(09. 07.)

Kis. Fiz. 3. Gyak.

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Termodinamika

B-M, G-L

Állapotjelző: leírják a rendszer pill.-nyi állapotát

→ intenzív: kiegyenlítődnek a foly. során: p, T, μ

→ extenzív: összeadódnak a foly. során: N, V, S

Állapotegyenletek:

$\frac{1}{273K} = \frac{1}{T_z}$

B-M: $T = áll.$; $pV = áll.$

hőm. kül. (°C)

G-L II: $V = áll.$, $\frac{p}{T} = áll. \rightsquigarrow p' = p_0(1+\beta\Delta t)$

G-L I: $p = áll.$, $\frac{V}{T} = áll. \rightsquigarrow V = V_0(1+\beta\Delta t)$

$p_0, V_0, 0°C$

$\downarrow$ izoV

p', V_0, t

$\downarrow$ izoT

p, V, t

$$p'V_0 = p_0V_0(1+\beta\Delta t) = pV$$

$\frac{R}{N_A}$ Boltzmann-áll.

$$\frac{p_0V_0}{T_z}(T_z+t) = pV \implies pV = nRT = \frac{N}{N_A}RT = Nk_BT$$

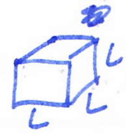
abs. rugalmas ütközés

Kinetikus gázelmélet:

Feltételezés: - ütközéskor csak imp. átadás ⇝ hard wall ⇒

⇒ impulzus előjelet vált

- sok atom, sok ütközés
- kocka alakú edény, L oldalhossz



- 1 atom, Δt idő, ütközés száma?

2 ütk. közt $\Delta s = 2L$

$$N = \frac{v_x \Delta t}{2L} = \frac{\frac{|p_x|}{m}\Delta t}{2L}$$

- 1 ütközés alatt átadott impulzus?

$$dp_x = 2|p_x|$$

- Δt idő alatt átadott impulzus ⇒ $N \cdot dp_x = |p_x|^2 \frac{\Delta t}{mL}$

További feltételezések:

- függetlenek
- ∅ kh.
- pontszerűek

↳ (- f=3)

- Átlagolás N db részecskére: $\sum_{i=1}^{N} \frac{|p_{ix}|^2 \Delta t}{mL} = F\Delta t = p L^2 \Delta t$

$\frac{\Delta p}{\Delta t}$ $\quad$ $\frac{F}{A} = p$ $\quad$ $[L^3 = V]$

$$p = \frac{1}{V} \sum_{i=1}^{N} \frac{|p_{ix}|^2}{m}$$

ezt átlagoljuk

- Átlagos impulzus: $\overline{p_x^2} = \frac{1}{N} \sum_{i=1}^{N} p_{ix}^2$

$$p = \frac{N}{V} \frac{\overline{p_x^2}}{m}$$

- nincs kitüntetett irány: $\overline{p_x^2} = \overline{p_y^2} = \overline{p_z^2} = \frac{1}{3}\overline{p^2}$

- így a nyomás: $p = \frac{N}{V} \frac{\overline{p^2}}{3m} = \frac{2}{3} \frac{N}{V} \frac{\overline{p^2}}{2m}$

$$\varepsilon = \frac{p^2}{2m} = \frac{1}{2}\frac{m^2v^2}{m} = \frac{1}{2}mv^2 \quad \text{[kinetikus energia]}$$

$$\bar{\varepsilon} = \frac{1}{N}\sum_{\alpha=1}^{N} |p_\alpha|^2/2m$$

- $p = \frac{2}{3}\frac{N}{V}\frac{\overline{p_x^2}}{2m} = \frac{2}{3}\frac{N}{V}\bar{\varepsilon}$ — kinetikus energia-átlag
- $pV = Nk_BT \Rightarrow \frac{2}{3}\bar{\varepsilon} = k_BT \Rightarrow \boxed{\bar{\varepsilon} = \frac{3}{2}k_BT}$

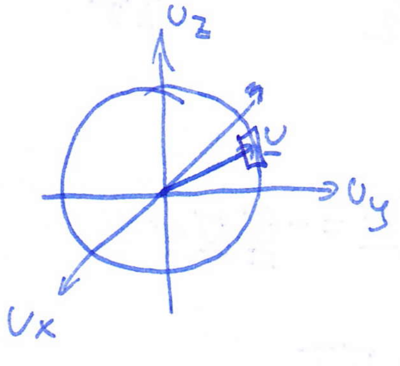
$\boxed{1/1}$ N részecske

belső E: $U = N\bar{\varepsilon} = N\frac{f}{2}k_BT$

$pV = \frac{2}{3}N\bar{\varepsilon} = \frac{2}{3}U \longrightarrow \boxed{p = \frac{2}{3}\frac{U}{V}}$

TRANSZPORT:

molekula $\rightsquigarrow$ random mozgás   kérdés: kül. mennyiségek hogyan változnak?



dP = az a valség, hogy a mol.-a hala-dási iránya (azaz $\underline{v}$ vektor) az adott irány körüli $d\Omega$ térszögben van?

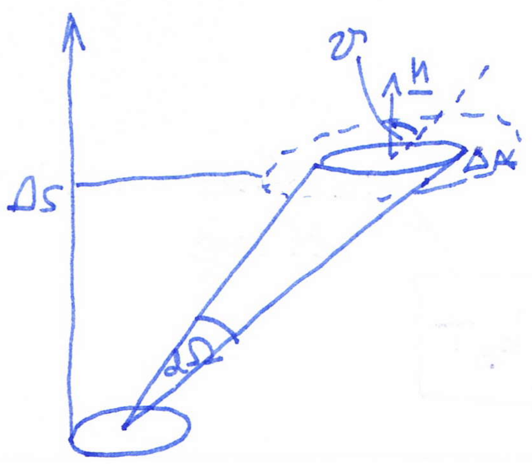
$$dP = \frac{dA}{4v^2\pi} = \frac{d\Omega}{4\pi} \qquad dA = r^2\sin\vartheta\, d\vartheta\, d\varphi \rightsquigarrow d\Omega = \sin\vartheta\, d\vartheta\, d\varphi$$

$$dN = N\,dP = N\frac{d\Omega}{4\pi}$$

kérdés: hány molekula ütközik adott idő alatt adott felületen?

$$(v, v+dv) \rightsquigarrow dN_v \qquad dN_{v,\vartheta,\varphi} = dN_v\frac{d\Omega}{4\pi}$$

kérdés: A→B-be mennyi részecske jut el?



$\Delta A, \Delta t$

max. Δs táv.-ra lehetnek

$\Delta s = v\cos\vartheta\,\Delta t$

$\Delta V = \Delta A\,\Delta s$

ΔA-n mennyi molekula halad át?

$$\Delta z_{v\vartheta\varphi} = dN_{v\vartheta\varphi}\frac{\Delta V}{V} = dN_v\,\frac{\Delta A\,\Delta t}{4\pi V}\,v\cos\vartheta\sin\vartheta\,d\vartheta\,d\varphi$$

haladási irányszögben van

$$n_v = \frac{N}{V}$$

$$\Delta z_{\Delta A,\Delta t} = \frac{\Delta A\,\Delta t}{4\pi}\,\underbrace{\frac{n_v}{N}\int_0^{v_{max}} v\,dN_v}_{\text{sebesség átlag } \bar v}\;\underbrace{\int_0^{\pi/2}\cos\vartheta\sin\vartheta\,d\vartheta}_{1/2}\;\underbrace{\int_0^{2\pi}d\varphi}_{2\pi}$$

$$\Delta z_{\Delta A,\Delta t} = \frac{1}{4}\,n_v\,\bar v\,\Delta A\,\Delta t$$

$$\boxed{\underline{j} = \frac{1}{4}\,n_v\,\bar v}$$

molekula áramlás sűrűség

$$\underline{j}_{el} = \underline{\underline{\sigma}}\,\underline{E} = -\underline{\underline{\sigma}}\,\underline{\nabla}\phi$$

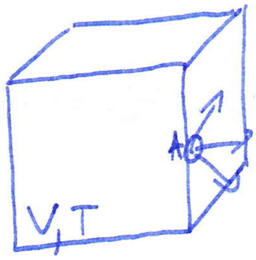
$$\underline{j}_Q = -\lambda\,\underline{\nabla}T$$

$$\underline{j} = -D\,\underline{\nabla}n$$

hajtóerő

Onsager: $\underline{j}_i = \sum_j L_{ij}\,\underline{X}_j$

1/6



$\Delta N_{\Delta t} = \frac{1}{4} n_V A \bar{v}$ — Alyuk, ha arra vagyunk kíváncsiak

falnak ütközők

• részecskeszám vált? $\longrightarrow \frac{dN}{dt} = V\frac{dn_V}{dt} = -\frac{1}{4} n_V \bar{v} A$

fogy

• $\Delta t = ?$ ha $p_2 = \frac{1}{2} p_1$

$$\frac{dn_V}{n_V} = -\frac{A\bar{v}}{4V}dt \quad / \int$$

$$\int \frac{1}{n_V} dn_V = -\int \frac{A\bar{v}}{4V} dt$$

$$\ln \frac{n_V}{n_{V_0}} = -\frac{A\bar{v}}{4V} t = -\frac{t}{\tau}$$

$$N(t) = V n_V(t) \longleftarrow \boxed{n_V(t) = n_{V_0} e^{-\frac{t}{\tau}}}$$

$$n_V(T_{1/2}) = \frac{n_{V_0}}{2} \quad n_V(\tau) = \frac{n_{V_0}}{e}$$

$$\frac{n_{V_0}}{2} = n_{V_0} e^{-\frac{T_{1/2}}{\tau}} \Rightarrow \boxed{T_{1/2} = \tau \ln 2}$$

1/17

T=áll. $V^{(1)}$ H_2 p_{kezd}	T=áll. $V^{(2)}$ H_2 $2p_{kezd}$

$p = \frac{3}{2} p_{kezd}$ lesz az egyensúlyi nyomás.

~~üreg~~

$$N_{lyuk} = \frac{1}{4} n_v A \bar{v}$$

- $$\frac{dN^{(1)}}{dt} = \underbrace{-\frac{1}{4} n_v^{(1)} \bar{v} A}_{\text{kiáramlás}} + \underbrace{\frac{1}{4} n_v^{(2)} \bar{v} A}_{\text{visszaáramlás}} \qquad N^{(2)} = N - N^{(1)}$$

- $$\frac{dN^{(2)}}{dt} = -\frac{dN^{(1)}}{dt}$$

$$\cancel{V^{(1)} \frac{dn_v^{(1)}}{dt} = -\frac{1}{4} \bar{v} A \left(1 + \frac{V^{(1)}}{V^{(2)}}\right)} \qquad \bar{v}^{(1)} = \bar{v}^{(2)} \text{ mert } T^{(1)} = T^{(2)} \,!$$

$$V^{(1)} \frac{dn_v^{(1)}}{dt} = -\frac{1}{4} \bar{v} A \left(1 + \frac{V^{(1)}}{V^{(2)}}\right) n_v^{(1)} + \frac{1}{4} \bar{v} A \frac{N}{V^{(2)}}$$

egyensúly: $\left.\frac{dn_v^{(1)}}{dt}\right|_{egyens.} = 0 \longrightarrow \left(1 + \frac{V^{(1)}}{V^{(2)}}\right) n_v^{(1)} = \frac{N}{V^{(2)}}$

$$n_v^{(1)} = \frac{N}{V^{(1)} + V^{(2)}} = n_v^{(2)}$$

nyomás? $\Rightarrow$ $pV = N k_B T \qquad p = \frac{N^{(1)} + N^{(2)}}{V^{(1)} + V^{(2)}} k_B T \left[= \sum_i \frac{N_i}{V_i} k_B T \right]$ ← n tartályra

$$p_{kezd} = \frac{N^{(1)}_{kezd}}{V^{(1)}} k_B T = \frac{N^{(2)}_{kezd}}{2V^{(2)}} k_B T \qquad V^{(1)} = V^{(2)} = V$$

$$\hookrightarrow N^{(1)}_{kezd} = \frac{1}{2} N^{(2)}_{kezd} = \frac{N}{3} \Longrightarrow \boxed{p = \frac{3}{2} p_{kezd}}$$

$\boxed{1/}$ $m_{H_2} = 2\,\frac{g}{mol}$ Dalton tv.: $p_{vég} = p_{vég\,H_2} + p_{vég\,O_2}$

$m_{O_2} = 32\,\frac{g}{mol}$

$$\frac{dN_H^{(1)}}{dt} = -\frac{1}{4} n_{vH}^{(1)} \bar{v}_H A + \frac{1}{4} n_{vH}^{(2)} A \bar{v}_H$$

$$\frac{dN_O^{(1)}}{dt} = -\frac{1}{4} n v_O^{(1)} \bar{v}_O A + \frac{1}{4} n v_O^{(2)} A \bar{v}_O$$

k.F.: $n_{vH\,kezd}^{(1)} = n_{vH\,kezd}$

$n_{vH\,kezd}^{(2)} = 0$

$$\frac{dN_H^{(1)}}{dt} = 0$$

$N_H^{(2)} = N_H - N_H^{(1)}$

(09.14.)

Előző gyak:

$$pV = nRT = N k_B T$$

$$pV = \frac{2}{3} U = \frac{2}{3} N \bar{\varepsilon}$$

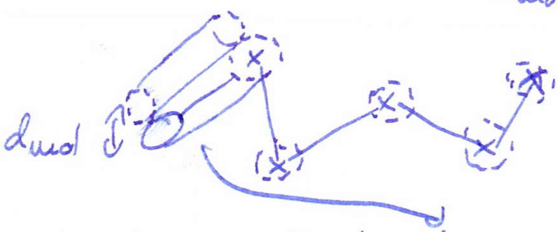
$$\underline{j}_{mol} = \frac{1}{4} n_V \bar{v}$$

$$\underline{j}_n = -D \underline{\nabla} n = -D \underline{\nabla} c$$

Szabad úthossz: $\langle l \rangle$ ütk. nélkül megtett táv

molekula → gömb → d_{mol}.

ütk. → rug.



$\langle l \rangle \gg d_{mol}$

Δt: $s = \bar{v} \Delta t$ henger térfogaton belüli cuccokkal ütk.

ütk. azokkal: $\Delta V = d_{mol}^2 \pi s = d_m^2 \pi \bar{v} \Delta t$

hatáskeresztmetszet:

$$\sigma = d_m^2 \pi - b^2 \pi$$

$$d\sigma = 2\pi d_m \, dd_m = 2\pi b \, db$$

Δt ütk. száma: $\underset{\text{ütk. száma}}{\Delta z} = \Delta V n_V = n_V d_m^2 \pi \bar{v} \Delta t$

$$\left[\text{2 ütk. eltelt idő: } \Delta\tau = \frac{\Delta t}{\Delta z} = \frac{1}{n_V d^2 \pi \bar{v}} \right]$$

$$\Downarrow$$

$$\langle l \rangle = \Delta\tau \bar{v} = \frac{1}{n_V d^2 \pi \sqrt{2}}$$

a többi mol. is zizeg

$$p = \frac{N k_B T}{V} = n_v k_B T$$

$$\hookrightarrow n_v = \frac{p}{k_B T}$$

$$\boxed{\langle l \rangle = \frac{k_B T}{\sqrt{2}\,\pi d_m^2 p}}$$

$\boxed{T1/4}$ $V = 1l$ gömb $\quad V = \frac{4R^3\pi}{3} \Longrightarrow R = 0{,}062\,m$

$T = 300K$

$H_2 \longrightarrow d_m = 2\,Å$

$p_{max} = ?$ hogy $\underset{4}{\langle l \rangle} > d_{edény}$

$$\frac{k_B T}{\sqrt{2}\,\pi d_m^2 p} \qquad p < \frac{k_B T}{2\sqrt{2}\,R d_m^2 \pi}$$

$$\boxed{p < 0.188\ Pa}$$

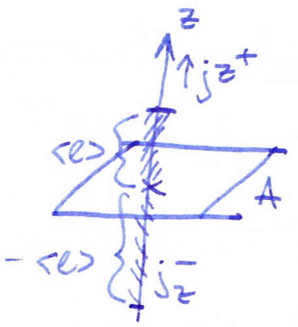
Diffúzió

→ magasabb konc. ↔ alacsonyabb

→ $|j|$ függ az ütközések számától

Tfh: $d, m, \bar{v}$ azonosak $\Longrightarrow \langle l \rangle$ is azonos

1D: $n_v = n_v(z)$; vál ΔA fel $\perp z$

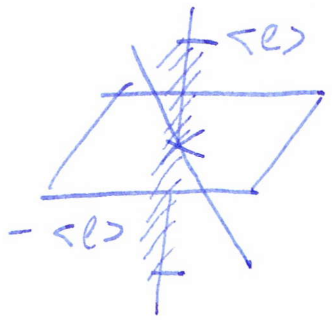


$$j_{diff} = j_z^+ - j_z^- = \frac{1}{4}\bar{v}\overbrace{\left(n_v^+ - n_v^-\right)}^{\text{sűrűség kül.}}$$

$j_{mol} = \frac{1}{4} n_v \bar{v}$, tfh. lineárisan vált. $\langle l \rangle$-en belül

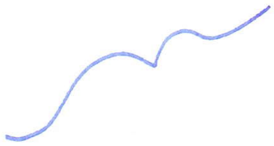
$n_v^+ \approx n_v(z - \langle l \rangle)$

$n_v^- \approx n_v(z + \langle l \rangle)$



$$n_v^+ - n_v^- \approx n_v\left[(z-\langle l\rangle)-(z+\langle l\rangle)\right] = -\frac{dn_v}{dz}\cdot 2\langle l\rangle$$

$$\left[\begin{array}{l} n_v^+ \approx n_v(z-\langle l\rangle) \\ n_v^- \approx n_v(z+\langle l\rangle) \end{array}\right.$$



$$j_{diff.} = -\frac{1}{2}\bar v\langle l\rangle\frac{dn_v}{dz}$$

$\frac{1}{3}$ valójában

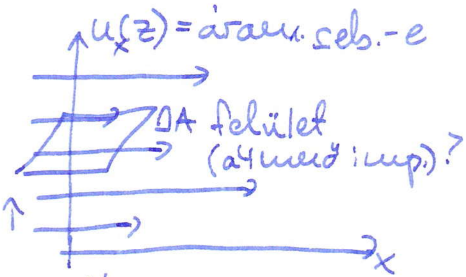
$$j_{diff.} = -D\nabla n \qquad \text{(Fick I.)}$$

diff.-ós áll.

ideális gáz $D=\frac{1}{3}\bar v\langle l\rangle$

Viszkozitás

áramlás irányára ⊥ vált. a seb.



imp. addódik át a kül. részek közt

$$\Delta p = \Delta p_x^+ - \Delta p_x^- = m(u_x^+ - u_x^-)$$

$\cancel{\Delta p = \Delta p_x t}$

$$j^z_{imp} = \frac{1}{4}n_v\bar v m(u_x^+ - u_x^-)$$

$$u_x^+ - u_x^- \approx \underbrace{u_x(z-\langle l\rangle)-u_x(z+\langle l\rangle)}_{-\frac{du_x}{dz}\langle l\rangle 2}$$

$$j^z_{imp} = -\underbrace{\frac{1}{3}n_v\bar v m\langle l\rangle}_{\eta:\ \text{viszkozitás}}\frac{du_x}{dz}$$

Newton-tv.: $j_z^{imp} = -\eta\frac{du_x}{dz}$

ideális g.: $\eta = \frac{1}{3}n_v\bar v\langle l\rangle m$

$\boxed{T1/5}$ ideális, D, η $\qquad \eta(T,V)$

$D(T,V)$

a, $T =$ áll.

b, $p =$ áll. $\qquad D = \frac{1}{3} \langle \ell \rangle \bar{v}$

$n_V = \frac{N}{V} \qquad \langle \ell \rangle = \frac{1}{\sqrt{2} n_V \sigma} \qquad \bar{v} = \sqrt{\frac{8 k_B T}{m \pi}}$ $\quad$ $M_{x}-B$

$$D(T,V) = \frac{1}{3} \frac{V}{\sqrt{2} N \sigma} \sqrt{\frac{8 k_B T}{m \pi}}$$

$$\eta(T,V) = +\frac{1}{3} \frac{1}{\sqrt{2} \sigma} \sqrt{\frac{8 k_B T}{m \pi}} \, m$$

a, $T =$ áll. $\quad \underset{\text{növeljük}}{V \to n \cdot V} \quad D \sim V$

$\eta \sim 1 \sim V^0$ (nem függ V-től)

