmondás}$$

2, ha $\omega_1 < \omega_0 < \omega_2$:

$$A_1\omega_0^2 - A_1\omega_1^2 = A_2\omega_2^2 - A_2\omega_0^2$$

$$\omega_0^2 = \frac{A_1\omega_1^2 + A_2\omega_2^2}{A_1 + A_2} \Rightarrow f_0 \approx 402\,Hz$$

3, ha $\omega_0 < \omega_1$ és $\omega_0 < \omega_2$:

$$A_1\omega_1^2 - A_1\omega_0^2 = A_2\omega_2^2 - A_2\omega_0^2$$

$$\omega_0^2 = \frac{A_1\omega_1^2 - A_2\omega_2^2}{A_1 - A_2} \Rightarrow f_0 \approx 394\,Hz$$

b, $f_3 = 395\,Hz$

$f_0 = 394\,Hz$: $A_1(\omega_1^2 - \omega_0^2) = A_3 \cdot (\omega_3^2 - \omega_0^2)$

$$A_3 = A_1 \cdot \frac{\omega_1^2 - \omega_0^2}{\omega_3^2 - \omega_0^2} \approx 54\,mm$$

$f_0 = 402\,Hz$: $A_1(\omega_0^2 - \omega_1^2) = A_3 \cdot (\omega_0^2 - \omega_3^2)$

$$A_3 = A_1 \cdot \frac{\omega_0^2 - \omega_1^2}{\omega_0^2 - \omega_3^2} \approx 2{,}6\,mm$$

c,

$$\frac{F_0}{m} = A_1(\omega_1^2 - \omega_0^2) \Rightarrow F_0 = mA_1(\omega_1^2 - \omega_0^2) \approx 0{,}84\,N \quad \text{ha } f_0 = 394\,Hz$$

$$F_0 = mA_1(\omega_0^2 - \omega_1^2) \approx 0{,}28\,N \quad \text{ha } f_0 = 402\,Hz$$