1.

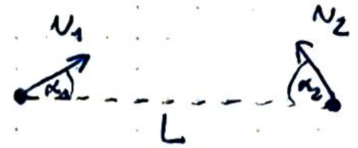
$L = 500\,m$

$v_1 = 30\,km/h$

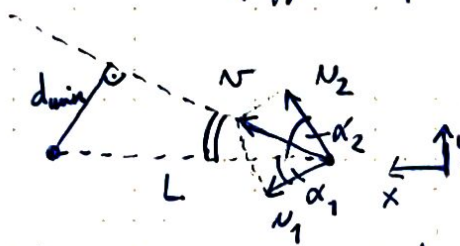
$v_2 = 10\,km/h$

$\alpha_1 = 30°$

$\alpha_2 = 45°$



Átülve a v_1-gyel mozgó csónakba:



$d_{min} = L \cdot \sin ∢$

$v_x = v \cdot \cos ∢ = v_1 \cos\alpha_1 + v_2 \cos\alpha_2 = 33{,}05\,\frac{km}{h}$

$|v_y| = v \cdot \sin ∢ = |v_2 \sin\alpha_2 - v_1 \sin\alpha_1| = 7{,}93\,\frac{km}{h}$

$\Big\}\ \mathrm{tg}\,∢ = 0{,}24 \rightarrow ∢ \approx 13{,}5°$

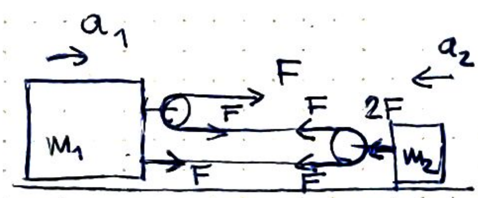
$d_{min} = L \cdot \sin ∢ \approx 117\,m \approx 120\,m$

2.

$m_1 = 4\,kg$

$m_2 = 2{,}5\,kg$

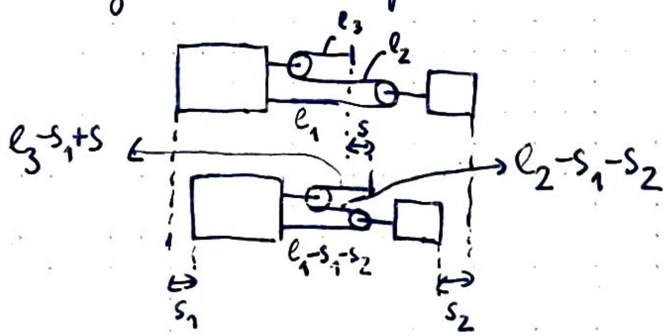
$F = 2{,}5\,N$



a) $3F = m_1 a_1 \rightarrow a_1 = \frac{3F}{m_1} \approx 1{,}9\,\frac{m}{s^2}$

$2F = m_2 a_2 \rightarrow a_2 = \frac{2F}{m_2} = 2\,\frac{m}{s^2}$

b) Vizsgáljuk meg a testek helyzetét t idő múlva.

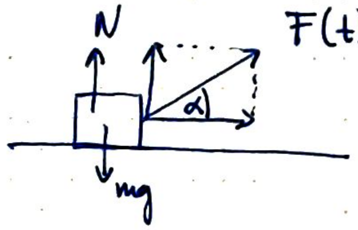


kötélhossz állandó: $\ell_1+\ell_2+\ell_3 = \ell_1 - s_1 - s_2 + \ell_2 - s_1 - s_2 + \ell_3 - s_1 + s$

$$s = 3s_1 + 2s_2$$

Mivel $s \sim a$: $\underline{\underline{a}} = 3a_1 + 2a_2 \approx \underline{\underline{9{,}7\,m/s^2}}$

(F3.)