$D \to n \cdot D \qquad \eta \to \eta$

b, $p =$ áll. $\quad p = \frac{N}{V} k_B T \qquad \sqrt{T} \sim \sqrt{pV} \sim \sqrt{V}$

$D \sim V^{3/2} \qquad \eta \to V^{1/2}$

$D \to n^{3/2} D \qquad \eta \to n^{1/2} \eta$

Hőáram / Hővezetés

$T = T(z) \qquad \bar{\varepsilon} = \frac{f}{2} k_B T \qquad$ linearizálás

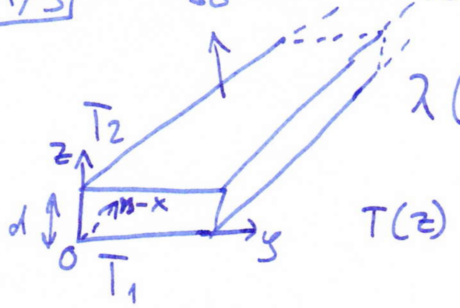
$$j_z^{hő} = \frac{1}{4} n_V \bar{v} (\bar{\varepsilon}_+ - \bar{\varepsilon}_-) = \frac{f k_B}{8} n_V \bar{v} (T_+ - T_-) \approx -\frac{f}{6} k_B n_V \bar{v} \langle \ell \rangle \frac{dT}{dz}$$

hőáramsűrűség: $\underline{j}_Q = -\lambda \underline{\nabla} T$

hővezetési eh.

ideális gáz: $\lambda = \frac{f}{6} k_B n_V \bar{v} \langle \ell \rangle \propto \frac{1}{\sigma} \sqrt{\frac{T}{m}}$

T1/9 egy szelet felületeit



$\lambda(T, \underline{r}) = \lambda$ (nem függ T-től, $\underline{r}$-től)

$T(z)$ kf: $T(0)=T_1$

$T(d)=T_2$

$$j_Q = -\lambda \frac{dT}{dz}$$

$J_Q = \int dA\, j_Q = \frac{dQ}{dt}$ ⇒ időegység alatt felületen átvitt hő

$$\left.\begin{array}{l} j_Q = -\lambda \frac{dT}{dz} \\ J_Q = \int dA\, j_Q = \frac{dQ}{dt} \end{array}\right\} \boxed{\frac{dQ}{dt} = -\lambda A \frac{dT}{dz}}$$

hővez. alap egyenlete

stac. eset → hő $\emptyset$ halmozódik fel (kialakult a végső hőm.-i profil)

$$J_Q = \text{áll.} \rightarrow \frac{dQ}{dt} = \underset{\text{áll.}}{\underbrace{\;\;}} \Rightarrow -\lambda A \frac{dT}{dz} = \text{áll.} \Rightarrow \frac{dT}{dz} = \text{áll.} = c$$

$$\frac{dT}{dz} = c_T$$

$$dT = c_T\, dz \quad /\int$$

$\boxed{T(z) = c_T z + T(0)}$ $T(0)=T_1$ $T(d)=T_2$

$$T(d) = \underbrace{T_2 = T_1}_{} + c_T d$$

$$c_T = \frac{T_2 - T_1}{d}$$

$$\boxed{T(z) = \frac{T_2 - T_1}{d} z + T_1}$$

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$\boxed{T1/10}$ Hővezetés alap tv.-e

$z = 5\,cm$ $\qquad \lambda = 2{,}1\cdot 10^{-2}\,\frac{J}{s\,cm\,°C}$

$T_e = -10°C$ $\qquad \rho = 0{,}92\,\frac{g}{cm^3}$

$T_0 = 0°C$

$L_0 = 335\,\frac{J}{g}$

$$\frac{dQ}{dt} = -\lambda A \frac{dT}{dz}\ ; \quad \underset{\text{múltkor}}{T(z)} = T_1 + \frac{T_2 - T_1}{d} z = c + c'z \quad \forall z(t)$$

$$\frac{dT}{dz} = \frac{T_0 - T_e}{z(t)} \qquad dQ = -L_0\, dm = -L_0 \rho A\, dz$$

$$\Downarrow$$

$$L_0 \rho \cancel{A} \frac{dz}{dt} = \lambda \cancel{A} \frac{T_0 - T_e}{z(t)}$$

$t(0) \Rightarrow$ ~~vastag~~ még nincs jég

$$\int z\, dz = \frac{T_0 - T_e}{L_0 \rho} \lambda\, dt \quad + C$$

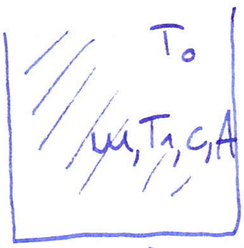
$$\frac{z^2}{2} = \frac{\lambda}{L_0 \rho}(T_0 - T_e)t \leadsto t(z) = \frac{\rho L_0}{2\cdot\lambda(T_0 - T_e)} z^2 \qquad t(5\,cm) = 5.1\ \text{óra}$$

$\boxed{T1/11}$

T_0

$T_1 > T_0$

m, c



$$j_Q = \alpha (T - T_0)$$

$$\text{hővez. alap tv.-e:}\ \boxed{\frac{dQ}{dt} = A\alpha (T - T_0)} \qquad dQ = ? \quad dQ = \underbrace{\overset{\text{hűl}}{-}\,c\,m\,dT}_{\text{hőelvonás}}$$

$$-\frac{c\,m\,dT}{dt} = A\alpha (T - T_0) \qquad \tilde{T} = (T - T_0)$$

$d\tilde{T} = dT$ mert T_0 egy const

$$-c\,m \frac{d\tilde{T}}{dt} = A\alpha \tilde{T}$$

$$\int \frac{d\tilde{T}}{\tilde{T}} = -\frac{A\alpha}{c\,m}\int dt \rightarrow \ln \tilde{T} = -\frac{\alpha A}{c\,m} t + \ln C$$

$$T - T_0 = e^{-\frac{\alpha A}{c\,m}t}\, C \qquad \text{k.f.}: T(0) = T_1 \leadsto \boxed{C = T_1 - T_0}$$

$$\boxed{T(t) = T_0 + (T_1 - T_0)e^{-\frac{\alpha A}{c\,m}t}}$$

Fundamentális termodinamika

↓

tapasztalati úton
építjük fel

$\boxed{0.\text{ Főtv.}}$: $A \Leftrightarrow B$, $B \Leftrightarrow C$ $\Rightarrow A \Leftrightarrow C$

Tap: • Testen végzett mechanikai munka elvész → hővé alakul
• Meleg testtel munkát lehet végeztetni

•

hőmérséklet emelkedés ~ hőtartalom ~ hő (energia)

→ környezettől elzárt → ∅ hőcsere → adiabatikusan elszigetelt
↳ meghatározott állapotváltozáshoz munka rendelhető

U_A , U_B

$W_{adiab.} = \Delta U$

$\delta W_{adiab} = dU$

→ (ha nem elszigetelt) ⇒ munka: makroszk.-an átadott E
hő: Q ——— " ——— E

$\delta Q = \delta W_{adiab} - \delta W$

⇒ $\boxed{1.\text{ Fő T.}}$ $dU = \delta W + \delta Q$ (⇐ energiamegm.)

dU: belső E vált.-a

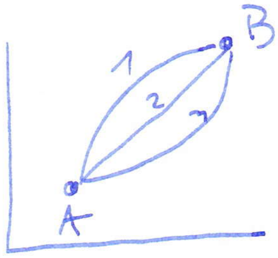
δQ: felvett/leadott hő (előjel függő)

δW: általunk végzett munka ⟹ $\delta W_{\text{általunk}} = -\delta W_{\text{rendszer által}}$

Kis.Fiz.3.Gy.-

d = állapotjelzők alapján e.é.-en meghatározható a megvált.

δ = állapotvált. útjától is függ



$dU = dU_1 = dU_2 = dU_3$

$\delta W_1 \neq \delta W_2$

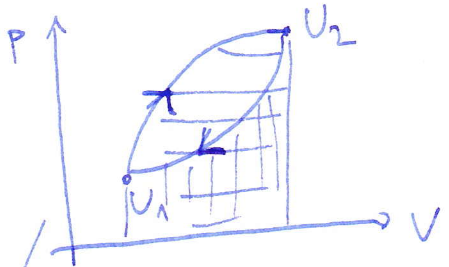
körfolyamat

$\boxed{dU = 0} \longrightarrow \delta Q + \delta W = 0$

$\delta Q = -\delta W$

Munkák:

mech.-i munka: térfogati munkavégzés



$\delta W_{mech.} = -p\,dV$

$\downarrow$

$W_{mech.} = -\int_{V_1}^{V_2} p\,dV$ (felső útra, mert $V_1 \to V_2$)

feltételezés: $\Delta p \ll p_{abs.}, p_{külső}$; $p \approx$ áll. $\Big\}$ ⇒ reverzibilisnek tekinthető emiatt

↓

rendszer által végzett munka a 2 terület kül.-e

Hő: $\delta Q \sim dT$

$\delta Q = K\,dT$ $K = ? \longrightarrow$ hőkapacitás (sok mindentől függ)

$K = c \cdot m$

↳ fajhő → anyagra jell.

$\boxed{K = c\,n}$ => gázok tömegét nehéz mérni
↳ moláris hőkap.

$$\boxed{c = \frac{1}{n}\frac{\delta Q}{dT} = \frac{1}{n}\frac{dU + p\,dV}{dT}}$$ => függ az úttól, mert tartalmazza a munkát is

c_v = áll. térf-on mért fajhő => $dV = 0$

$$\delta Q = dU = n c_v dT \qquad dU = \left.\frac{\partial U}{\partial T}\right|_V dT + \underbrace{\left.\frac{\partial U}{\partial V}\right|_T dV}_{dV=0 \text{ akkor eltűnik}}$$ (láncszabály, teljes deriváltakra csak)

d miatt teljes differenciál

$$\boxed{c_v = \frac{1}{n}\left.\frac{\partial U}{\partial T}\right|_V}$$

c_p = áll. nyom.-on mért fajhő => $dp = 0$

$$c_p = \frac{1}{n}\left.\frac{dU + p\,dV}{dT}\right|_p$$

<u>entalpia</u>: $H = U + pV$ (Legendre-transzformált)

$$dH = dU + \underbrace{\cancel{V\,dp}}_{dp=0} + p\,dV$$

$$\boxed{c_p = \frac{1}{n}\left.\frac{\partial H}{\partial T}\right|_p} \left(= \frac{1}{n}\left.\frac{\delta Q}{\delta T}\right|_p\right)$$

Kis.Fiz.3.Gy.-

c_p, c_v nem fgtl.-ek

$$dU = \left.\frac{\partial U}{\partial T}\right|_V dT + \left.\frac{\partial U}{\partial V}\right|_T dV$$

$$\delta Q = dU + p\,dV = \underbrace{\left.\frac{\partial U}{\partial T}\right|_V dT}_{n c_v\, dT} + \underbrace{\left.\frac{\partial U}{\partial V}\right|_T dV + p\,dV}_{dV\left[\left.\frac{\partial U}{\partial V}\right|_T + p\right]}$$

$$c_p = \frac{1}{n}\left.\frac{\delta Q}{\partial T}\right|_p = \frac{1}{n}\left\{ n c_v + \left[\left.\frac{\partial U}{\partial V}\right|_T + p\right]\left.\frac{\partial V}{\partial T}\right|_p \right\}$$

$$\boxed{c_p - c_v = \frac{1}{n}\left[\left.\frac{\partial U}{\partial V}\right|_T + p\right]\left.\frac{\partial V}{\partial T}\right|_p}$$

$$\boxed{c_p - c_v = R}$$

Izobár térf. hőtág. e.h.

$$\beta_p = \frac{1}{V}\left.\frac{\partial V}{\partial T}\right|_p$$

$$c_p - c_v = \frac{1}{n}\left[\left.\frac{\partial U}{\partial V}\right|_T + p\right]\beta_p V$$

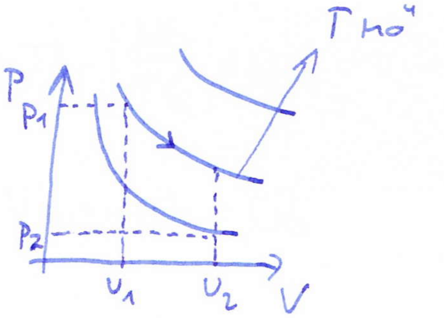
$$c_p = c_v + R = \frac{f+2}{2} R \qquad \gamma = \frac{c_p}{c_v} = \frac{f+2}{f}$$

↳ id. gáz. U-ból

ÁLLAPOTVÁLTOZÁSOK

(id. gázok)

i, izoT; $T = \text{áll.}$



$pV = nRT = \text{áll.} \rightsquigarrow p(V) \sim \frac{1}{V}$

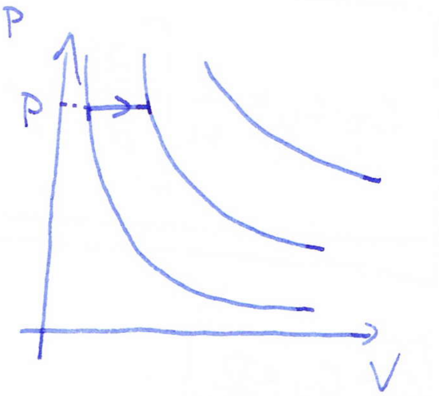
ideális g.: $U(T) = U = \text{áll.}$ $\underline{\underline{dU = 0}}$

végzett munka: $W = -\int_{V_1}^{V_2} p\,dV = -\int_{V_1}^{V_2} \frac{nRT}{V} dV = \boxed{-nRT \ln \frac{V_2}{V_1} = -nRT \ln \frac{p_1}{p_2}}$

$p = \frac{nRT}{V} = p(V)$

$Q = -W$

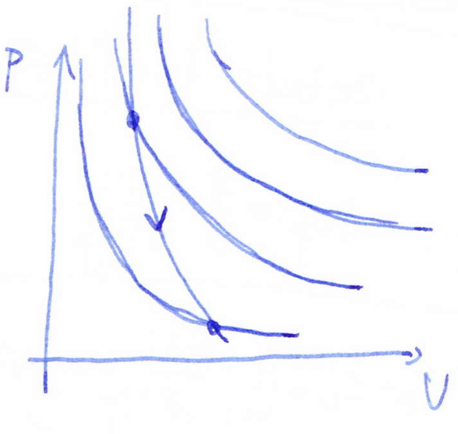
ii, izop; $p = \text{áll.}$



$\boxed{W = \cancel{\int p\,dV} - p(V_2 - V_1)}$

$Q = U_2 - U_1 + p(V_2 - V_1) = nc_p(T_2 - T_1)$

iii, adiabatikus $\rightsquigarrow$ nincs hőcsere $\delta Q = 0$



$dU + p\,dV = 0$

$n c_v dT + p\,dV = 0$

$\Downarrow$

$n c_v dT + nRT \frac{dV}{V} = 0$

$R = c_p - c_v; \ \gamma = \frac{c_p}{c_v}$

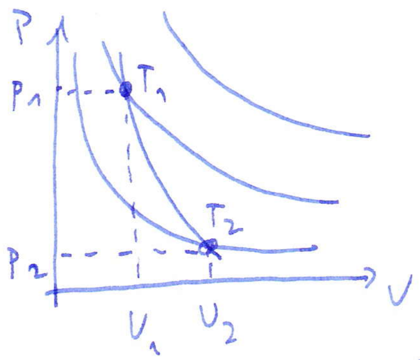
$\frac{dT}{T} + (\gamma - 1) \frac{dV}{V} = 0 \ / \cdot \int$

$\ln T + (\gamma - 1) \ln V = \text{const.}$

$\boxed{TV^{\gamma - 1} = \text{const.}}$

$pV^{\gamma} = \text{const.}$

Biz.: (adiabata meredekebb mint izoterma)



izot: $pV = p_0V_0$

adiab: $pV^{\gamma} = p_0V_0^{\gamma}$

meredekség: $\left.\dfrac{\partial p}{\partial V}\right|_{T}^{V_0} = -\dfrac{p_0}{V_0}$

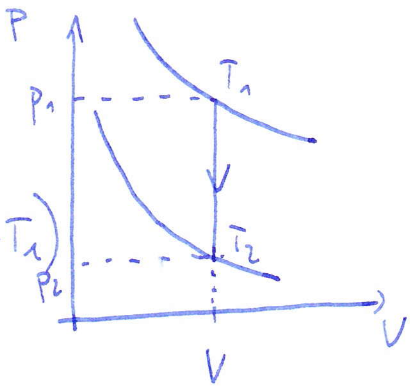
$\left.\dfrac{\partial p}{\partial V}\right|_{adiab.}^{V_0} = -\gamma\dfrac{p_0}{V_0} = \gamma$

γ-szorosa (és $\gamma > 1$)

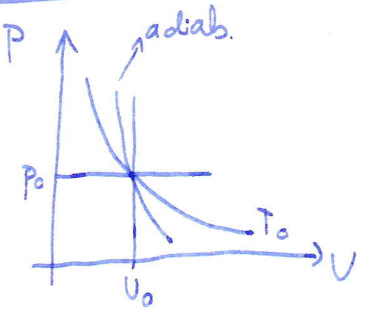
iv) izoV, $V = áll. \rightarrow dV = 0$

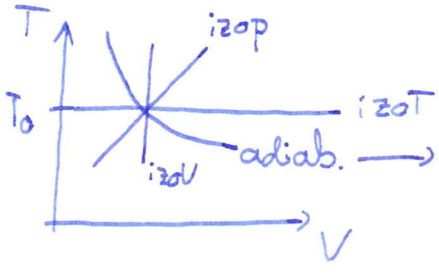
$\delta W = 0$

$Q = U_2 - U_1 = n c_V (T_2 - T_1)$



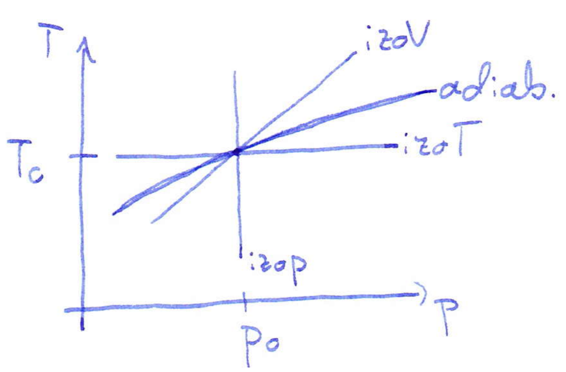
T2/1





$\longrightarrow TV^{\gamma-1} = const.$

id. gáz: $pV = nRT \quad T = \dfrac{pV}{nR}$

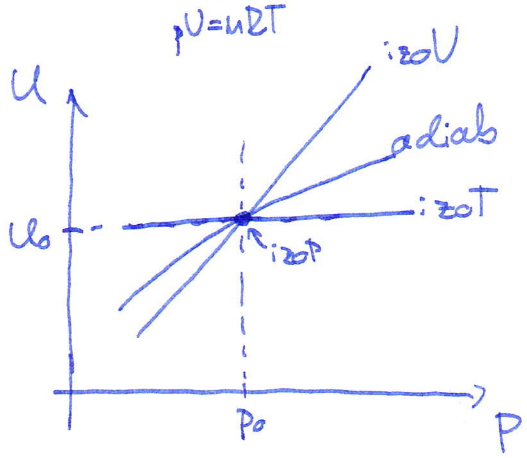


(09.28.)

T2/2

$$U=\frac{f}{2}k_BT = \frac{f}{2}k_B\frac{pV}{Rn}$$

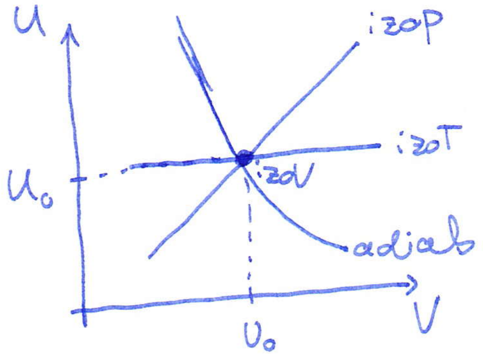
$pV=nRT$

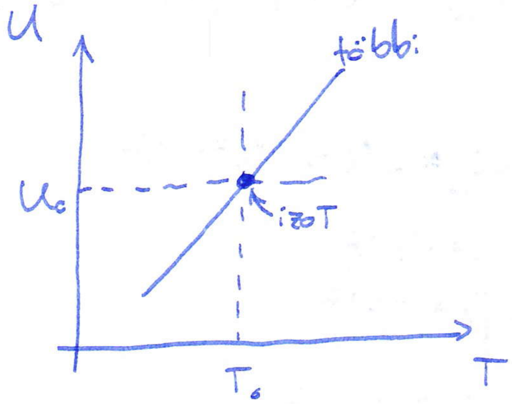


$pV^\gamma=$ áll

↳ $pV=\text{const}\cdot V^{1-\gamma}\propto U$

$\text{const}\; p^{1-\frac{1}{\gamma}}\propto U$





T2/3 p=áll.

