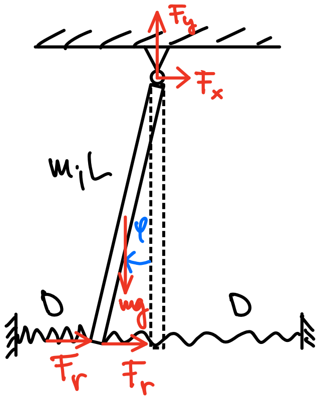
F1.



A felfüggesztési ponton átmenő tengelyre:

$$\frac{1}{3}mL^2\cdot\ddot{\varphi} = -mg\cdot\frac{L}{2}\cdot\sin\varphi - 2F_r\cdot L\cdot\cos\varphi$$

$\varphi\to 0$: $\sin\varphi\approx\varphi$; $\cos\varphi\approx 1$; $F_r\approx D\cdot L\varphi$

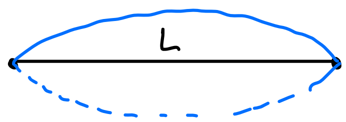
$$\frac{1}{3}m\cancel{L^2}\ddot{\varphi} = -mg\frac{\cancel{L}}{2}\cdot\varphi - 2D\cancel{L^2}\cdot\varphi$$

$$\ddot{\varphi} = -\underbrace{\frac{\frac{3mg}{2}+6DL}{mL}}_{\omega^2}\cdot\varphi$$ harmonikus rezgés

$$\omega = \sqrt{\frac{3}{2}\frac{g}{L}+\frac{6D}{m}}$$

F2.

alaphang:



$\lambda_0 = 2L = \frac{c}{f_0} \Rightarrow f_0 = \frac{c}{2L}$

valamint: $c = \sqrt{\frac{F}{\rho}}$

Tehát: $f_0 = \frac{1}{2L}\cdot\sqrt{\frac{F}{\rho}}$ $\rho = \frac{m}{L}$ → változtatás során állandó

$$\left.\begin{aligned} L &\to 0{,}65L \\ F &\to 1{,}7F \end{aligned}\right\} \Rightarrow f_0' = \frac{1}{2\cdot 0{,}65L}\cdot\sqrt{\frac{1{,}7F}{\rho}} = \underbrace{\frac{1}{2L}\sqrt{\frac{F}{\rho}}}_{f_0}\cdot\frac{\sqrt{1{,}7}}{0{,}65} \approx 2f_0$$

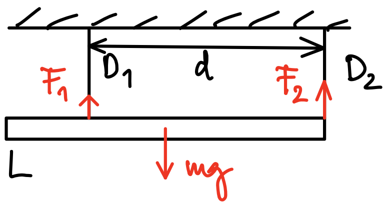
F3

a, $\ell; D$

$D = \frac{EA}{\ell}$ → $\ell_1 = \frac{\ell}{3}$: $D_1 = \frac{EA}{\ell_1} = \frac{3EA}{\ell} = 3D$

$\ell_2 = \frac{2\ell}{3}$: $D_2 = \frac{EA}{\ell_2} = \frac{3EA}{2\ell} = \frac{3}{2}D$

b,



vízszintes marad: $\ell_1+\Delta\ell_1 = \ell_2+\Delta\ell_2$ (1)

$mg = F_1+F_2 = D_1\Delta\ell_1 + D_2\Delta\ell_2$ (2)

A rúd jobb oldali végére:

$$mg\frac{L}{2} = F_1 \cdot d = D_1 \Delta l_1 \cdot d \quad (3)$$

(1): $\Delta l_2 = l_1 - l_2 + \Delta l_1$

(2): $mg = D_1 \cdot \Delta l_1 + D_2(l_1 - l_2) + D_2 \Delta l_1 \Rightarrow \Delta l_1 = \dfrac{mg + D_2(l_2 - l_1)}{D_1 + D_2}$