$F(t) = k\cdot t$

$$a(t) = \frac{kt\cdot\cos\alpha}{m}$$

a) $mg = N + F(t)\cdot\sin\alpha \rightarrow N = mg - k\cdot t\sin\alpha$

Elválás pillanatában: $N = 0$: $mg = k\cdot t_e\cdot\sin\alpha$

$$t_e = \frac{mg}{k\cdot\sin\alpha}$$ indulástól számítva

$N_0 = 0$:

$$v(t) = \int_0^t a(t')\,dt' = \frac{k\cdot\cos\alpha}{m}\int_0^t t'\,dt' = \frac{k\cos\alpha}{2m}t^2$$ elválásig

$$\underline{\underline{v(t_e)}} = \frac{k\cdot\cos\alpha}{2m}\,\frac{m^2g^2}{k^2\sin^2\alpha} = \underline{\underline{\frac{mg^2\cos\alpha}{2k\sin^2\alpha}}}$$

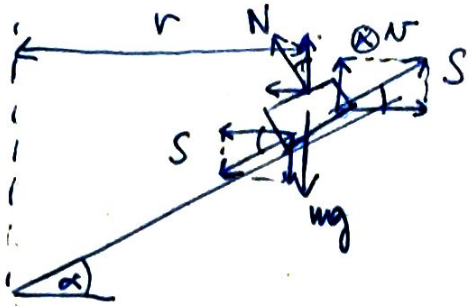
b) $$s(t) = \int_0^t v(t')\,dt' = \frac{k\cos\alpha}{2m}\int_0^t t^2\,dt = \frac{k\cos\alpha}{6m}t^3$$

$$\underline{\underline{s(t_e)}} = \frac{k\cos\alpha}{6m}\cdot\frac{m^3g^3}{k^3\sin^3\alpha} = \underline{\underline{\frac{m^2g^3\cos\alpha}{6k^2\sin^3\alpha}}}$$

F4) $r = 50\,m$

$\alpha = 30°$

$\mu_0 = 0{,}4$



Minimális v esetén S „felfelé" mutat (pl.: v=0-nál lecsúszna),
maximális v esetén S „lefelé" mutat (ne csússzon ki).

$$N\cos\alpha \pm S\sin\alpha = mg \quad (1) \quad /\cdot\cos\alpha \rightarrow N\cos^2\alpha \pm S\sin\alpha\cos\alpha = mg\cos\alpha$$

$$N\cdot\sin\alpha \mp S\cos\alpha = m\frac{v^2}{r} \quad (2) \quad /\cdot\sin\alpha \rightarrow N\sin^2\alpha \mp S\cdot\sin\alpha\cos\alpha = m\frac{v^2}{r}\sin\alpha \quad \Big\} \oplus$$

$$S \le \mu_0 N \quad (3)$$

$$N = mg\cos\alpha + m\frac{v^2}{r}\sin\alpha$$

(1):

$$\pm S = \frac{mg - N\cos\alpha}{\sin\alpha} = \frac{mg - mg\cos^2\alpha - m\frac{v^2}{r}\cos\alpha\sin\alpha}{\sin\alpha} = mg\sin\alpha - m\frac{v^2}{r}\cos\alpha$$

(3):

$$mg\sin\alpha - m\frac{v^2}{r}\cos\alpha \le \mu_0 mg\cos\alpha + \mu_0\frac{mv^2}{r}\sin\alpha$$

$$v \ge \sqrt{\frac{\sin\alpha - \mu_0\cos\alpha}{\cos\alpha + \mu_0\sin\alpha}\cdot gr} \approx 8{,}5\,\frac{m}{s} = v_{min}$$

Másrészt:

(3): $$m\frac{v^2}{r}\cos\alpha - mg\sin\alpha \le \mu_0 mg\cos\alpha + \mu_0\frac{mv^2}{r}\sin\alpha$$

$$v \le \sqrt{\frac{\mu_0\cos\alpha + \sin\alpha}{\cos\alpha - \mu_0\sin\alpha}\cdot gr} \approx 25\,\frac{m}{s} = v_{max}$$