γ ismert

$\Delta W\propto\Delta Q\propto\Delta U$

$$\gamma=\frac{c_p}{c_v}=\frac{f+2}{f}\longrightarrow f\gamma=f+2 \qquad f(\gamma-1)=2 \Rightarrow f=\frac{2}{\gamma-1}\Rightarrow\frac{f}{2}=\frac{1}{\gamma-1}$$

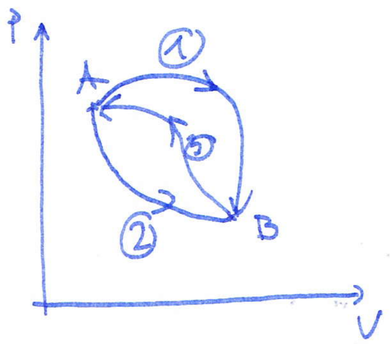
$$\Delta W_g=p\Delta V=nR\Delta T\longrightarrow\Delta T=\frac{\Delta W}{nR} \qquad \Delta U=\frac{f}{2}nR\Delta T=\frac{f}{2}\Delta W\Rightarrow\boxed{\Delta U=\frac{1}{\gamma-1}\Delta W}$$

$$\Delta Q=c_pn\Delta T=c_p\frac{\Delta W}{R} \qquad c_v=\frac{1}{n}\frac{\partial U}{\partial T}\Big|_V=\frac{f}{2}R \qquad c_p-c_v=R$$

$$c_p=c_v+R=\frac{f}{2}R+R=R\left(\frac{f}{2}+1\right)=\gamma c_v=R\left(1+\frac{1}{\gamma-1}\right)=R\frac{\gamma}{\gamma-1}$$

$$\boxed{\Delta Q=\Delta W\frac{\gamma}{\gamma-1}=\gamma\Delta U}$$

T2/4



$\Delta Q_1 = 100J$

$\Delta W_{1,g} = 30J$

$\Delta Q_2 = ?$

a, $\Delta W_{2,g} = 10J$

b, $\Delta Q_3 = ?$

$\Delta W = 20J$

Mj.: ΔU fgtl úttól, 1,2,3 esetben u.a., irány ±1

$$\Delta U = \Delta W_{mi} + \Delta Q = \Delta Q - \Delta W_g = \underline{\underline{70J}} = \Delta U_{1,2,3}$$

a) $\Delta Q_2 = \Delta U + \Delta W_g = 80J$

b, $\Delta U = -70J$

$\Delta W_{3,g} = -20J$

$\Delta Q_3 = -80J$

T2/5 $\kappa_T = \frac{1}{p}$ Kompressz. $\kappa = -\frac{1}{V}\frac{\partial V}{\partial p}\Big|_{\text{állapotvált.}}$

$\kappa_{ad} = \frac{1}{\gamma p}$

hogy κ ⊕ legyen norm. anyagokra

izoT ⇒ $V(p) = \frac{Nk_BT}{p}$

$$\frac{\partial V}{\partial p}\Big|_T = -Nk_BT/p^2 = -\frac{V}{p}$$

$$\boxed{\kappa_T = \frac{1}{p}}$$

adiab ⇒ $pV^\gamma = \text{const}$ $/\cdot\frac{d}{dV}$

HF $\frac{d}{dp}$

$$\frac{\partial p}{\partial V}V^\gamma + p\gamma V^{\gamma-1} = 0$$

$$\hookrightarrow \frac{\partial p}{\partial V} = -\frac{\gamma p}{V} \Rightarrow \frac{\partial V}{\partial p} = -\frac{V}{\gamma p} \rightsquigarrow \boxed{\kappa_{ad} = \frac{1}{\gamma p}}$$

$\boxed{T2/6}$

$p = p(T,V)$ ✓

β_p, κ_T ✓

$c = \frac{1}{m}\frac{\delta Q}{dT}$

$\kappa_T = -\frac{1}{V}\frac{\partial V}{\partial p}\Big|_T$

$\beta_T = \frac{1}{V}\frac{\partial V}{\partial T}\Big|_p$

$\frac{\partial p}{\partial T}\Big|_V = ?$

$\boxed{\text{izop} \Rightarrow}$ $dp = 0 = \frac{\partial p}{\partial T}\Big|_V dT + \frac{\partial p}{\partial V}\Big|_T dV \quad / \cancel{\frac{d}{dT}}\ \frac{\partial}{\partial T}$

$$0 = \frac{\partial p}{\partial T}\Big|_V \underbrace{\frac{\partial T}{\partial T}}_{1} + \underbrace{\frac{\partial p}{\partial V}\Big|_T}_{\frac{1}{\cancel{V}\kappa_T}} \underbrace{\frac{\partial V}{\partial T}\Big|_p}_{\cancel{V}\beta_T} \Rightarrow \boxed{\frac{\partial p}{\partial T}\Big|_V = \frac{\beta_p}{\kappa_T}}$$

$\boxed{\text{izoV} \Rightarrow}$ $dV = 0 = \frac{\partial V}{\partial T}\Big|_p dT + \frac{\partial V}{\partial p}\Big|_T dp \quad / \frac{d}{dT}$

$\boxed{\text{izoT} \Rightarrow}$ $dT = 0 = \frac{\partial T}{\partial p}\Big|_V dp + \frac{\partial T}{\partial V}\Big|_p dV \quad / \frac{d}{dp}$

$\boxed{T2/7}$

$V, \overset{\text{áll}}{p_0}, T_1 \to T_2$

$\Delta U = ?$

$pV = nRT = Nk_BT = \text{const} \Rightarrow NT = \text{const}$

áll const

$\boxed{N \sim \frac{\text{const}}{T}}$

$U = \frac{f}{2}Nk_BT = \frac{f}{2}k_B \cdot \text{const}$ $\boxed{\Delta U = 0}$

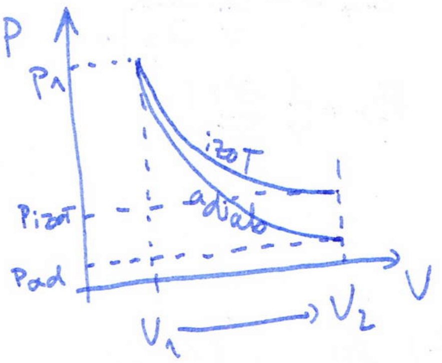
amennyivel nő az ~~belső~~ energiája a bent levő részecskéknek, pont annyi megy ki:

2/8 $V_1 \nearrow V_2$

V_1, γ

ad, T

$V_2 = ?$



izoT: $p_1 V_1 = p_{2izoT} V_2$

ad: $p_1 V_1^{\gamma} = p_{2ad} V_2^{\gamma} = \underbrace{\frac{p_{2izoT}}{2} V_2^{\gamma}}_{\text{feladat adja meg}}$

$$\frac{ad}{izoT} \Rightarrow V_1^{\gamma-1} = \frac{1}{2} V_2^{\gamma-1} \longrightarrow \boxed{V_2 = V_1 2^{\frac{1}{\gamma-1}}}$$

$\boxed{\text{Val. gázok}}$: vdW

molok k.h. } kereknek vesszük őket
véges méret }

1 mol. <u>Mol. kiterj.:</u> mozgáshoz rendelt térfogat lecsökken, kisebb

$$V \to \underset{\text{1 mol.}}{V_M} - b$$

b ← molok által elfoglalt

<u>Vonzás:</u> falnak átadott imp. csökken

$p \to p + p_{kM}$ ← kohéziós nyomás

visszahúzó mol-ák } számával: $p_{kM} \sim \rho^2 \sim \frac{1}{V_M^2}$
ütköző mol-ák }

$$\boxed{p_{kM} = \frac{a}{V_M^2}}$$

<u>vdW:</u> $(p + p_{kM})(V_M - b) = RT$

$$\left(p + \frac{a}{V_M^2}\right)(V_M - b) = RT \quad V_M = \frac{V}{n}$$

<u>n mol:</u> $\boxed{\left(p + \frac{n^2 a}{V^2}\right)(V - nb) = nRT}$

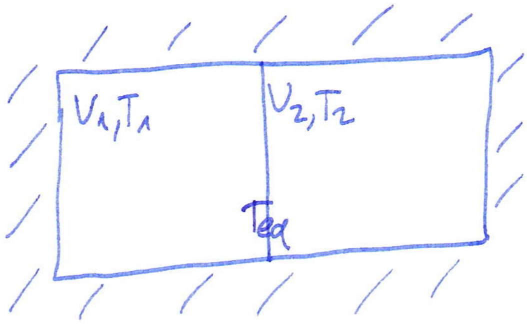
$$U_{kM} = \int_b^{V_M} p_{kM}\, dV'_M = \int_b^{V_M} \frac{a}{V'^2_M}\, dV'_M = -\frac{a}{V_M} + \underbrace{\frac{a}{b}}_{:= const}$$

$$U_k = \int_{b/n}^{V/n} \frac{n^2 a}{V^2}\, dV = -\frac{n^2 a}{V}$$

$$U_{vdW} = U_{id} + U_k = \boxed{\frac{f}{2} nRT - \frac{n^2 a}{V}}$$

$\boxed{T2/9}$

$$U = c_v m T - \frac{m^2}{M^2}\frac{a}{V}$$



→ szőrös
=
hőszigetelt
:D

a, $V_1, V_2 =$ áll.

$U = U_1 + U_2 =$ áll.

$$\cancel{c_v m} T_1 - \cancel{\frac{m^2}{M^2}\frac{a}{V_1}} + \cancel{c_v m} T_2 - \cancel{\frac{m^2}{M^2}\frac{a}{V_2}} = \cancel{c_v} T_{eq} \cancel{m} \cdot 2 - \cancel{\frac{m^2}{M^2}\frac{a}{V_1}} - \cancel{\frac{m^2}{M^2}\frac{a}{V_2}}$$

$$\boxed{T_{eq} = \frac{T_1 + T_2}{2}}$$

(10.05.)

Valódi gázok ⇝ VDW

↳ k.h. figyelembevétele

↳ korrekciók: véges kiterjedés; vonzás ⇝ nyomás

1 mol V_M

$\boxed{V_M \to V_M - b}$ → mol.-ák által kiszorított V

vonzás $p \to p + p_{bM}$

falnak ütköző mol.-ák száma ~ $\frac{1}{V}$; visszahúzó mol.-ák száma ~ $\frac{1}{V}$ } $p_{bM} \sim \frac{1}{V^2} \sim \frac{1}{V_M^2}$

$$\boxed{p_{bM} = \frac{a}{V_M^2}}$$

$$(p - p_{bM})(V - b) = RT$$

$$\boxed{\left(p + \frac{n^2 a}{V^2}\right)(V - nb) = nRT}$$

$$\left(p + \frac{a}{V_M^2}\right)(V_M - b) = RT$$

$$V_M = \frac{V}{n}$$

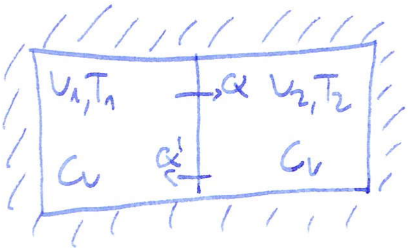
$$U_{bM} = \int_b^{V_M} p_{bM}\, dV_M = a \int_b^{V_M} \frac{1}{V_M^2}\, dV_M = -\frac{a}{V_M} + \underbrace{\cancel{\frac{a}{b}}}_{\text{const}:=0}$$ → ez nem vezet jó mo.-ra ↯

$$U_b = \int_{\frac{b}{n}}^{\frac{V}{n}} \frac{n^2 a}{V^2} \frac{dV}{n} = -\frac{n^2 a}{V} + \text{const}$$

$$\boxed{U_{vdW} = U_{id} + U_a = \frac{f}{2} nRT - \frac{n^2 a}{V}}$$

T2/9

$U = c_v m T - \frac{m^2}{M^2}\frac{a}{V}$



a, $V_i = \text{const}$

$U = U_1 + U_2 = \text{áll.}$

$\cancel{c_v} \cancel{m} T_1 - \cancel{\frac{m^2}{M^2}\frac{a}{V_1}} + \cancel{c_v}\cancel{m} T_2 - \cancel{\frac{m^2}{M^2}\frac{a}{V_2}} =$

$= 2\cancel{c_v}\cancel{m} T_{eq} - \cancel{\frac{m^2}{M^2}\frac{a}{V_1}} - \cancel{\frac{m^2}{M^2}\frac{a}{V_2}}$

$$\boxed{T_{eq} = \frac{T_1 + T_2}{2}}$$

b, lebontjuk a falat

$V_i \neq \text{const}$

$U = \text{const}$

$V = V_1 + V_2 = \text{const}$

$2m$ gázunk lesz $\quad c_v m T_1 - \frac{m^2}{M^2}\frac{a}{V_1} + c_v m T_2 - \frac{m^2}{M^2}\frac{a}{V_2} =$

$= 2m c_v T_{eq}' - \frac{(2m)^2}{M^2}\frac{a}{V_1 + V_2}$

$$\Rightarrow \boxed{T_{eq} = \frac{T_1 + T_2}{2} - \frac{m a}{2M^2 c_v}\left(\frac{1}{V_1} + \frac{1}{V_2} - \frac{4}{V_1 + V_2}\right)}$$

T2/10

$V = V_0(1 - ap + bT) \quad a, b = ?$

$\rightsquigarrow c, \beta_p, \kappa_T$

$a(p, T, \dots)$

$\left.\frac{\partial V}{\partial p}\right|_T = -V_0 a$

$\left.\frac{\partial V}{\partial T}\right|_p = V_0 b$

$\kappa_T = -\frac{1}{V_0}\left.\frac{\partial V}{\partial p}\right|_T \Rightarrow a = \kappa_T$

$\beta_p = \frac{1}{V_0}\left.\frac{\partial V}{\partial T}\right|_p \Rightarrow b = \beta_p$

$$\Rightarrow \boxed{V = V_0(1 - \kappa_T p + \beta_p T)}$$

T2/11

$\beta_p = \frac{3aT^3}{V} \quad \kappa_T = \frac{b}{V} \quad a, b = \text{const.}: V$

$\beta_p = \frac{1}{V}\left.\frac{\partial V}{\partial T}\right|_p = \frac{3aT^3}{V} \longrightarrow \left.\frac{\partial V}{\partial T}\right|_p = 3aT^3 \quad / \int dT$

$V(p, T) = \frac{3}{4}aT^4 + f(p)$

$$\frac{\partial U}{\partial p}\Big|_T = -b = \frac{df(p)}{dp} \quad \Big/ \int dp$$

$$f(p) = -bp + U_0$$
↳ csak konst.

$$\boxed{U(p,T) = \frac{3}{4}aT^4 - bp + U_0}$$

$\boxed{\text{T2/12}}$

$[c_p - c_V](T, V, \beta_p)$ I. főt. $\delta Q = dU + \cancel{d}p\,dV$

$$U = c_V n T - \frac{n^2 a}{V}$$

$$dU = \frac{\partial U}{\partial T}\Big|_V dT + \frac{\partial U}{\partial V}\Big|_T dV$$

$$\delta Q = \left[\frac{\partial U}{\partial V}\Big|_T + p\right] dV + \underbrace{\frac{\partial U}{\partial T}\Big|_V}_{c_V} dT \quad \Big/ \frac{\partial}{\partial T} \leftarrow c = \frac{1}{n}\frac{\delta Q}{dT}$$

$$c_p - c_V = \frac{1}{n}\left[\frac{\partial U}{\partial V}\Big|_T + p\right]\frac{\partial V}{\partial T}\Big|_p$$

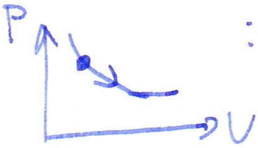
$$c_p - c_V = \frac{1}{n}\left[\frac{n^2 a}{V^2} + p\right] V\beta_p$$

$$\to \frac{\partial U}{\partial V}\Big|_T = \frac{n^2 a}{V^2}$$

$$\left(p + \frac{n^2 a}{V^2}\right)(V - nb) = nRT$$

$$\boxed{c_p - c_V = \frac{V\beta_p}{n}\,\frac{nRT}{V - nb}}$$

↝ állapotváltozók



izoT: $\Delta T = 0$ $\quad pV = nRT = \text{áll.}$

$U(T) = \text{áll}$ $\quad \Delta U = 0 = n c_V \Delta T \Rightarrow \boxed{\Delta Q = -\Delta W_{mi}}$ → mi végezzük

$$\boxed{W_{mi} = -\int_{V_1}^{V_2} p\,dV = -nRT\int_{V_1}^{V_2}\frac{dV}{V} = -nRT\ln\frac{V_2}{V_1}}$$

$$\Delta Q_{fel} = nRT\ln\frac{V_2}{V_1}$$

$$W_g = nRT\ln\frac{V_2}{V_1}$$

izoV: $dV = 0 \Rightarrow \boxed{W = 0}$

$\Delta U = \Delta Q$

$\Delta Q = U_2 - U_1 = c_V n (T_2 - T_1) = \frac{c_V}{R} V_1 (p_2 - p_1) = \Delta U$

izop: $p = \text{áll.}$

$\boxed{\Delta W_{mi}} = -p(V_2 - V_1) = p(V_1 - V_2)$

$\Delta Q = \underbrace{U_2 - U_1}_{} + p(V_2 - V_1) = n c_p \Delta T$

$\Delta U = n c_V \Delta T$

$\boxed{\Delta W_{mi} = p(V_1 - V_2)}$. $\boxed{\Delta Q = p \frac{c_p}{R} (V_2 - V_1)}$

$\boxed{\Delta U = p \frac{c_V}{R} (V_2 - V_1)}$

adiab: $\delta Q = 0 \rightarrow \Delta Q = 0$

$\Delta W = \Delta U$

$pV^\gamma = \text{cost} := k \mapsto p_i = \frac{k_i}{V_i^\gamma}$

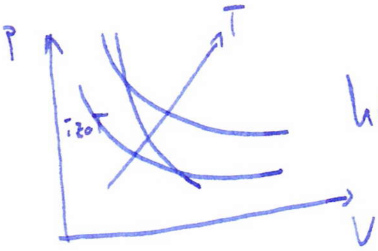
$W_{mi} = -\int_{V_1}^{V_2} p\,dV = -\int_{V_1}^{V_2} \frac{k_i}{V^\gamma} dV = -\frac{k_2 V_2^{1-\gamma} - k_1 V_1^{1-\gamma}}{1-\gamma}$

$\boxed{W_{mi} = -\frac{p_2 V_2 - p_1 V_1}{1-\gamma}}$

Á.V.	ΔQ	ΔU	ΔW_{mi}
$p = \text{const}$	$p_1 \frac{c_p}{R} (V_2 - V_1)$	$p_1 \frac{c_V}{R} (V_2 - V_1)$	$p_1 (V_1 - V_2)$
$V = \text{const}$	$V_1 \frac{c_V}{R} (p_2 - p_1)$	$V_1 \frac{c_V}{R} (p_2 - p_1)$	0
$T = \text{const}$	$nRT \ln \frac{V_2}{V_1}$	0	$-nRT \ln \frac{V_2}{V_1}$
$\delta Q = 0$	0	$-\frac{p_1 V_1 - p_2 V_2}{\gamma - 1}$	$-\frac{p_1 V_1 - p_2 V_2}{\gamma - 1}$

T3/1 1 mol id. gáz $V=\frac{k}{\sqrt{p}}$ - módon váltjuk a

$\rightsquigarrow p=\frac{k^2}{V^2}$



hűl!