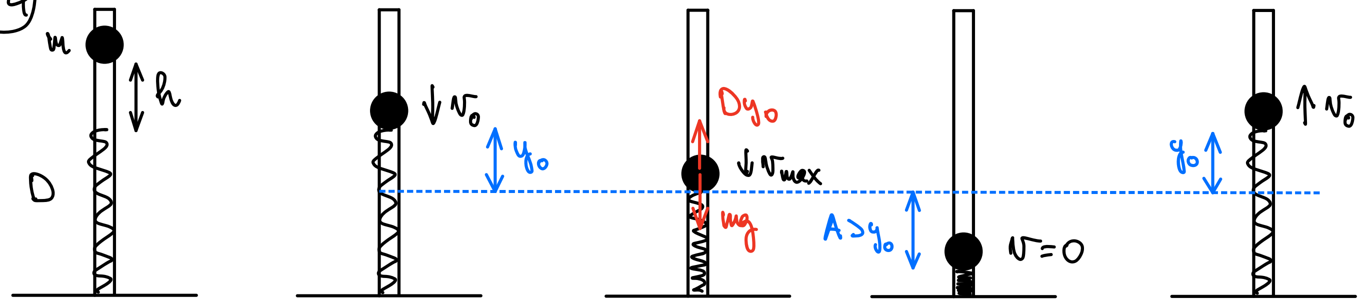
(3): $mg\dfrac{L}{2} = D_1 \cdot \dfrac{mg + D_2(l_2 - l_1)}{D_1 + D_2} \cdot d$

$$d = \frac{mg}{mg + D_2(l_2 - l_1)} \cdot \frac{D_1 + D_2}{D_1} \cdot \frac{L}{2}$$

$$d = \frac{mg}{mg + \frac{\cancel{3}}{2}D \cdot \frac{l}{\cancel{3}}} \cdot \frac{\overset{3}{\cancel{\frac{9}{2}}}\cancel{D}}{\cancel{3}\cancel{D}} \cdot \frac{L}{2} =$$

$$= \frac{mg}{mg + \frac{Dl}{2}} \cdot \frac{3}{4}L = \frac{3}{4}L \cdot \frac{1}{1 + \frac{Dl}{2mg}} = 0{,}625L \approx 0{,}63L$$

F4



$$D = \frac{3mg}{2h}$$

a, Mechanikai energia megmarad: $mgh = \frac{1}{2} m v_0^2 \Rightarrow v_0 = \sqrt{2gh}$

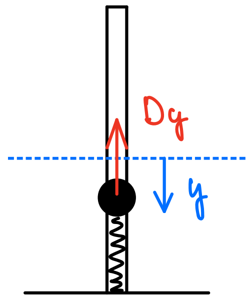
b, Amikor a test sebessége maximális, a rá ható erők eredője nulla (egyensúlyi helyzet):

$$mg = Dy_0 \Rightarrow y_0 = \frac{mg}{D} = \frac{2}{3} h$$

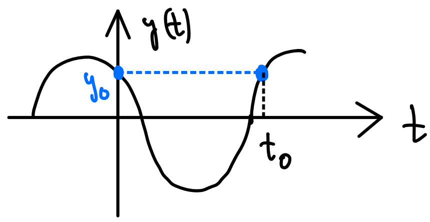
c) Legyen a test kitérése az egyensúlyi helyzethez képest y:

$$m\ddot{y} = -Dy \Rightarrow \ddot{y} = -\frac{D}{m} \cdot y$$

harmonikus rezgés: $\omega = \sqrt{\frac{D}{m}} = \sqrt{\frac{3g}{2h}}$



d, Kezdeti feltételekkel a függvény az alábbi:



kezdőfázis: $\pi/2 < \varphi < \pi$

$$y(t) = A \sin(\omega t + \varphi)$$
$$v(t) = A\omega \cdot \cos(\omega t + \varphi)$$

Tehát:
$$\left.\begin{aligned} y(t=0) &= y_0 = A \sin\varphi \\ v(t=0) &= -v_0 = A\omega \cdot \cos\varphi \end{aligned}\right\} \Rightarrow \operatorname{tg}\varphi = -\frac{\omega y_0}{v_0} = -\frac{1}{\sqrt{3}} \Rightarrow \varphi = \frac{5\pi}{6}$$

$$A = \frac{y_0}{\sin\varphi} = 2y_0 = \frac{4}{3} h$$

$$y(t) = \frac{4}{3} h \cdot \sin\left(\omega t + \frac{5\pi}{6}\right) \qquad v(t) = \sqrt{\frac{8}{3} gh} \cdot \cos\left(\omega t + \frac{5\pi}{6}\right)$$

e) $y(t_0) = y_0 = 2y_0 \cdot \sin\left(\omega t_0 + \frac{5\pi}{6}\right) \Rightarrow \sin\left(\omega t_0 + \frac{5\pi}{6}\right) = \frac{1}{2} \Rightarrow \omega t_0 + \frac{5\pi}{6} = 2\pi + \frac{\pi}{6}$

$$\Rightarrow t_0 = \frac{4}{3} \frac{\pi}{\omega} = \frac{4}{3} \cdot \pi \cdot \frac{T}{2\pi} = \frac{2}{3} T$$