$c=?$ $\quad c=\frac{1}{n}\frac{\delta Q}{dT}$

$\delta Q = dU + p\,dV$

$\begin{cases}\delta Q = n c\, dT \\ dU = n c_v\, dT\end{cases}$

$$dU=\frac{\partial U}{\partial V}\Big|_T dV+\frac{\partial U}{\partial T}\Big|_V dT$$

$$\delta Q=\frac{\partial U}{\partial T}\Big|_V dT+\left[\frac{\partial U}{\partial V}\Big|_T+p\right]dV$$

$$n c\,dT = n c_v\,dT + \left[\cancel{\frac{\partial U}{\partial V}\Big|_T}+p\right]dV$$

id.g. $\frac{\partial U}{\partial V}\Big|_T=0$

$$n c\,dT = n c_v\,dT + p\,dV \longrightarrow n c\,dT=\left[n c_v + p\frac{dV}{dT}\Big|_?\right]\cdot dT$$

$dV=\frac{dV}{dT}dT$

$pV=nRT \rightsquigarrow V=\frac{nRT}{p}$

$V=\frac{k}{\sqrt{p}} \quad p=\frac{k^2}{V^2}$

$$V=\frac{nRT}{k^2}V^2 \qquad \boxed{V=\frac{k^2}{nRT}}$$

$$\frac{dV}{dT}=-\frac{k^2}{nRT^2}=-\frac{V}{T}$$

$$c\cancel{n}\,dT = c_v\cancel{n}\,dT-p\frac{V}{T}dT$$

$\hookrightarrow \frac{\cancel{n}R\cancel{T}}{\cancel{T}}$

$\Downarrow$

$$\boxed{c=c_v-R}$$

T3/2 $c=$áll. c_p, c_v, V

előzőből: $n c\,dT=\left[n c_v + p\frac{dV}{dT}\Big|_{\text{á.v.}}\right]dT$

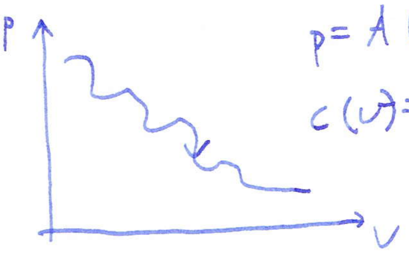
$$c\cancel{n}\,dT = c_v\cancel{n}\,dT+\frac{\cancel{n}RT}{V}\frac{dV}{dT}\Big|_{\text{á.v.}}dT$$

$$\frac{c-c_v}{R}\frac{dT}{T}=\frac{dV}{V} \quad /\int$$

$$\frac{c-c_v}{R}\ln T=\ln V+\ln k \qquad T=k\cdot V^{\frac{R}{c-c_v}} \cdot \quad \frac{pV}{nR}=T \qquad \frac{c-c_v}{R}\ln\frac{pV}{nR}=\ln V+\ln k$$

$$p=\underbrace{k V^{\frac{R}{c-c_v}-1} nR}_{k'}=k'\cdot V^{\frac{R}{c-c_v}-1}=k' V^{\frac{c_p-c_v}{c-c_v}}$$

$\boxed{T3/3}$



$p = f(V)$

$c(V) = c_V + R \dfrac{f(V)}{f(V) + V\dfrac{df}{dV}}$

$c(V) = c_V + R \dfrac{f(V)}{f(V) + V\dfrac{df}{dV}}$

jó-e erre előző feladat?

$p = \dfrac{k^2}{V^2}$

$\dfrac{dp}{dV} = -\dfrac{2k^2}{V^3}$

$\Rightarrow c = c_V + R \dfrac{\frac{k^2}{V^2}}{\underbrace{\frac{k^2}{V^2} - V\frac{2k^2}{V^3}}_{-1}} = c_V - R$

Tehát $c_{..}$-re jó

$\rightarrow$ korábbról

$c = c_V + \dfrac{1}{n} p \dfrac{dV}{dT}\Big|_{á.v.}$

$f(V)V = pV = nRT \longrightarrow T = \dfrac{f(V)V}{nR}$

$\dfrac{dT}{dV} = \left(\dfrac{dV}{dT}\right)^{-1} \qquad \dfrac{dT}{dV} = \dfrac{1}{nR}\left(V f'(V) + f(V)\right)$

$$\boxed{\dfrac{dV}{dT}\Big|_{á.v.} = \dfrac{nR}{V \cdot f'(V) + f(V)}}$$

$c = c_V + \dfrac{f(V)}{n} \dfrac{nR}{\dfrac{df}{dV} V + f(V)}$

$$\boxed{c(V) = c_V + R \dfrac{f(V)}{f(V) + V\dfrac{df}{dV}}}$$

$\square$

(10.12.)

$$c(V) = c_v + \frac{f(V)R}{\frac{df}{dV}V + f(V)} \quad \text{látszik}$$

b, p_m, V_m $\quad p = a - bV$ ált.

$T = max$

$$pV = nRT$$
$$\shortparallel$$
$$f(V)V$$

$$T(V) = \frac{pV}{nR} = \frac{f(V)V}{nR} = \frac{(a-bV)V}{nR}$$

$$\left.\frac{\partial T}{\partial V}\right|_{V_m} = 0 = \frac{a-2bV}{nR} \Rightarrow \boxed{V_m = \frac{a}{2b}}$$

$$p_m = p(V_m) = a - b\frac{a}{2b} = \boxed{\frac{a}{2}}$$

$\boxed{T3/2}$

$$c(V) = c_v + \frac{f(V)R}{\frac{df}{dV}V + f(V)} = const$$

$$c(V) = c_v + \frac{R}{1 + \underbrace{\frac{V}{p}\frac{dp}{dV}}_{const := A}} = const$$

$$\frac{V}{p}\frac{dp}{dV} = A$$

$$\frac{dp}{p} = A\frac{dV}{V} \quad /\cdot\int$$

$$\ln\frac{p}{p_0} = A\ln\frac{V}{V_0}$$

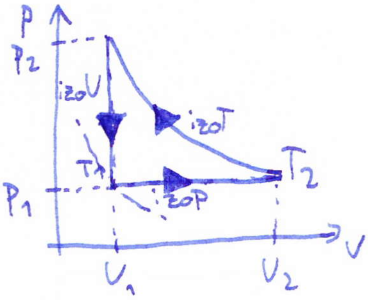
$$\boxed{p = p_0\left(\frac{V}{V_0}\right)^A}$$

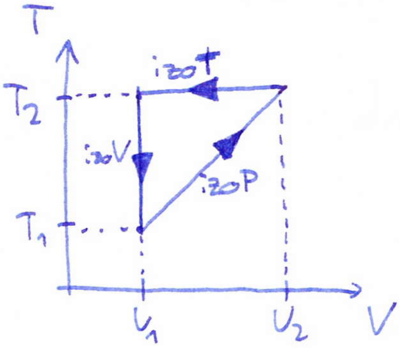
$$c(V) = c_v + \frac{R}{1+A}$$

$$A = \frac{R}{c - c_v} - 1 = \frac{c_p - c}{c - c_v}$$

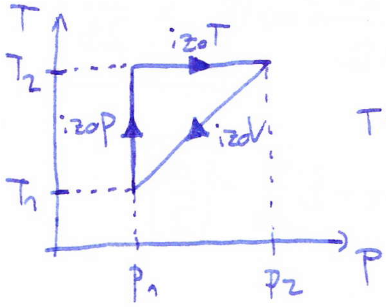
$$\boxed{p = p_0\left(\frac{V}{V_0}\right)^{\frac{c_p - c}{c - c_v}}}$$

T3/4-5

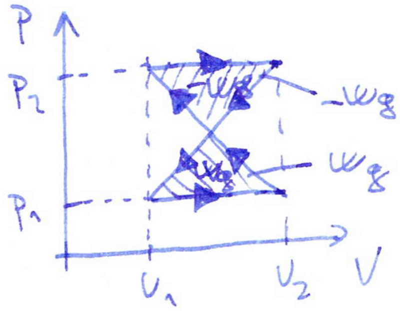




$$T = \frac{p}{nR} V$$



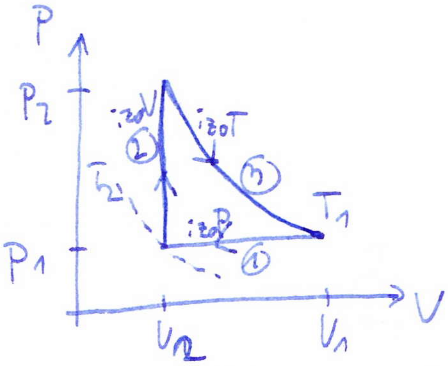
$$T = \frac{V}{nR} p$$



$\delta W_g = p\,dV$

$\Delta W = 0$

T3/6



a, $\Delta U_1 = ?$

b, $\Delta Q_2 = ?$

c, $\Delta W_g, \Delta Q_l = ?$

a, $\boxed{\Delta U_1} = \frac{f}{2} nR(T_2 - T_1) = \frac{f}{2} p_1 (V_2 - V_1) = \boxed{\frac{p_1 (V_2 - V_1)}{\gamma - 1}}$

b,

2, $dV = 0 \quad \Delta W_2 = 0$

$\Delta U_2 = \Delta Q_2$

$\Delta U_2 = -\Delta U_1 = \Delta Q_2$

c, $\Delta U = \oint dU = 0 \quad \oint \delta Q = \oint \delta W = \Delta W_1 + \Delta W_2 + \Delta W_3 = \boxed{p_1 (V_2 - V_1) + p_1 V_1 \ln \frac{V_1}{V_2}}$

$\Delta W_{1g} = p_1 (V_2 - V_1)$

$\Delta W_{2g} = 0$

$pV = nRT \rightsquigarrow p = \frac{nRT}{V}$

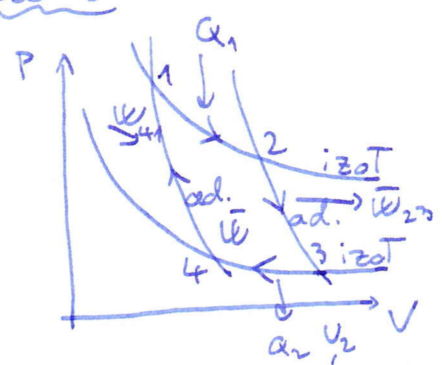
$\Delta W_{3g} = \int_{V_2}^{V_1} p\,dV = nRT_1 \int_{V_2}^{V_1} \frac{1}{V} dV = nRT_1 \ln \frac{V_1}{V_2} = \cancel{nR} \frac{p_1 V_1}{\cancel{nR}} \ln \frac{V_1}{V_2}$

Kis.Fiz.3.Gy.-

Hatásfok

Reverzibilis Carnot

$dU=0$



1 mol id.g.

$\bar{W}$ => gáz végzi

W => mi végezzük

1→2: $Q_{12} := Q_1 = -W_{12} = \int_{V_1} p(V)dV = RT_1 \ln\frac{V_2}{V_1} > 0$ hőfelv. $Q_{be} = Q_{fel}$

2→3: $Q_{23} = 0$ $\quad W_{23} = U_3 - U_2 = c_V(\underbrace{T_3 - T_2}_{<0})$ $\Rightarrow \bar{W}_{23} > 0$

3→4: $Q_2 := Q_{34} = -W_{34} = RT_2 \ln\frac{V_4}{V_3} < 0$ hőlead. $(-Q_2 = Q_{le} = Q_{ki})$

4→1: $Q_{41} = 0$

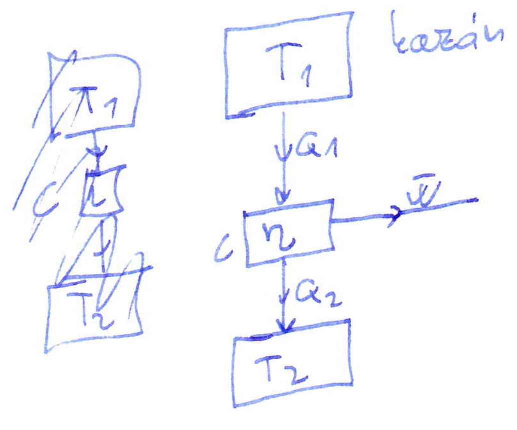
$W_{41} = c_V(T_2 - T_3) > 0$

$\bar{W} = -(W_{12} + \cancel{W_{23}} + W_{34} + \cancel{W_{41}})$

$$\boxed{\bar{W} = RT_1 \ln\frac{V_2}{V_1} + RT_2 \ln\frac{V_4}{V_3}} > 0 \qquad \bar{W} = Q_1 + Q_2$$

$\Downarrow$

$\Rightarrow$ gáz hőfelv. árán munkát végez



$$\eta := \frac{\bar{W}}{Q_1} = \frac{\bar{W}_{gáz}}{Q_{fel}} \leq \frac{Q_1 + Q_2}{Q_1} = 1 + \frac{Q_2}{Q_1}$$

$$\frac{Q_2}{Q_1} = \frac{RT_2 \ln\frac{V_4}{V_3}}{RT_1 \ln\frac{V_2}{V_1}} = -\frac{T_2}{T_1} \Rightarrow 0 = \frac{Q_1}{T_1} + \frac{Q_2}{T_2}$$

red. hő

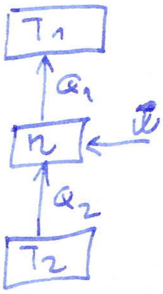
$$\eta_c = 1 - \frac{T_2}{T_1} = \boxed{1 - \frac{T_{kisebb}}{T_{nagyobb}} = \eta_c}$$

$0 = \frac{Q_1}{T_1} + \frac{Q_2}{T_2}$

II. főt. (Planck)

$\eta < 1$

hőszivattyú / hűtőgép



Teljesítménytényező:

$$k_{h.sz.} = \frac{|Q_1^f|}{W} = \frac{1}{\eta} \qquad Q_1^f = -Q_1$$

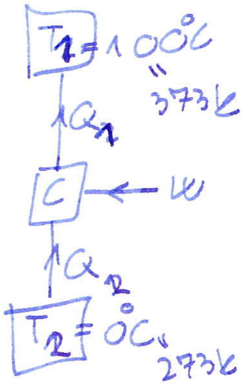
általánosan:

$$\boxed{\eta = 1 - \frac{Q_{le}}{Q_{fel}} = 1 + \frac{Q_2}{Q_1}}$$

$$k_{h.g.} = \frac{Q_2}{W} = \frac{T_2}{T_1 - T_2}$$

Carnot: fgtl. az anyagi minőségtől

$\boxed{T3/7}$



$T_1 = 100°C = 373K$, $T_2 = 0°C = 273K$

$L_f = 2{,}25 \cdot 10^6 \frac{J}{kg}$ $\quad m_g = 1kg$ gőz

$L_o = 3{,}35 \cdot 10^5 \frac{J}{kg}$ $\quad m = ?$

$$\eta_c = 1 - \frac{T_2}{T_1} \qquad \eta = \frac{\Delta W}{Q_1} = \frac{Q_1 - |Q_2|}{Q_1} = 1 - \frac{|Q_2|}{Q_1}$$

$$\Rightarrow \frac{|Q_2|}{Q_1} = \frac{T_2}{T_1} \qquad Q_1 = L_f \cdot m_g \qquad Q_2 = L_o \cdot m_j$$

$$\frac{|Q_2|}{Q_1} = \frac{T_2}{T_1} = \frac{L_o m_j}{L_f m_g} \Rightarrow m_j = \frac{T_2 L_f m_g}{T_1 L_o} \cong \boxed{4{,}92\,kg}$$

! KELVIN !

$$\Delta W = \eta Q_1 = \left(1 - \frac{T_2}{T_1}\right) L_f m_g \approx \boxed{603\,kJ}$$

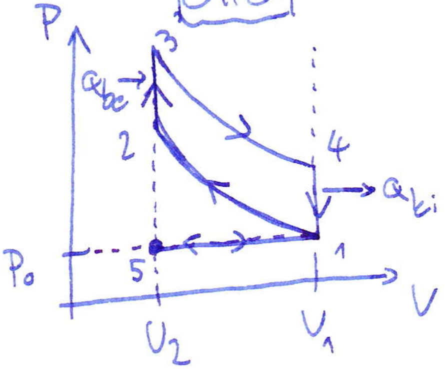
$1kWh = 50Ft$

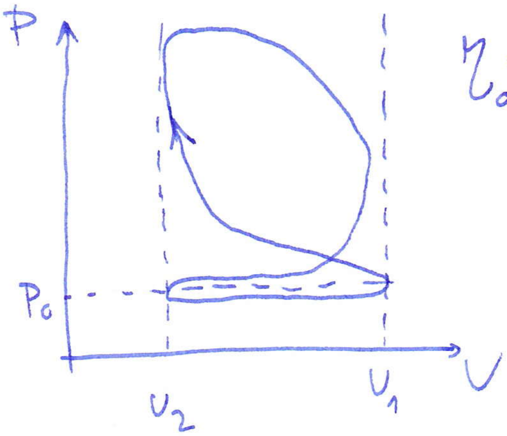
$[W] = \left[\frac{J}{s}\right] \qquad J = W \cdot s \qquad Wh = 3600\,Ws = 3600\,J$

$1kWh = 3600\,kJ \longrightarrow$ $\boxed{9\,Ft}$

Kis.Fiz.3.Gy.-

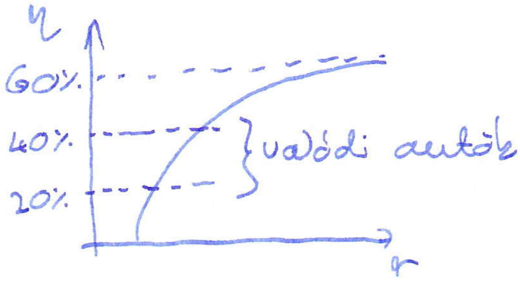
Otto:



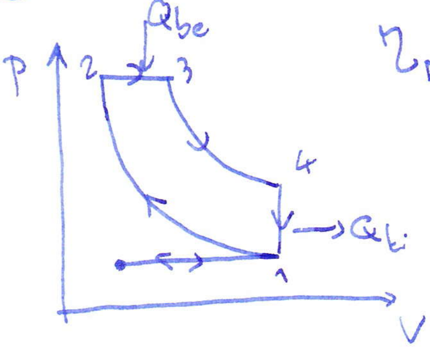


$$\eta_O = 1 - \frac{1}{(V_1/V_2)^{\gamma-1}} = 1 - \frac{1}{r^{\gamma-1}}$$

anyagi paraméter =>
=> oktánszám



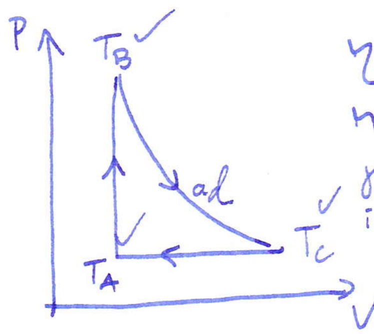
DIESEL:



$$\eta_D = 1 - \frac{T_1}{\gamma T_2} \frac{(T_4/T_1 - 1)}{(T_3/T_2 - 1)}$$

(10.19.)

T3/8



$\eta = ?$

$n = 1$ mol

γ ✓

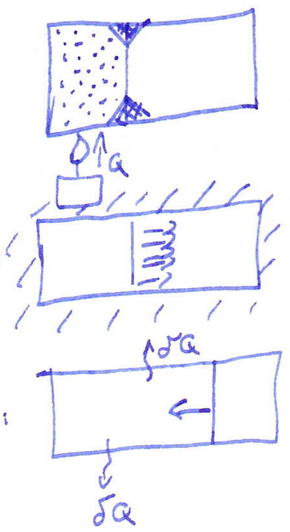
id. g.

$$\eta = \frac{W}{Q_{fel}} = \frac{Q_{fel} - |Q_{le}|}{Q_{fel}} = 1 - \frac{|Q_{le}|}{Q_{fel}}$$

A→B:

B→C:

C→A:



$$|Q_{le}| = n\cancel{c}_p (T_C - T_A)$$

$$Q_{fel} = n c_V (T_B - T_A)$$

$$\frac{c_p}{c_V} = \gamma \Rightarrow \eta = 1 - \gamma \frac{T_C - T_A}{T_B - T_A}$$

$T_C = ?$

$$T_C \Rightarrow T_C V_C^{\gamma-1} = T_B V_B^{\gamma-1} \longrightarrow \boxed{T_C = T_B \left(\frac{V_B}{V_C}\right)^{\gamma-1}} = T_B \left(\frac{T_A}{T_C}\right)^{\gamma-1} \Rightarrow T_C = \left(T_B T_A^{\gamma-1}\right)^{\frac{1}{\gamma}}$$

$V_C = ?$

$$\frac{V_B}{V_C} = \frac{T_A}{T_C}$$

$$\eta = 1 - \gamma \frac{\left(T_B T_A^{\gamma-1}\right)^{\frac{1}{\gamma}} - T_A}{T_B - T_A}$$

T3/9 n mol, id. g.

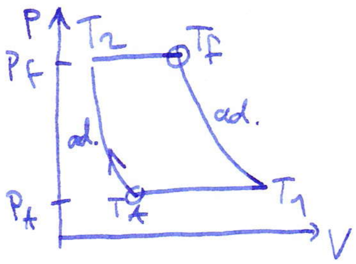
$\eta = ?$

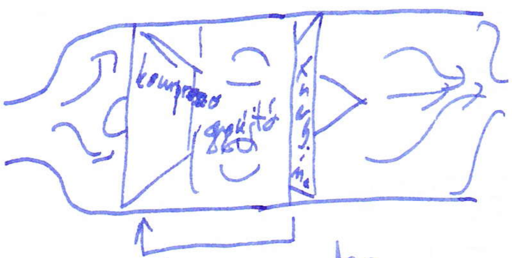
$\eta(f) = ?$

max $\eta = ?$

Joule ciklus

Brayton —"—





W a comp.-ba

$$\eta = 1 - \frac{|Q_{le}|}{Q_{fel}} = 1 - \frac{\cancel{c}_p (T_1 - T_A)}{\cancel{c}_p (T_f - T_2)}$$

$T_1, T_2 = ?$

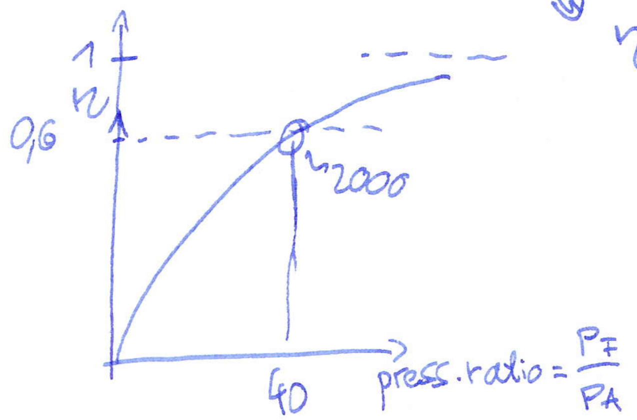
$$TV^{\gamma-1}=\text{const} \longrightarrow \underline{p^{\frac{1-\gamma}{\gamma}}T=\text{const}} \qquad \frac{T_1}{T_F}=\left(\frac{p_A}{p_F}\right)^{\frac{1-\gamma}{\gamma}}=:x \qquad \frac{T_2}{T_1}=\frac{1}{x}$$

$$\Longrightarrow \eta=1-\frac{T_F x - T_A}{T_F - T_A/x}=1-x=\boxed{1-\left(\frac{p_A}{p_F}\right)^{\frac{1-\gamma}{\gamma}}}$$

$$\gamma=\frac{f+2}{f} \rightsquigarrow \boxed{\frac{1-\gamma}{\gamma}=\frac{2}{2+f}} \qquad \frac{p_A}{p_F}<1$$

$$\Downarrow$$

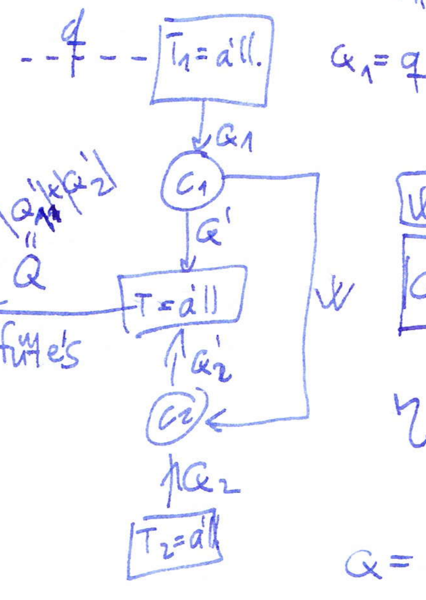
$$\eta\uparrow \Longleftrightarrow f\downarrow$$



úgy kapunk maxot, ha egy atomos $f=3$ gázt veszünk, illetve $\eta\uparrow \Longleftrightarrow \frac{p_A}{p_F}\downarrow$

T3/10

$\cancel{T_1>T_2>T_3}$ $\quad T_1>T>T_2$ $\quad \boxed{\eta_1=\frac{W}{Q_1}=1-\frac{|Q_{le}|}{Q_{fel}}=1-\frac{|Q_1'|}{Q_1}=1-\frac{T}{T_1}}$



$Q_1=q$

$$\boxed{W=Q_1-|Q_1'|=Q_1\left(1-\frac{T}{T_1}\right)}$$

$$\boxed{|Q_1'|=Q_1\frac{T}{T_1}}$$

$$\eta_2=1-\frac{T_2}{T}=\frac{W}{|Q_2'|} \qquad \boxed{|Q_2'|=\frac{W}{1-\frac{T_2}{T}}=\frac{Q_1\left(1-\frac{T}{T_1}\right)}{1-\frac{T_2}{T}}}$$

$$Q=|Q_1'|+|Q_2'|=\ldots=\underbrace{Q_1}_{q}\frac{1-\frac{T_2}{T_1}}{1-\frac{T_2}{T_1}}$$

$$\boxed{Q=Q_1\frac{1-\frac{T_2}{T_1}}{1-\frac{T_2}{T}}}>q \qquad T_2<T<T_1 \qquad \frac{T_2}{T_1}<\frac{T_2}{T}$$

ZH IDÁIG

Kis.Fiz.3.Gy.-

II. Főtétel

Clausius: $\nexists$ olyan folyamat amely pusztán abból állna, hogy a hő a hidegebb helyről a melegebb felé áramlik

$\Rightarrow$ $\exists$ olyan foly.-ok, amelyek az E megm.-nak megfelel, mégsem mennek végbe.

Entrópia: „rendezetlenség mértéke" $\Rightarrow$ egyre több szabadsági fokra ~~oszlik~~ oszlik szét az energia

I. F.T.: E megm. $\Rightarrow$ mi mehet végbe

II. F.T.: $\Rightarrow$ mi fog végbemenni:

S: állapot fgv., extenzív

rev. foly: red. hő = entrópia megvált.-a   hőfelv. $S\uparrow$ hőleadás $S\downarrow$

$$dS = \frac{\delta Q^{rev.}}{T} \quad /\int$$

$$S_A - S_B = \int_B^A \frac{\delta Q^{rev.}}{T}$$

rev. körfoly.: $\boxed{dS = 0}$ $\quad \oint dS = 0 = \oint \frac{\delta Q^{rev.}}{T}$

irrev. foly.: $dS \geq 0$ entrópia produkció

$$\oint \frac{Q^{irrev.}}{T} < 0 \qquad \frac{\delta Q^{irrev}}{T} < dS \qquad \underbrace{\frac{dQ}{T} \leq dS}$$

rev. 0   irrev. >0

$$\Downarrow$$

$$dS = \frac{\delta Q}{T} + dS_{prod.} = \frac{\delta Q}{T} + d\sigma$$

izolált rsz., term. egyensúly $\Leftrightarrow$ S max. értékét

$\rightsquigarrow dS = 0$

$$\boxed{dS = \frac{\delta Q}{T} = \frac{dU - \delta W}{T} = \frac{c_V n\,dT}{T} + \frac{p\,dV}{T}} \Rightarrow \boxed{dU = T\,dS - p\,dV}$$ I. fundamentális egyenlet

$pV = nRT \quad /.d$

$Vdp + dVp = nRdT \quad /:T$

$$\boxed{\frac{dp}{p} + \frac{dV}{V} = \frac{dT}{T}}$$

T4/1

$p_1 = 2 \cdot 10^6\,\mathrm{Pa}$

$T = 27\,°C = \text{áll.}$

$V_1 = 1$ liter $\neq$ áll.

idg

$p_2 = 10^5\,\mathrm{Pa}$

$\Delta S = ?$

$$dS = \frac{\delta Q}{T}$$

$$T\,dS = \delta Q = c_V n\,dT + \frac{nRT}{V}dV \to dS = c_V n\frac{dT}{T} + n\frac{R}{V}dV \quad /\int$$

$$\delta Q = dU + p\,dV$$

$dU = c_V n\,dT$, $p = \frac{nRT}{V}$

$$\boxed{\Delta S = S_2 - S_1 = \underbrace{c_V n \ln\frac{T_2}{T_1}}_{\text{izoT} \Rightarrow 0} + nR\ln\frac{V_2}{V_1}}$$

izoT

$$\Delta S = nR\ln\frac{V_2}{V_1} \rightsquigarrow V_i = \frac{nRT}{p_i}$$

?

$$\Delta S = nR\ln\frac{p_1}{p_2}\,;\quad n = \frac{p_1V_1}{RT}$$

$$\boxed{\Delta S = \frac{p_1V_1}{T}\ln\frac{p_1}{p_2}} \approx 20\,\frac{\mathrm{J}}{\mathrm{K}}$$

$\boxed{T4/2}$ $m = 1\,g$ $N_2 \Rightarrow f = 5$

$\Delta S = ?$

izop $\Rightarrow p = \text{áll.}$

$V_1 = 1$ liter

$V_2 = 5$ liter

$p = p_1 = p_2$

$T_i = \frac{p}{nR} V_i$

$\frac{T_2}{T_1} = \frac{V_2}{V_1}$

$$dS = nc_v \frac{dT}{T} + nR\frac{dV}{V}$$

$$S_2 - S_1 = nc_v \ln\frac{T_2}{T_1} + nR\ln\frac{V_2}{V_1} = \underbrace{(nc_v + nR)}_{nc_p} \ln\frac{V_2}{V_1} = \boxed{nc_p \ln\frac{V_2}{V_1}} = 3{,}34\,\frac{J}{K}$$

$$\left.\begin{aligned} \frac{c_p}{c_v} &= \gamma \sim f \\ c_p - c_v &= R \end{aligned}\right\} \quad c_p = \frac{f+2}{2}R = \frac{7}{2}R \qquad m \Rightarrow n \qquad M_{N_2} = 14\,\frac{g}{mol}$$

$\boxed{T4/3}$ m, M, γ, idg.

a, $S(T,V) = ?$

b, ad $\quad \delta Q = 0 \Leftrightarrow dS = 0 \Rightarrow S = S_0$

$c_p - c_v = R$

$c_p = R + c_v = \gamma c_v$

$c_v = \frac{R}{\gamma - 1}$

$$dS = nc_v \frac{dT}{T} + nR\frac{dV}{V} \qquad \int_{\alpha_0}^{\alpha_x}$$

$$\boxed{S = S_0 + nR\ln\left[\left(\frac{T}{T_0}\right)^{\frac{1}{\gamma-1}}\frac{V}{V_0}\right]}$$

$\Downarrow$

$\ln[\;] = 0$, $[\;] = 1$

$$S - S_0 = nc_v \ln\frac{T}{T_0} + nR\ln\frac{V}{V_0}$$

adiabatára $\left(\frac{T}{T_0}\right)^{\frac{1}{\gamma-1}} \frac{V}{V_0} = 1$

$\Downarrow$

$$\boxed{TV^{\gamma-1} = \text{const}}$$

$\boxed{T4/4}$ $S(T,V) = nc_v \ln T + nR\ln V + S_0$

a, $S_0(n)$ $\quad$ S extenzív $\quad dU = \text{int} * d(\text{ext})$

b, $S_0(n) = ?$

$S = \sum S_i = n\bar{S}_i \qquad \bar{S}_i = \frac{1}{n}\sum_i S_i$

iterózsa.

független n-től ✓

$$S(T,V,n) = n\Big(c_v \ln T + R\ln \underbrace{\frac{V}{n}}_{V_M} + \underbrace{R\ln n + \frac{S_0(n)}{n}}_{s_0}\Big) = n\left(c_v \ln T + R\ln V_M + s_0\right)$$

$$\boxed{S_0(n) = -nR\ln n + ns_0}$$

$\boxed{T4/5}$ (10.26.)

$\cancel{\frac{T_2}{T_1}}$ $\frac{T_2}{T_1}=\frac{m_g L_f}{m_j L_o}$

$m=1g$

$T_0=0°C$ víz $\nearrow T_1=100°C$ izop (gőz)

273K 373K

$\Delta S=?$

$dS=\frac{\delta Q}{T}$

melegítés $\boxed{\Delta S_1=\int_{T_0}^{T_1}\frac{\delta Q}{T}=cm\int_{T_0}^{T_1}\frac{dT}{T}=cm\ln\frac{T_1}{T_0}}$

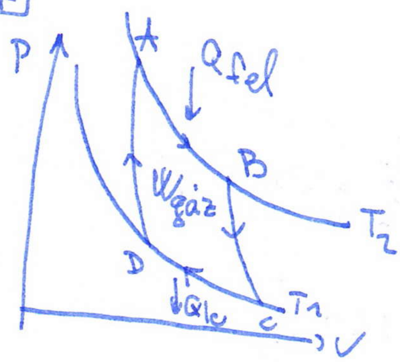
$\delta Q=cm\,dT$

$c=4{,}18\frac{J}{kgK}$

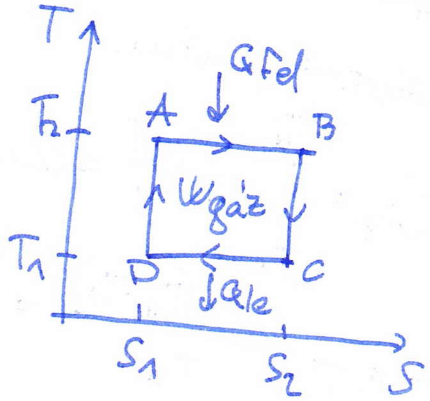
gőzölés $\boxed{\Delta S_2=\int\frac{\delta Q}{T}=\frac{L_f m}{T_1}}$ $\Delta S=\Delta S_1+\Delta S_2$

$\delta Q=L_f m$

$\boxed{T4/17}$



$\Rightarrow$



ad: $\delta Q=0\rightarrow dS=0$

$\delta Q=dU+pdV$ $\oint\delta Q=\oint dU+\oint pdV$

$dS=\frac{\delta Q}{T}\Rightarrow\delta Q=TdS$

körfoly. $\oint dU=0$

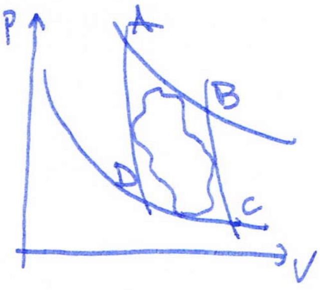
$\oint TdS=\underbrace{\oint pdV}_{W_{gáz}}$

$=1-\frac{|Q_{fel}|}{Q_{le}}=1-\frac{T_1\Delta S}{T_2\Delta S}=1-\frac{T_1}{T_2}$

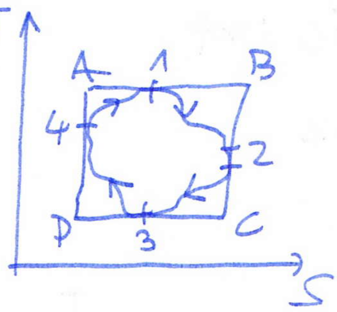
kis T_1

$$\cancel{\eta = 1 - \frac{|Q_{le}|}{Q_{fel}}}$$

amorfra:



$\Rightarrow$



$$\int_4^1 \underset{0}{\underset{\vee}{TdS}} + \int_1^2 \underset{0}{\underset{\vee}{TdS}} > 0 \quad \text{hőfelv.}$$

$\longrightarrow$ hő hőm. $\Rightarrow$ nő S

$$\int_2^3 \underset{0}{\underset{\wedge}{TdS}} + \int_3^4 \underset{0}{\underset{\wedge}{Tds}} < 0 \quad \text{hőleadás}$$

$\longleftarrow$ hő csökk $\Rightarrow$ S csökk.

$\boxed{T4/6}$ $m = 1kg$

$T_0 = 273K$ víz

$T_H = 373K$ hőtart. $\rightarrow$ áll. hőm.

a, $dS = \frac{\delta Q}{T}$ $\quad \Delta S_{víz} = \int_{T_0}^{T_H} cm \frac{dT}{T} = cm \ln \frac{T_H}{T_0} = \boxed{1306 \frac{J}{K}}$

b, $\Delta S_{tart.} = \frac{\Delta Q}{T_H} = \underbrace{-}_{\text{hőleadás}} \frac{cm(T_H - T_0)}{T_H} = \boxed{-1121 \frac{J}{K}}$

c) teljes entrópia változás

$$\Delta S = \Delta S_{víz} + \Delta S_{tart.} = cm\left[\ln\left(\frac{T_H}{T_0}\right) + \frac{T_0}{T_H} - 1\right] = \boxed{185 \frac{J}{K}} > 0$$

d, $T_i = 323K$ $\Rightarrow$ $\Delta S_{víz} = \int_{T_0}^{T_i} \frac{\delta Q}{T} + \int_{T_i}^{T_H} \frac{\delta Q}{T} = \left(\int_{T_0}^{T_i} + \int_{T_i}^{T_H}\right) \frac{\delta Q}{T} = \int^{T_H} \frac{\delta Q}{T} = \boxed{cm \ln \frac{T_H}{T_0}}$

T_H

mindegy hány lépés

$\Delta S_{tart}^{(1)} = \frac{\Delta Q^{(1)}}{T_i} = \frac{-cm(T_i - T_0)}{T_i}$

$\Delta S_{tart}^{(2)} = \frac{\Delta Q^{(2)}}{T_H} = \frac{-cm(T_H - T_i)}{T_H}$

$\Delta S'_{tart} = cm\left(\frac{T_0}{T_i} + \frac{T_i}{T_H} - 2\right) = \boxed{-1208 \frac{J}{K}}$ $\nearrow$ $\boxed{\Delta S'' = 98 \frac{J}{K}}$

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$$\Delta S'_{tart} = cm\left(\frac{T_0}{T_H} - 1 - \left[1+\frac{T_0}{T_H} - \frac{T_0}{T_i} - \frac{T_i}{T_H}\right]\right) = \Delta S_{tart} - cm\underbrace{\left[1+\frac{T_0}{T_H}-\frac{T_0}{T_i}-\frac{T_i}{T_H}\right]}_{1+\frac{T_0}{T_H} - \frac{T_0T_H - T_i^2}{T_iT_H}}$$

$$\left.\frac{\partial \Delta S'_{tart}}{\partial T_i}\right|_{min} = 0 = \left[+\frac{T_0}{T_i^2} - \frac{1}{T_H}\right]\cancel{(cm)}$$

$$\frac{T_0}{T_i^2} = \frac{1}{T_H} \rightarrow \boxed{T_i^* = \sqrt{T_0 T_H}}$$

e, N tartály

$\Delta T = \frac{T_H - T_0}{N}$, $N \to \infty$, $\Delta T \to 0$

$$\Delta S_{víz} = \int_{T_0}^{T_H} \frac{\delta Q}{T} = cm \ln\frac{T_H}{T_0}$$

$$\Delta S''_{tart} = -\lim_{N\to\infty}\left(cm\left[\frac{\Delta T}{T_1} + \frac{\Delta T}{T_2} + \ldots + \frac{\Delta T}{T_N}\right]\right) = -cm\lim_{N\to\infty}\sum_{i=1}^{N}\frac{\Delta T}{T_i} =$$

$\sum \to \int$; $\Delta T \to dT$ ha $N\to\infty$; $\Delta T \to 0$

$$= -cm\int_{T_0}^{T_H}\frac{dT}{T} = -\Delta S_{víz} \qquad \Delta S_{tot} = 0$$

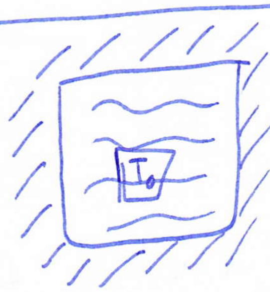
T4/9

$m_1 = 0{,}2\,kg$, $T_1 = 373\,K$ } Fe

$m_2 = 0{,}5\,kg$, $T_2 = 285\,K$ } H_2O

$c_1 = 0{,}46\,\frac{J}{gK}$

$c_2 = 4{,}18\,\frac{J}{gK}$



$$c_1 m_1 (T_1 - T_k) = c_2 m_2 (T_k - T_2)$$

$$c_1 m_1 T_1 - c_1 m_1 T_k = c_2 m_2 T_k - c_2 m_2 T_2$$

$$c_1 m_1 T_1 + c_2 m_2 T_2 = T_k (c_1 m_1 + c_2 m_2)$$

$$T_k = \frac{c_1 m_1 T_1 + c_2 m_2 T_2}{c_1 m_1 + c_2 m_2} = \boxed{288{,}7\,K}$$

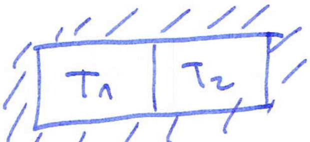
$$\Delta S = \Delta S_{Fe} + \Delta S_{H_2O} = \int_{T_1}^{T_k}\frac{c_1 m_1}{T}dT + \int_{T_2}^{T_k}\frac{c_2 m_2}{T}dT =$$

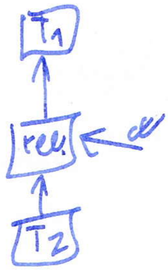
$$= c_1 m_1 \ln\frac{T_k}{T_1} + c_2 m_2 \ln\frac{T_k}{T_2} = 3.41\,\frac{J}{K}$$

$\boxed{T4/10}$ $c = 100\ \frac{J}{K}$

$T_1 = 273\,K$

$T_2 = 373\,K$

a,  $T_k = ?$

b,  $T_k' = ?$

$$\Delta U_b = \Delta U_{b,1} + \Delta U_{b,2} = c(T_k' - T_1) + c(T_k' - T_2) = 2c\left(\sqrt{T_1T_2} - \frac{T_1+T_2}{2}\right) \leq 0$$

a, $c(T_k - T_1) = c(T_2 - T_k)$

$$T_k = \frac{cT_1 + cT_2}{2c} = \boxed{\frac{T_1+T_2}{2}}$$

c, $\Delta U_i = ?$

$\Delta S_i = ?$

$\Delta U = 0$

$c(T_k - T_1) = c(T_2 - T_k)$

b, rev. $\Delta S = 0$

$$dS = \frac{\delta Q}{T} \quad \delta Q = dU - \delta W_{mi} = dU + p\,dV \quad dU = c_v\,dT$$

$$dS = c_v \frac{dT}{T} + \frac{p\,dV}{T} \quad / \int$$

$dV = 0$

$$\Delta S_i = c_v \ln \frac{T_i^{(vég)}}{T_i^{(kezd)}}$$

$i = 1,2$

$T_i^{(vég)} = T_k'$; $T_i^{(kezd)}$: T_1, T_2

$$\Delta S = 0 = \Delta S_1 + \Delta S_2 = c \ln \frac{T_k'}{T_1} + c \ln \frac{T_k'}{T_2} = c \ln \underbrace{\frac{T_k'^2}{T_1T_2}}_{1}$$

$$\boxed{T_k' = \sqrt{T_1T_2}}$$

a, $\Delta U_a = 0 = \Delta U_{a,1} + \Delta U_{a,2} \Rightarrow \Delta U_{a,1} = -\Delta U_{a,2}$ $\quad \Delta U_{a,1} = c_v(T_k - T_1) =$

$$= c_v\left(\frac{T_1+T_2}{2} - T_1\right) = c_v\left(\frac{T_2 - T_1}{2}\right)$$

$$\Delta S_a = \Delta S_{a,1} + \Delta S_{a,2}$$

$$\Delta S_{a,1} = \int \frac{\delta Q}{T} = c_v \int_{T_1}^{T_2} \frac{dT}{T} = c_v \ln \frac{T_k}{T_1}$$

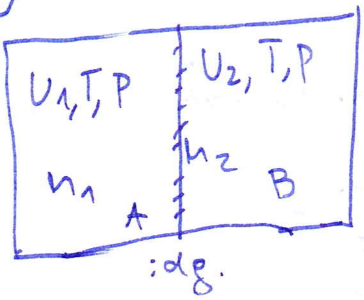
$$\Delta S_{a,2} = c_v \ln \frac{T_k}{T_2}$$

$$\Delta S_a = c_v \ln \frac{T_k^2}{T_1T_2} = 2c_v \ln \frac{T_k}{\sqrt{T_1T_2}} = 2c_v \ln \frac{\frac{T_1+T_2}{2}}{\sqrt{T_1T_2}} \geq 0$$

Kis.Fiz.3.Gy.-

(11.02.)

T4/8



a, Biz. $T, p \neq$ vált

b, $\Delta S = ?$

c, ΔS, ha $A = B$

jel: $i = 1,2$ tart.

A, B gáz

kezd: $pV_1 = n_A R T_1$; $n_1 = n_A$

$pV_2 = n_B R T_2$; $n_2 = n_B$

T áll., mert izolált a rendszer

a, Dalton-tv.

$$\left.\begin{array}{l} p_A = \dfrac{N_A k_B T}{V_1+V_2} \\[2ex] p_B = \dfrac{N_B k_B T}{V_1+V_2} \end{array}\right\} p^* = p_A + p_B = \frac{N_A k_B T + N_B k_B T}{V_1+V_2} = \frac{pV_1 + pV_2}{V_1+V_2} = p \quad \blacksquare$$

b, $$dS = c_V n \frac{dT}{T} + \frac{p\,dV}{T} \overset{p = \frac{nRT}{V}}{=} c_V n \frac{dT}{T} + \frac{nR\cancel{T}}{\cancel{T}}\frac{dV}{V} \quad / \int$$

$$\Delta S_j = \cancel{c_{V_j} n_j \ln \frac{T_j^{vég}}{T_j^{kezd}}} + n_j R \ln \frac{V_j^{vég}}{V_j^{kezd}}$$

$V_A^{kezd} = V_1$

$V_B^{kezd} = V_2$

$V_A^{vég} = V_B^{vég} = V_1 + V_2$

$$\Delta S = \Delta S_A + \Delta S_B = R\left(n_A \ln \frac{V_1+V_2}{V_1} + n_B \ln \frac{V_1+V_2}{V_2}\right) = R\left(n_A \ln \frac{n_A+n_B}{n_A} + n_B \ln \frac{n_A+n_B}{n_B}\right) \geq 0$$

$V_1 = \dfrac{n_A RT}{p}$ $\quad V_2 = \dfrac{n_B RT}{p}$

$n_A = n_1$

$n_B = n_2$

azaz nem hülyeség $\Delta S \geq 0$

c, $\Delta S = 0$ nagyon sok a részecske, gyakorlatilag mindegy, hogy ott van-e a fal, vagy sem

$\longleftrightarrow$ Gibbs-paradoxon

Gibbs-paradoxon:

Boltzmann: $S = k_B \ln W$

W: adott makroáll.-hoz tartozó mikroáll.-ok száma

legyen 2 golyó, piros, kék 2g.
vagy makro

1.	2.
p	p
p	k
k	p
k	k

4 mikroáll

túlszámolás.
Ha 1. 2. nem
kül., akkor
pk u.a. mint
kp mégis
2x vettem

$W_{kezd} = W_1 \cdot W_2$

$W_{vég} =$

$$\Delta S = k_B \ln \frac{W_{vég}}{W_{kezd}} = k_B \ln \frac{W_{vég}}{W_1 W_2}$$

$$\Delta S = k_B \left(\ln \left(\frac{N_A + N_B}{N_A}\right)^{N_A} \ln \left(\frac{N_A + N_B}{N_B}\right)^{N_B} \right)$$

$$S^* = k_B \ln W^* = k_B \ln \frac{W}{\prod_j N_j!} \quad \text{megkül.-tetlen}$$

$$\Rightarrow \boxed{T4/8} \quad \Delta S^* = k_B \ln \frac{W^*_{vég}}{W^*_{kezd}} = k_B \ln \left(\frac{W_{vég}}{W_{kezd}} \cdot \frac{N_A! N_B!}{N_A! N_B!} \right) = \Delta S$$

azonos gázok: $\Delta S^* = k_B \ln \left(\frac{W_{vég}}{W_{kezd}} \cdot \frac{N_A! N_B!}{\underbrace{(N_A + N_B)!}} \right)$

↳ nem tudjuk melyik honnan jött

Kis.Fiz.3.Gy:

Stirling-formula:

$$\ln N! \approx N \ln N - N$$

$$\Delta S^* = \Delta S(\ldots) = k_B \ln 1 = 0$$

u.o. gázok keverése

$$N_A \ln \frac{N_A}{N_A+N_B} + N_B \ln \frac{N_B}{N_A+N_B}$$

ha nem u.o. gázok keverednek, és $p=\text{const}$ $T=\text{const}$ akkor is változik az S, mert több lesz a mikroállapotok száma $\Rightarrow$ keveredési entrópia

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$$\boxed{\text{FUNDAMENTÁLIS EGYENLETEK}}$$

kémiai pot., Maxwell-rel.

Kémiai pot.

1 részecske kivételéhez szükséges energia: $\mu \rightarrow$ intenzív

ext. párja: N

kontinuum anyag

1 mol kivételéhez....

I. Fund. egy.: $dU = TdS - pdV + \mu dN \rightsquigarrow U(S,V,N) = ?$

$$\mu = \left.\frac{\partial U}{\partial N}\right|_{S,V}$$

U extenzív

Több komponens: $\sum_i \mu_i dN_i$

$$dU = \underbrace{\left.\frac{\partial U}{\partial S}\right|_{V,N}}_{T} dS + \underbrace{\left.\frac{\partial U}{\partial V}\right|_{S,N}}_{-p} dV + \underbrace{\left.\frac{\partial U}{\partial N}\right|_{S,V}}_{\mu} dN$$

Euler-egyenlet: Áll.: $U(S,V,N)$ homogén, elsőrendű fgv.-e a változóinak

homogén Lextenzív, elsőrendű, nincs olyan, hogy pl. 2-t összeadok és 3x-es lesz

$\downarrow$

$$U(\lambda S,\lambda V,\lambda N)=\lambda U(S,V,N) \quad \Big/ \frac{d}{d\lambda}$$

$$\frac{dU}{d\lambda}=\underbrace{\frac{\partial U}{\partial \lambda S}}_{T}\underbrace{\frac{\partial \lambda S}{\partial \lambda}}_{S}+\underbrace{\frac{\partial U}{\partial \lambda V}}_{-p}\underbrace{\frac{\partial \lambda V}{\partial V}}_{V}+\underbrace{\frac{\partial U}{\partial \lambda N}}_{\mu}\underbrace{\frac{\partial \lambda N}{\partial \lambda}}_{N}=U(S,V,N)$$

$$\underbrace{\frac{\partial U}{\partial S}}_{T}=\underbrace{\frac{\partial \lambda U}{\partial \lambda S}}_{T} \qquad U(S,V,N)=TS-pV+\mu N$$

$$dU=TdS+SdT-pdV-Vdp+\mu dN+d\mu N$$

$\to$ int. menny.-ek diff.-jai

Gibbs-Duhem rel.

$$SdT-Vdp+Nd\mu=0$$

$$\Rightarrow d\mu=-\frac{S}{N}dT+\frac{V}{N}dp$$

összefüggnek $\Rightarrow$
$\Rightarrow$ pusztán intenzívvel nem lehet leírni a r.sz.-t, kell min. 1 extenzív is

ξ_i: ext., X_i: int.

:D kanonikusan konjugált :D

$$\left.\boxed{\begin{aligned} U(\xi_i) &= \sum_i X_i \xi_i \\ dU(\xi_i) &= \sum_i X_i \, d\xi_i \end{aligned}}\right\} \text{kiskanonikus sokaság}$$

Legendre-trafó. $\to$ d(int)

Helmholtz szabadenergia: $\left.\boxed{F = U - TS}\right\}$ kanonikus sokaság

$$dF = dU - TdS - SdT = \cancel{TdS} - pdV + \mu dN - \cancel{TdS} - SdT =$$

$$= \boxed{-SdT - pdV + \mu dN = dF}$$

Entalpia: $H = U + pV$

$$dH = dU + dpV + pdV = TdS + Vdp + \mu dN$$

Gibbs: $G = H - TS$

$$dG = dH - TdS - SdT = -SdT + Vdp + \mu dN$$

Termodin.-i pot.: $\left.\begin{aligned} \phi &= U - TS - \mu N \\ d\phi &= -SdT - pdV - Nd\mu \end{aligned}\right\}$ Nagykanonikus sokaság

$\boxed{T5/1}$ $dU = TdS - pdV + \mu dN$

$$\left.\frac{\partial U}{\partial S}\right|_{V,N} = T \qquad \left.\frac{\partial U}{\partial V}\right|_{S,N} = -p \qquad \left.\frac{\partial U}{\partial N}\right|_{V,S} = \mu$$

$\overset{(T,V,N)}{dF} = -SdT - pdV + \mu dN$

$$\left.\frac{\partial F}{\partial T}\right|_{V,N} = -S \qquad \left.\frac{\partial F}{\partial V}\right|_{T,N} = -p \qquad \left.\frac{\partial F}{\partial N}\right|_{T,V} = \mu$$

$\cancel{dG = -SdT}$

$dH = TdS + Vdp + \mu dN$

$$\left.\frac{\partial H}{\partial S}\right|_{p,N} = T \qquad \left.\frac{\partial H}{\partial p}\right|_{S,N} = V \qquad \left.\frac{\partial H}{\partial N}\right|_{S,p} = \mu$$

$dG = -SdT + Vdp + \mu dN$

$$\left.\frac{\partial G}{\partial T}\right|_{p,N} = -S$$

$$\left.\frac{\partial G}{\partial p}\right|_{T,N} = V$$

$$\left.\frac{\partial G}{\partial N}\right|_{T,p} = \mu$$

$\boxed{T5/8}$ $pV = A(T) + B(T)p + C(T)p^2 \longrightarrow V = \frac{A}{p} + B + Cp$

T fix, $p_0 \nearrow p_1$

$\Delta G, \Delta S = ?$ $\quad \left.\frac{\partial G}{\partial p}\right|_{T,N} = V = \frac{A}{p} + B + Cp \quad \Big/ \int_{p_0}^{p_1}$

$\Delta G = ?$ $\boxed{\Delta G = A(T)\ln\frac{p_1}{p_0} + B(T)(p_1 - p_0) + \frac{C(T)}{2}(p_1^2 - p_0^2)}$

Kis.Fiz.3.Gy.-

T5/1

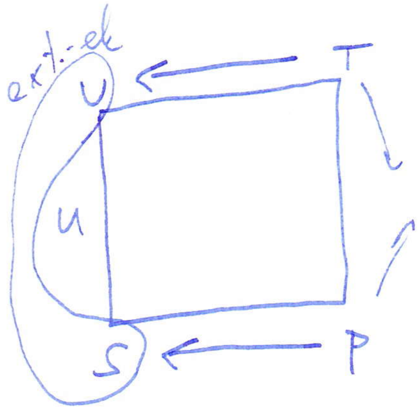
$$-\Delta S = \frac{\partial G}{\partial T}\Big|_{p,N}$$

$$\boxed{\Delta S = \frac{dA(T)}{dT}\ln\left(\frac{p_0}{p_1}\right) + \frac{dB(T)}{dT}(p_0-p_1) + \frac{dC(T)}{dT}\left(p_0^2-p_1^2\right)}$$

T5/2 Mx-rel. $\Longleftrightarrow$ Young-tétel a Fund.e.-re

$$dU = TdS - pdV + \mu dN = \frac{\partial U}{\partial S}\Big|_{V,N} dS + \frac{\partial U}{\partial V}\Big|_{S,N} dV - \left(\frac{\partial U}{\partial N}\Big|_{S,V} dN\right)$$

$$\frac{\partial}{\partial V}\underbrace{\frac{\partial U}{\partial S}}_{T}\Big|_V = \frac{\partial}{\partial S}\underbrace{\frac{\partial U}{\partial V}}_{-p}\Big|_S \Rightarrow \boxed{\frac{\partial T}{\partial V}\Big|_S = -\frac{\partial p}{\partial S}\Big|_V}$$



ezekkel az int.-ekkel

vízszintes nyilak $\times(-1)$

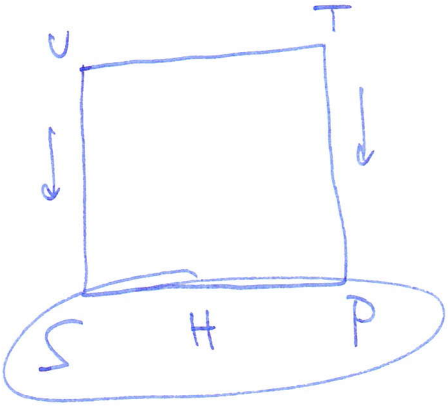
függőleges nyilak $\times(+1)$

ferde nyilak $\times(\sqrt{\pi}\gamma)$

$$dH = TdS + Vdp$$

$$\frac{\partial}{\partial p}\frac{\partial H}{\partial S}\Big|_p = \frac{\partial}{\partial S}\frac{\partial H}{\partial p}\Big|_S$$

$$\boxed{\frac{\partial T}{\partial p}\Big|_S = \frac{\partial V}{\partial S}\Big|_p}$$



T5/1

$$dF = -S\,dT - p\,dV$$

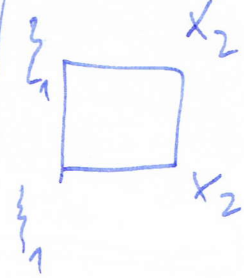
$$\frac{\partial}{\partial V}\underbrace{\frac{\partial F}{\partial T}\Big|_V}_{-S} = \frac{\partial}{\partial T}\underbrace{\frac{\partial F}{\partial V}\Big|_T}_{-p}$$

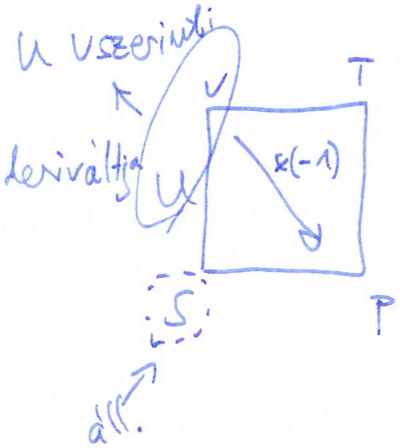
$$\boxed{\frac{\partial S}{\partial V}\Big|_T = \frac{\partial p}{\partial T}\Big|_V}$$

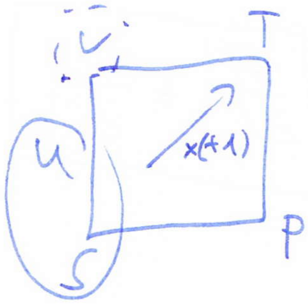
$$dG = -S\,dT + V\,dp \;(+\mu\, dN)$$

$$\frac{\partial}{\partial p}\underbrace{\frac{\partial G}{\partial T}\Big|_p}_{-S} = \frac{\partial}{\partial T}\underbrace{\frac{\partial G}{\partial p}\Big|_T}_{V}$$

$$\boxed{-\frac{\partial S}{\partial p}\Big|_T = \frac{\partial V}{\partial T}\Big|_p}$$







(11.09.)

T5/3

$dU=0 \quad \frac{\partial U}{\partial V}\Big|_T = T\frac{\partial p}{\partial T} - p \qquad dU = \frac{\partial U}{\partial V}\Big|_T dV + \frac{\partial U}{\partial T}\Big|_V dT = dTS - pdV \quad /\,{:}dV|_T$

$$\frac{\partial U}{\partial V}\Big|_T = T\underbrace{\frac{\partial S}{\partial V}\Big|_T} - p \Rightarrow \boxed{\frac{\partial U}{\partial V}\Big|_T = T\frac{\partial p}{\partial T}\Big|_V - p}$$

ez áll, így az a tag is itt van

$-p \to X \qquad dU = \sum_i X_i d\xi_i = TdS + Xd\xi \quad /\,{:}d\xi|_T$

$V \to \xi$

$$\frac{\partial U}{\partial \xi}\Big|_T = T\frac{\partial S}{\partial \xi}\Big|_T + X$$

$$dU = \underbrace{\frac{\partial U}{\partial S}\Big|_\xi}\, dS + \underbrace{\frac{\partial U}{\partial \xi}\Big|_S}\, d\xi$$

$$\frac{\partial}{\partial \xi} = \frac{\partial}{\partial S}$$

$$\frac{\partial S}{\partial \xi}\Big|_T = -\frac{\partial X}{\partial T}\Big|_\xi$$

$$\Downarrow$$

$$\boxed{\frac{\partial U}{\partial \xi}\Big|_T = -T\frac{\partial X}{\partial T}\Big|_\xi + X}$$

$TdS = dU^{(T,S)} - Xd\xi$

$$dS = \frac{1}{T}\frac{\partial U}{\partial T}\Big|_\xi dT - \frac{1}{T}\left(\frac{\partial U}{\partial \xi}\Big|_T - X\right)d\xi$$

$$\frac{\partial^2 S}{\partial T \partial \xi} = \frac{\partial^2 S}{\partial \xi \partial T}$$

$$\Downarrow$$

$$\frac{\partial}{\partial \xi}\left(\frac{1}{T}\frac{\partial U}{\partial T}\Big|_\xi\right) = \frac{\partial}{\partial T}\left(\frac{1}{T}\frac{\partial U}{\partial \xi}\Big|_T - \frac{X}{T}\right)$$

Young tétel nem csak a potenciálokra működik, így nem kell a Maxwellt levezetni:

T5/5 $\quad S=S(p,T)$ mutassuk: $TdS = n\,c_p\,dT - \beta_p TVdp$

$c_p = \frac{\partial H}{\partial T}\Big|_p \frac{1}{n} \quad c_V = \frac{\partial U}{\partial T}\Big|_V \frac{1}{n} \quad \beta_p = \frac{1}{V}\frac{\partial V}{\partial T}\Big|_p \quad \kappa_T = -\frac{1}{V}\frac{\partial V}{\partial p}\Big|_T \longleftrightarrow \kappa_S = -\frac{1}{V}\frac{\partial V}{\partial p}\Big|_S$

$$dS = \underbrace{\frac{\partial S}{\partial p}\Big|_T} dp + \frac{\partial S}{\partial T}\Big|_p dT \qquad \frac{\partial S}{\partial T}\Big|_p \Rightarrow \frac{\partial S}{\partial T}\Big|_p \cdot \frac{\partial H}{\partial H}\Big|_p = nc_p\frac{\partial S}{\partial H}\Big|_p$$

$$\boxed{\frac{1}{T}nc_p}$$

$dH = dU + pdV + dpV$, $\;TdS$

$$\left(\frac{\partial S}{\partial H}\right)^{-1} = \frac{\partial H}{\partial S} \Rightarrow \frac{\partial H}{\partial S}\Big|_p = T$$

$$-\frac{\partial V}{\partial T}\Big|_p = -V\beta_p$$

$$\Rightarrow \boxed{TdS = nc_pdT - V\beta_pTdp}$$

FÁZISÁTALAKULÁSOK

jel: G ⇝ mol N → n (1 molra nézzük U-t)

fázis ⇒ gáz, foly., szilárd db. ⇝ anyag homogén része
↓
fázishatár zárja le

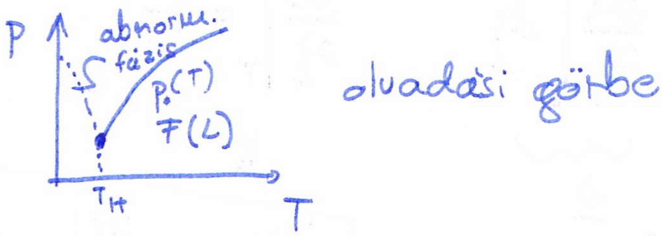
komponens ⇒ anyagok: MgAl

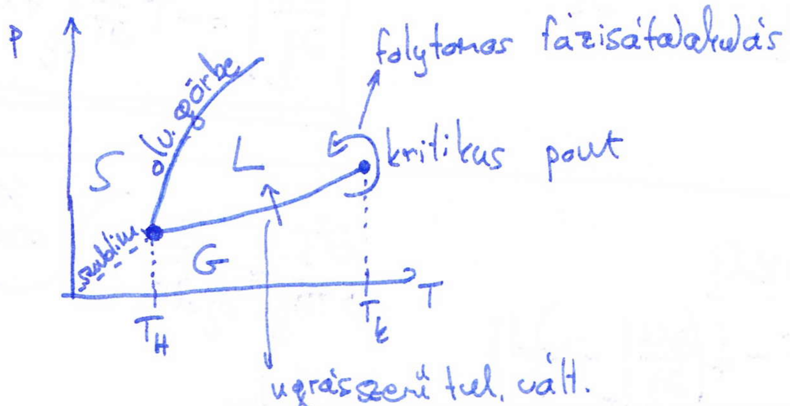
fázishatáron áthaladva ugrásszerűen változhatnak a tul.ok

TAPASZTALAT: olvadás során T nem vált.

↳ betáplált energia a mol. kötéseket szakítja fel

⇒ pot. E ↑, ∅T





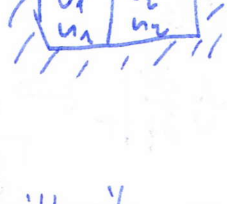
Fázisegyensúly feltétele:

a, izolált r.sz.

$U = U_1 + U_2$

$V = V_1 + V_2$

$n = n_1 + n_2$



$dS = 0$ ⇝ nem nő tovább → egyensúly

$S = S_1(U_1, V_1, n_1) + S_2(U_2, V_2, n_2)$

$U_2 = U - U_1$

...

$$0 = dS(\underbrace{U, V, n}_{\text{paramétereket}}, \overbrace{U_1, V_1, n_1}^{\text{változók}}) = \left[\frac{\partial S_1}{\partial U_1}\bigg|_{V_1, n_1} - \frac{\partial S_2}{\partial U_2}\bigg|_{V_2, n_2}\right] dU_1 + \left[\frac{\partial S_1}{\partial V_1}\bigg|_{n_1, U_1} - \frac{\partial S_2}{\partial V_2}\bigg|_{n_2, U_2}\right] dV_1 +$$

$dU_2 = -dU_1$

$$\ldots + \left[\frac{\partial S_1}{\partial n_1}\Big|_{\cancel{V_1},U_1} - \frac{\partial S_2}{\partial n_2}\Big|_{V_2,U_2} \right] dn_1 = 0$$

$\sum_i x_i = 0\ ;\ x_i \geqq 0$

$\Downarrow$

$\forall x_i = 0$

$$\frac{\partial S_1}{\partial U_1}\Big|_{V_1,n_1} = \frac{\partial S_2}{\partial U_2}\Big|_{V_2,n_2} \qquad \frac{\partial S}{\partial U}\Big|_{V,n} = \frac{1}{T}$$

$$\frac{\partial S_1}{\partial V_1}\Big|_{U_1,n_1} = \frac{\partial S_2}{\partial V_2}\Big|_{U_2,n_2} \qquad \frac{\partial S}{\partial V}\Big|_{U,n} = \frac{p}{T}$$

$$\frac{\partial S}{\partial n}\Big|_{U,V} = -\frac{\mu}{T}$$

$\Rightarrow \boxed{\begin{matrix} T_1 = T_2 \\ p_1 = p_2 \\ \mu_1 = \mu_2 \end{matrix}} \Rightarrow$ int. menny.-ek egyenlőek

$\Downarrow$

egyensúly

$\to$ $\forall$ két fázist u.a. "szereti"

$\mu(T,p)$

---

b, izoT - izop r.sz.

$\boxed{dG = 0}$ $\quad G(T,p,n_1) = G_1(T,p,n_1) + G_2(T,p,n_2)$

$n = n_1 + n_2$

$dn_2 = \cancel{dn} - dn_1$

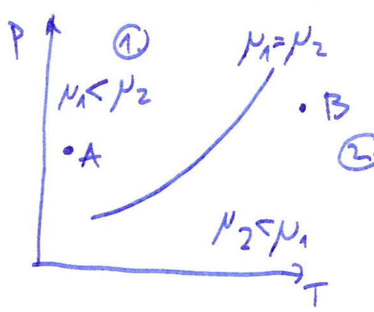
$$0 = dG = \left[\frac{\partial G_1}{\partial T}\Big|_{p,n_1} - \frac{\partial G_2}{\partial T}\Big|_{p,n_2} \right] dT + [\ldots] dp + \boxed{\frac{\partial G_1}{\partial n_1}\Big|_{p,T} dn_1 + \frac{\partial G_2}{\partial n_2}\Big|_{p,T} dn_2}$$

$$0 = dG = \underbrace{\frac{\partial G_1}{\partial n_1}\Big|_{T,p} dn_1 + \frac{\partial G}{\partial n_2}\Big|_{T,p} dn_2}_{\left[\frac{\partial G_1}{\partial n_1} - \frac{\partial G_2}{\partial n_2} \right] dn_1}$$

$$\boxed{\mu_1 = \mu_2}$$

Fázisdiagramm:

egyensúlyi áll.-at leíró $p = f(T)$ fgv. ahol $\boxed{\mu_1 = \mu_2}$



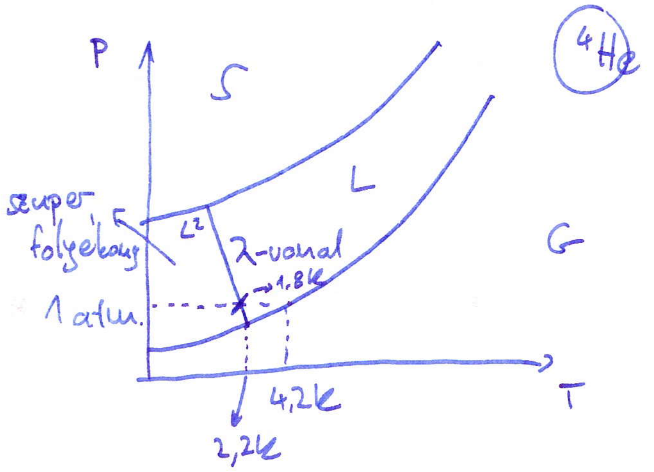
A-ban $\exists$ ② (gyors átmenet)

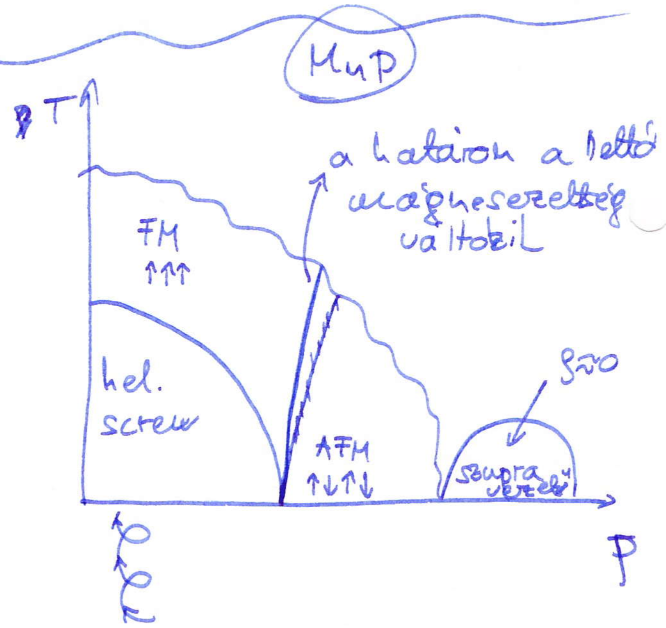
$$G = \mu_1 n_1 + \mu_2 n_2$$

$$dG = \mu_1 dn_1 + \mu_2 \underbrace{dn_2}_{= -dn_1} = [\mu_1 - \mu_2] dn_1 < 0$$

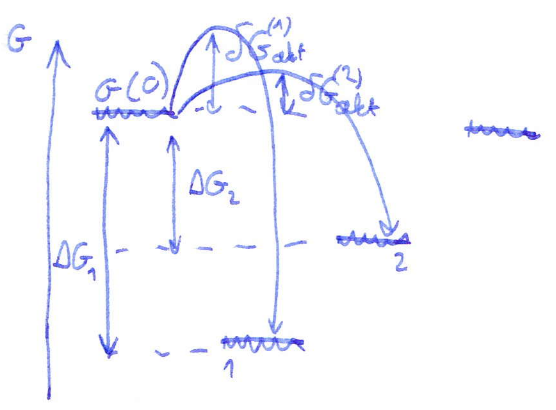
$$G_{min} = \mu_1 n_1^* + \mu_2 n_2^* = \mu_1 n$$

n_1 max, $n_2 = 0$





Merre megy a r.sz?



küll.-ő energiájú állapotok

végső áll → Hamilton-elv

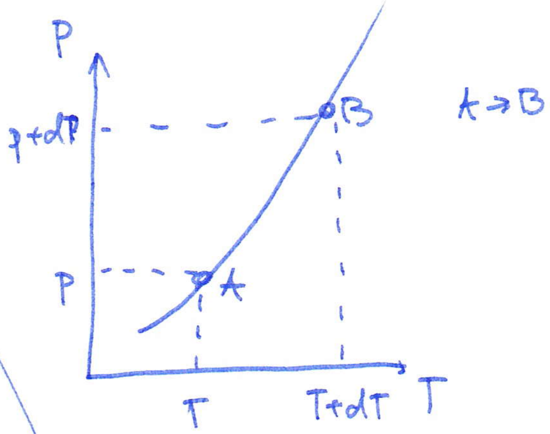
$\Downarrow$ $\Downarrow$

$|\Delta G| = max$

DE nem erre indul, hanem

$\delta G = min. \rightsquigarrow$ legexotermebb átalakulás felé

Clausius–Clapeyron: $\mu(T,p)$



$p = f(T)$

$$d\mu_1 = \frac{\partial \mu_1}{\partial T}\Big|_p dT + \frac{\partial \mu_1}{\partial p}\Big|_T dp =$$

$$= d\mu_2 = \frac{\partial \mu_2}{\partial T}\Big|_p dT + \frac{\partial \mu_2}{\partial p}\Big|_T dp$$

$$\frac{dp}{dT} = \frac{\frac{\partial \mu_2}{\partial T} - \frac{\partial \mu_1}{\partial T}}{\frac{\partial \mu_2}{\partial p} - \frac{\partial \mu_1}{\partial p}} = \frac{S_{M_2} - S_{M_1}}{V_{M_2} - V_{M_1}} \overset{C.C.}{\rightsquigarrow} S_{M_2} - S_{M_1} = \frac{Q_{M12}}{T} = \frac{L}{T}$$

$G = \mu n$ (Euler alak) $\Rightarrow \mu = \frac{G(T,p)}{n}$ $\cancel{\frac{\partial \mu}{\partial T}}$ $\frac{\partial \mu}{\partial T} = -\frac{S}{n} = -S_M$

$$\frac{\partial \mu}{\partial p} = \frac{V}{n} = V_M$$

(11.23.)

T5/4

$dT(\beta_p, \kappa_T, c)$

$dV\downarrow,\ \delta Q = 0 \Longrightarrow dS = 0$

I. f.t.: $dU = T\cancel{dS} - p\,dV \quad dU = -p\,dV$

$$U(T,V) = \underbrace{\frac{\partial U}{\partial T}\Big|_V}_{C_V} dT + \frac{\partial U}{\partial V}\Big|_T dV$$

$$\Big\} \quad -p\,dV = C_V\,dT + \frac{\partial U}{\partial V}\Big|_T dV =$$

$\frac{\partial V}{\partial V}$-vel bővítés is jó!

$$= C_V\,dT + T\underbrace{\frac{\partial p}{\partial T}\Big|_V}_{?} dV - p\,dV \Rightarrow$$

emlék: $\frac{\partial U}{\partial V}\Big|_T = T\frac{\partial p}{\partial T}\Big|_V - p$

$$\Rightarrow C_V\,dT = -T\frac{\beta_p}{\kappa_T}\,dV$$

$$\Downarrow$$

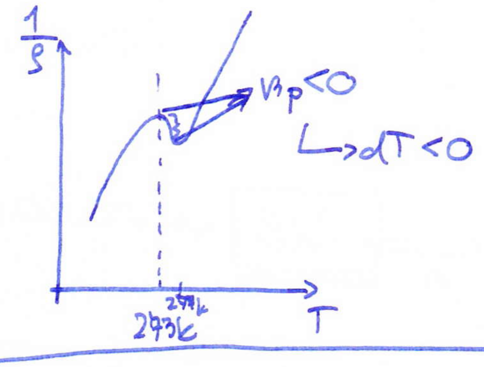
$$\boxed{dT = -T\frac{\beta_p}{\kappa_T C_V}\,dV}$$

Trükk: (?-hoz) $dp(T,V) = \frac{\partial p}{\partial T}\Big|_V dT + \frac{\partial p}{\partial V}\Big|_T dV = 0$ ha izobar

$\downarrow$ el kéne tüntetni $\longrightarrow$ $\Big/ \frac{\partial}{\partial T}$

$$0 = \frac{\partial p}{\partial T}\Big|_V \underbrace{\frac{\partial T}{\partial T}}_{1} + \underbrace{\frac{\partial p}{\partial V}\Big|_T}_{-\frac{1}{V\kappa_T}} \underbrace{\frac{\partial V}{\partial T}\Big|_p}_{V\beta_p} \Rightarrow \frac{\partial p}{\partial T}\Big|_V = \frac{\beta_p}{\kappa_T}$$

Víz:



T5/6 $U(S,V)$ ✓ $p, T, H(S,V)$

$dU = T\,dS - p\,dV \quad dU = \frac{\partial U}{\partial S}\Big|_V dS + \frac{\partial U}{\partial V}\Big|_S dV \Rightarrow$ $\boxed{\begin{array}{l} \frac{\partial U}{\partial S}\Big|_V = T(S,V) \\ \frac{\partial U}{\partial V}\Big|_S = -p(S,V) \end{array}}$

$H = U + pV = U - \frac{\partial U}{\partial V}\Big|_S V$

$F = U - TS = U - \frac{\partial U}{\partial S}\Big|_V S$

$G = H - TS = U - TS + pV =$

$\phi = U - \mu N$

$\boxed{T5/7}$ m, T, V, dV ✓ c_p, β_p ✓ $dS = ?$ $p = \text{áll}?$

$$S(T,p) \Rightarrow dS = \underbrace{\frac{\partial S}{\partial T}\bigg|_p}_{?} dT + \underbrace{\frac{\partial S}{\partial p}\bigg|_T}_{0} dp$$

0 mert izobár

$\delta Q = m c_p dT = T dS$

$\hookrightarrow \frac{\partial S}{\partial T} = \frac{m c_p}{T}$

$$dS = \frac{m c_p}{T} dT = \frac{m c_p}{T} dV \underbrace{\frac{dT}{\partial V}}_{\frac{1}{V\beta_p}}$$

ez zavar meg → def. alapján bővítünk dV-vel

$$dS(m, T, V, dV, c_p, \beta_p) = \boxed{\frac{m c_p}{T V \beta_p} dV}$$

! $\boxed{T5/9}$!

$$f = \underset{\overset{\vee}{0}}{a} T \left(\frac{\ell}{\ell_0} - \left(\frac{\ell_0}{\ell} \right)^2 \right)$$

a, $U(\ell) = U$

b, $dU, dF, dG = ?$

c, $T = \text{áll.}$, $\Delta W = ?$, $\Delta Q_{\ell} = ?$ $\ell_0 \nearrow 2\ell_0$

d, $T \uparrow$, ha $\delta Q = 0$

f: intenzív $\quad p \to -f$

ℓ: extenzív $\quad V \to \ell$

a, $\frac{\partial U}{\partial \ell}\Big|_T = f - T \frac{\partial f}{\partial T}\Big|_\ell = f - T\frac{f}{T} = \boxed{0}$

$\Downarrow$

$\boxed{U(\ell) = U}$

volt: $\frac{\partial U}{\partial V}\Big|_T = T \frac{\partial p}{\partial T}\Big|_V - p$

b, $dU = \boxed{T dS + f d\ell}$

$dF = dU - d(TS) = \boxed{-S dT + f d\ell}$

$dG = dU - d(TS) - d(\underset{\overset{\Updownarrow}{-p}}{f}\ell) = \boxed{-S dT - \ell df}$

c, $dU = \delta Q + \delta W_{mi}$

$T = \text{áll.} \to U(\ell) = U$ $\quad dU = 0 \leadsto \delta Q = -\delta W_{mi} = \boxed{-f d\ell} \Rightarrow \delta Q < 0 \Rightarrow$ hűl

$W = \int f d\ell \to \delta W = f d\ell$

$$\Delta Q = -\Delta W = -\int_{\ell_0}^{2\ell_0} f d\ell = -\int_{\ell_0}^{2\ell_0} aT \left(\frac{\ell}{\ell_0} - \left(\frac{\ell_0}{\ell} \right)^2 \right) d\ell = aT \left[-\frac{\ell^2}{2\ell_0} - \frac{\ell_0^2}{\ell} \right]_{\ell_0}^{2\ell_0} = \boxed{-aT\ell_0 < 0}$$

d, $dU = \overset{0 \text{ adiab.}}{\delta Q} + \delta W = f d\ell$; $\quad dU_{(T,\ell)} = \underbrace{\frac{\partial U}{\partial T}\Big|_\ell}_{c_\ell} dT + \underbrace{\frac{\partial U}{\partial \ell}\Big|_T}_{0} d\ell$; $\quad f d\ell = c_\ell dT \to \boxed{\frac{\partial T}{\partial \ell}\Big|_S = \frac{f}{c_\ell}}$

$c_\ell > 0$

$f > 0$

$d\ell > 0$

$\Downarrow$

$dT > 0$

! $\boxed{T5/10}$! $\varepsilon_r(T)$, $\Delta Q = ?$

$\underline{P}$

$\underline{E} \nearrow (\underline{D}, \underline{E})$

$T =$ áll.

$dV = 0$

⇝ $\underline{E}$: int

TD: $\underline{P}$: ext. ⇝ makr. pol. $\}$ $-p \to \underline{E}$, $V \to \underline{P}$

(ED: $\underline{P}$: pol. sűrűség)

$$\delta Q = dU - \delta W = dU - \underline{E}\, d\underline{P} \qquad \underline{E} \parallel \underline{P} \rightsquigarrow \underline{E}\, d\underline{P} = E\, dP$$

$$dU(T,P,V) = \frac{\partial U}{\partial T}\Big|_{P,V} dT + \frac{\partial U}{\partial P}\Big|_{T,V} \partial P + \frac{\partial U}{\partial V}\Big|_{T,P} \overset{0}{\overset{\|}{dV}}$$

$$\frac{\partial U}{\partial P}\Big|_T = E - T \frac{\partial E}{\partial T}\Big|_P$$

$$\boxed{\delta Q = -T \frac{\partial E}{\partial T}\Big|_P dP} + \cancel{E\, dP} - \cancel{E\, dP}$$

pol. sűrűség

$$\underbrace{\underline{D} = \varepsilon \underline{E} \qquad \varepsilon = \varepsilon_0 \varepsilon_r \qquad \underline{D} = \varepsilon_0 \underline{E} + \underline{\mathcal{P}}}$$

$$\varepsilon \underline{E} = \varepsilon_0 \underline{E} + \underline{\mathcal{P}} \longrightarrow \underline{E} = \frac{\mathcal{P}}{\varepsilon_0(\varepsilon_r - 1)} = \frac{P}{V \varepsilon_0 (\varepsilon_r - 1)}$$

$$\frac{\partial E}{\partial T}\Big|_P = -\frac{P}{\varepsilon_0 V} \frac{1}{(\varepsilon_r(T) - 1)^2} \frac{\partial \varepsilon_r}{\partial T}\Big|_P$$

tfh. $\varepsilon_r(p) = \varepsilon_r$

$$\boxed{\delta Q = \frac{T}{\varepsilon_0 (\varepsilon_r - 1)^2 V} \frac{\partial \varepsilon_r}{\partial T}\Big|_P P\, dP} \Longrightarrow \Delta Q = \frac{T}{\varepsilon_0 (\varepsilon_r - 1)^2 V} \frac{\partial \varepsilon_r}{\partial T} \frac{P^2}{2}$$

$$\Longrightarrow \boxed{\Delta Q = \frac{1}{2} T \frac{\partial \varepsilon_r}{\partial T} \varepsilon_0 E^2 V}$$

⇔ ezért melegednek a kondik

T6/1

$p = \text{áll.}$

$L = \Delta Q_{\text{átalak}} = \Delta H$

I. f.t.: $dU = \delta Q - p\,dV$

$H = U + p\,dV \rightsquigarrow dH = \delta Q + V\,dp$

$\delta Q - p\,dV = dU = dH - d(pV) = dH - \overset{p=\text{áll}}{\cancel{dp}}V - p\,dV = dH - p\,dV$

$\delta Q = dH$

$\boxed{L = \Delta H}$ $\quad / \int_{\text{fázisátalak}}$

↓

látens hő

T6/2

1 kg víz

101,3 kPa = áll

$V = 220\,V$

$t = 34{,}2$ perc

$I = 5\,A$

$T = \text{áll.}$

$\Delta H, \Delta S, \Delta U = ?$

$\Delta H = \Delta Q = W_{\text{elektr.}} = \overbrace{UIt}^{\text{Joule-hő}} = \boxed{2.257\,MJ}$

$\Delta S = \int_{\text{kezd}}^{\text{vég}} dS = \int \frac{\delta Q}{T} = \frac{1}{T}\int \delta Q = \frac{\Delta Q}{T} \approx \boxed{6\,\frac{kJ}{K}}$

$\Delta U = \Delta Q - p\Delta V = ⊛ \quad V_{\text{gőz}} \gg V_{\text{víz}}$

↑ id. gáz

$\Delta V = V_{\text{gőz}} - V_{\text{víz}} = V_{\text{gőz}} = \frac{m}{M}\frac{RT}{p}$

$⊛ = \Delta Q - p\frac{m}{M}\frac{RT}{p} = \Delta Q - \frac{m}{M}RT \cong \boxed{2\,MJ}$

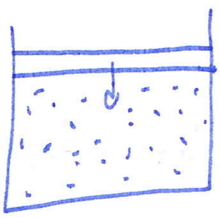
T6/3

$T_f = 100°C$ gőz, id.g.

$\Delta m = 0.7\,g$ víz

$p_{\text{külső}} = \text{áll.}$

$\Delta W_{m_i} = ?$



$\Delta W = -p_k \Delta V = -p_k (V_2 - V_1)$

$p_k V_1 = \frac{m_1}{M} RT_f$

$p_k V_2 = \frac{m_2}{M} RT_f$

$\Delta W = -p_k RT_f \frac{1}{M} \underbrace{(m_2 - m_1)}_{\Delta m = 0{,}7\,g}$

$= \boxed{120\,kJ}$

T6/4

$\mu_p(T)$

$p = \text{áll.}$

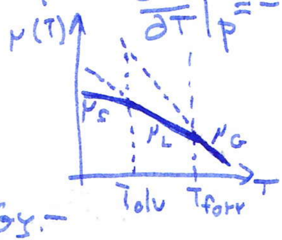
1 komp

$U_M = TS - pV + \mu$

$G = U - TS + pV = \mu n$

$\frac{\partial G}{\partial T}\Big|_p = -S$

$\boxed{\frac{\partial \mu}{\partial T}\Big|_p = -S_M}$

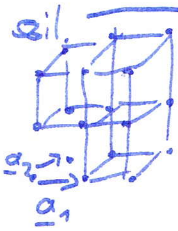


$\frac{\partial S_M^i}{\partial T} = 0$

$\hookrightarrow \mu_i(T) \sim T$

Kis Fiz. 3. Gy.-

szil.



$\boxed{S_M^S < S_M^L < S_M^G}$

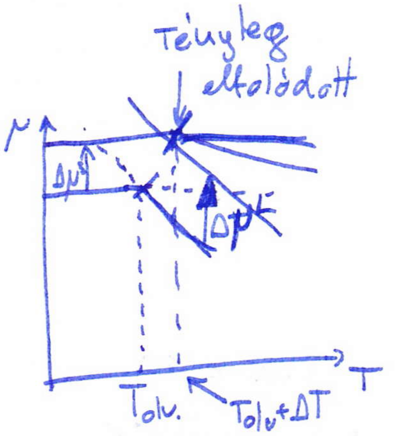
$< \underline{r}, \sigma$ nagy

$\exists\, \underline{a}, \sigma$ kicsi

↳ első szomsz. távolság

(11.30.)

TG/5



$T_{olv.}\uparrow$ ha $p\uparrow$

$V_M^S < V_M^L$

$$\left.\frac{\partial G}{\partial p}\right|_T = V \longrightarrow \left.\frac{\partial \mu}{\partial p}\right|_T = V_M$$

Euler: Legendre: $G = \mu n$

$$\left.\frac{\partial \mu}{\partial p}\right|^S < \left.\frac{\partial \mu}{\partial p}\right|^L$$

$$\left.\frac{\partial \mu}{\partial p}\right|_T = V_M \rightsquigarrow \Delta\mu_i = V_M^i \Delta p$$

víz: $V_M^{s,H_2O} > V_M^{L,H_2O}$ $\quad T_{olv}(p+\Delta p) < T_{olv}(p)$

TG/6

$$p = p_1 + \frac{L_M^{olv.}}{\Delta V_M^{olv.}} \ln \frac{T}{T_1}$$

$T \leftrightarrow p$

$T_1 \leftrightarrow p_1$

a, Biz.

b, lin. alizálni

Egyensúly: $p_1 = p_2 = p$

$T_1 = T_2 = T$

$\mu_1(p,T) = \mu_2(p,T) = \mu(p,T)$

$\hookrightarrow d\mu_1 = d\mu_2$ (egyensúlyi görbén)

$$\left.\frac{\partial \mu_1}{\partial p}\right|_T dp + \left.\frac{\partial \mu_1}{\partial T}\right|_p dT = \underbrace{\left.\frac{\partial \mu_2}{\partial p}\right|_T}_{V_{M2}} dp + \underbrace{\left.\frac{\partial \mu_2}{\partial T}\right|_p}_{S_{M2}} dT$$

$\mu = \frac{G}{n}$

$dG = dU + d(pV) - d(TS)$

$$(S_{M2} - S_{M1})\,dT = (V_{M2} - V_{M1})\,dp$$

$$\frac{dp}{dT} = \frac{T}{T}\cdot\frac{S_{M2}-S_{M1}}{V_{M2}-V_{M1}} = \frac{L_M^{olv.}}{T\Delta V_M^{olv}} \quad \underline{\text{C-C.}}\ !$$

akkor tudják integrálni, ha

$L_M^{olv}(T) = L_M^{olv}$

$\Delta V_M^{olv}(T) = \Delta V_M^{olv.}$ közelítés korlátjai

$\Downarrow$

$$dp = \frac{L_M}{\Delta V_M}\frac{dT}{T} \quad \int$$

$$dp' = \frac{L_M}{\Delta U_M}\frac{dT'}{T'} \quad \Big/ \int_{p_1}^{p} dp' \quad \int_{T_1}^{T} dT'$$

$$\boxed{p = p_1 + \frac{L_M}{\Delta U_M}\ln\frac{T}{T_1}}$$

ha $x \approx 0$

Linearizálás: kis hőm. vált. $T-T_1 \ll T_1 \Rightarrow \ln(1+x) \approx x$

ez már linearizált de nélkül $\ln\frac{\cdots}{\cdots}$ van

$$p = p_1 + \frac{L_M}{\Delta U_M}\ln\underbrace{\frac{T-T_1+T_1}{T_1}} \approx \boxed{p_1 + \frac{L_M}{\Delta U_M}\frac{T-T_1}{T_1}}$$ (feltétel: T közel T_1-hez)

$1+\underbrace{\frac{T-T_1}{T_1}}_{x}$ kéne: elég közel 0-hoz $\Rightarrow T-T_1 \ll T_1 \Rightarrow$

$\Downarrow$

$\frac{T-T_1}{T_1} \approx 0$

TG/9

$p_0 = 1$ bar

$T_{olv} = 83$ K

$L_M^{olv.} = 1176 \frac{J}{mol}$

$\Delta U_M^{olv.} = 3{,}5 \frac{cm^3}{mol}$

$p_0\uparrow$, $L_M^{olv} =$ áll.

$\Delta U_M \sim T^{3/2} \longrightarrow \Delta U_M = \Delta U_{M_0}\left(\frac{T}{T_{olv}}\right)^{3/2}$

$p' = ?$, hogy $T_{olv} \to 2T_{olv}$

$p_0 \uparrow p' = ?$

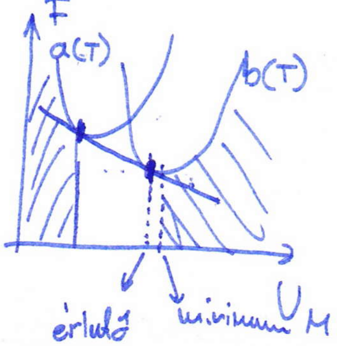
C-C. $$\frac{dp}{dT} = \frac{L_M (T_{olv})^{3/2}}{T^{5/2}\Delta U_{M_0}} \longrightarrow dp = \frac{L_M (T_{olv})^{3/2}}{T^{5/2}\Delta U_{M_0}}dT \quad \Big/ \int_{p_0}^{p'} dp \quad \int_{T_{olv}}^{2T_{olv}} dT$$

$$p' = p_0 + \frac{L_M T_{olv}^{3/2}}{\Delta U_{M_0}}\left[-\frac{2}{3}T^{-3/2}\right]_{T_{olv}}^{2T_{olv}} = p_0 + \frac{2 L_M \cancel{T_{olv.}^{3/2}}}{3\Delta U_{M_0}\cancel{T_{olv}^{3/2}}}\left[1-\left(\frac{1}{2}\right)^{3/2}\right] \approx \boxed{1{,}5\ \text{kbar}}$$

$\ominus$-ból $\oplus$

TG/10

FT.:
TG/7,8



p, T, μ ⇒ azonos

intenzívek azonosak a fázisokban → egyens.
a, b fázisok

T ✓

μ ✓

p-t kell meghatározni:

$$\frac{\partial F}{\partial V}\Big|_T = \boxed{\frac{\partial F_M}{\partial V_M}\Big|_T = -p}$$ (⇐ F.E. alapján)

↳ azonos meredekség, ha egyensúly

másszóval: érintők ∥-ak

$G = \mu n = F + pV$

$$\mu_a = \mu_b = F_M^i + pV_M^i \longrightarrow \boxed{-p = \frac{F_M^b - F_M^a}{V_M^b - V_M^a}}$$ ⇝ egyensúly

↓

összekötő egyenesen van

$V_M < V_M^a \Rightarrow$ csak a fázis

$V_M > V_M^b \Rightarrow$ csak b fázis

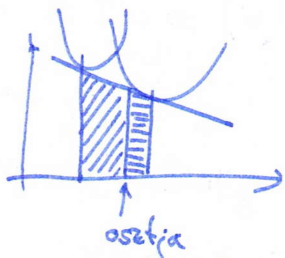
$V_M^a < V_M < V_M^b \Rightarrow$ a és b is jelen van

rel. koncentráció: $x \in [0,1]$: a; b: $1-x$

$$F_M = xF_{Ma}^* + (1-x)F_{Mb}^*$$

$$\frac{V_M - V_{Mb}}{V_{Ma} - V_M} = \frac{x}{1-x}$$

Mérleg szabály



ez működik komponensekre is, nem csak fázisokra

FÁZISÁTALAKULÁSOK

elsőrendű

μ elsőrendű deriváltjai ugranak

$$-\frac{\partial \mu}{\partial T}\Big|_p = S_M \qquad S_M^A \neq S_M^B$$

$$-\frac{\partial \mu}{\partial p}\Big|_T = V_M \qquad V_M^A \neq V_M^B$$

$\boxed{T6/11}$

másodrendű (folytonos)

μ másodrendű deriváltjai ugranak

$$\frac{\partial^2 \mu}{\partial p^2}\Big|_T = \frac{\partial V}{\partial p}\Big|_T = -V\kappa_T$$

$$\frac{\partial^2 \mu}{\partial T^2}\Big|_p = -\frac{c_p}{T}$$

$$\frac{\partial^2}{\partial T \partial p} = V\beta_p$$

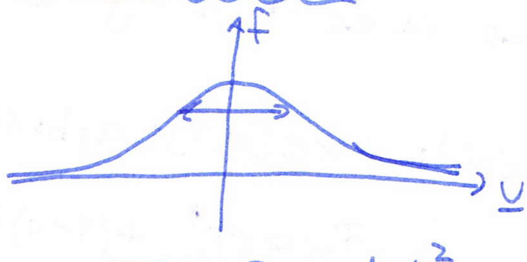
$\Downarrow$

mérhető mennyiségek ugranak

(μ folytonos $\Rightarrow$ törés)

Mx-seloszlás:

$$f(\underline{v}) = \sqrt{\frac{m}{2\pi k_B T}}^3 e^{-\frac{m\underline{v}^2}{2k_B T}}$$

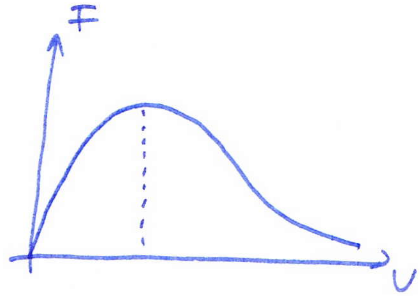


seb.nagys.-ra von.-ó eloszlás

$$F(v) = 4\pi v^2 f(\underline{v}) = 4\pi v^2 \sqrt{\frac{m}{2\pi k_B T}}^3 e^{-\frac{mv^2}{2k_B T}}$$

térszög, Jacobi

gömbhéjra int.-unk $v, v+dv$ közt



$F(v) = \max$

$\ln F(v) = \max$, mert 1-1 értékű fgv.-ek $\quad \ln F(v) = \max \quad \Big/ \frac{\partial}{\partial v^2}$

$\hookrightarrow v_c = \sqrt{\frac{2k_B T}{m}}$

$$\bar{v} = \int_0^\infty F(v)\,v\,dv = \sqrt{\frac{8k_BT}{m}}$$

$\downarrow$ várható érték

$$\overline{v^2} = 3\overline{v_x^2} = 3\frac{k_BT}{m}$$

$\downarrow$ izotróp a tér, 1 irányra számolunk

T1/3

$\varepsilon = \frac{1}{2}mv^2 \rightsquigarrow v = \sqrt{\frac{2\varepsilon}{m}}$ (dv miatt számoljuk)

$f(\varepsilon) \Rightarrow$ energia eloszlás $= ?$ (sűrűség)

$$\boxed{F(v)\,dv = f(\varepsilon)\,d\varepsilon}$$

$\downarrow$ normatartó leképezés miatt szorozzuk dv-vel

$$\frac{dv}{d\varepsilon} = \sqrt{\frac{2}{m}}\,\frac{1}{2}\sqrt{\frac{1}{\varepsilon}} \rightarrow dv = \frac{d\varepsilon}{\sqrt{2m\sqrt{\varepsilon}}}$$

$$\rightarrow A\cdot\left(\frac{v}{v_0}\right)^2 e^{-\left(\frac{v}{v_0}\right)^2}$$

$\downarrow$ dim.-tlanítjuk

A és v_0 új const.-ok $\longrightarrow$ még ~~...~~ constok: $B = \frac{A}{\varepsilon_0\sqrt{2m}}$

$\varepsilon_0 = \frac{1}{2}mv_0^2$

~~$f(\varepsilon) = B\cdot\sqrt{\varepsilon}$~~

$$f(\varepsilon)\,d\varepsilon = A\underbrace{\left(\frac{\varepsilon}{\varepsilon_0}\right)}_{v^2\text{-esből}}\underbrace{e^{-\varepsilon/\varepsilon_0}}_{\text{Gaussból}}\frac{d\varepsilon}{\sqrt{2m\varepsilon}} \Rightarrow \boxed{f(\varepsilon) = B\sqrt{\varepsilon}\,e^{-\varepsilon/\varepsilon_0}} \rightsquigarrow \varepsilon_0\ \max = ?$$

$$\left.\frac{df(\varepsilon)}{d\varepsilon}\right|_{\varepsilon_0} = 0 = Be^{-\varepsilon_0/k_BT}\underbrace{\left(\frac{1}{2\sqrt{\varepsilon_0}} - \frac{\sqrt{\varepsilon_0}}{k_BT}\right)}_{0} \Rightarrow \varepsilon_0 = \frac{k_BT}{2}$$

$$\bar{\varepsilon} = \int_0^\infty \varepsilon f(\varepsilon)\,d\varepsilon = B\int_0^\infty \varepsilon^{3/2} e^{-\varepsilon/k_BT}\,d\varepsilon \Rightarrow \text{parc.-an kiszámolni?}$$

$$\boxed{\bar{\varepsilon} = \frac{3}{2}k_BT}$$

T1/2 entrópia $\rightarrow$ fázisátal + mx. eloszlás