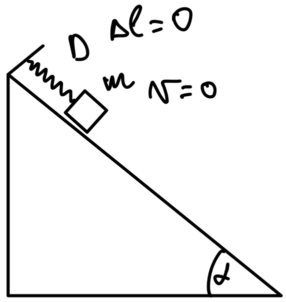
F1.

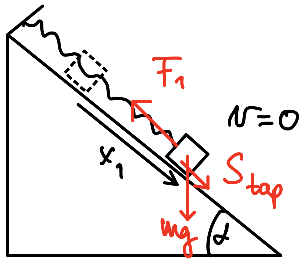


elindul, ha:

$$mg\sin\alpha > F_{tap}^{max} = \mu_0 mg\cos\alpha$$

$\mu_0 < \mathrm{tg}\,\alpha = 1$ → teljesül

a,



munkatétel:

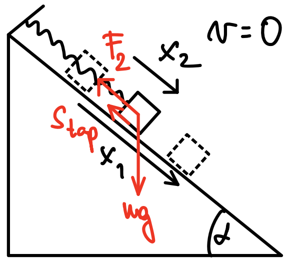
$$mg\cancel{x_1}\sin\alpha - \mu mg\cos\alpha\cdot\cancel{x_1} - \frac{1}{2}Dx_1^{\cancel{2}} = 0 \quad (x_1 \neq 0)$$

$$x_1 = \frac{2mg}{D}\cdot(\sin\alpha - \mu\cos\alpha) \approx 0{,}21\,m$$

b, visszaindul, ha

$$F_1 - mg\sin\alpha > S_{tap}^{max}$$

$$Dx_1 - mg\sin\alpha > \mu_0 mg\cos\alpha$$



mivel megáll, a kiindulási helyzethez képest lejjebb lesz ez az állapot

a, rész eredményével:

$$2\cancel{mg}\sin\alpha - \mu\cdot 2\cancel{mg}\cos\alpha - \cancel{mg}\sin\alpha > \mu_0\cancel{mg}\cos\alpha$$

$\mu_0 < \mathrm{tg}\,\alpha - 2\mu\cos\alpha \approx 0{,}65$ → teljesül

munkatétel:

$$-mg\sin\alpha(x_1-x_2) - \mu mg\cos\alpha(x_1-x_2) \overbrace{+\frac{1}{2}Dx_1^2 - \frac{1}{2}Dx_2^2}^{W_{mg}>0} = 0$$

$$-mg(\cancel{x_1-x_2})(\sin\alpha+\mu\cos\alpha) = -\frac{1}{2}D(\cancel{x_1-x_2})(x_1+x_2) \qquad (x_1 \neq x_2)$$

$$\underline{x_2} = \frac{2mg}{D}(\sin\alpha+\mu\cos\alpha) - x_1 =$$ ← a, rész eredményével

$$= \frac{2mg}{D}(\cancel{\sin\alpha}+\mu\cos\alpha - \cancel{\sin\alpha}+\mu\cos\alpha) = \frac{4mg}{D}\cdot\mu\cos\alpha \approx 0{,}14\,m$$

c, Nem indul el, ha

$$mg\sin\alpha - F_2 < S_{tap}^{max}$$

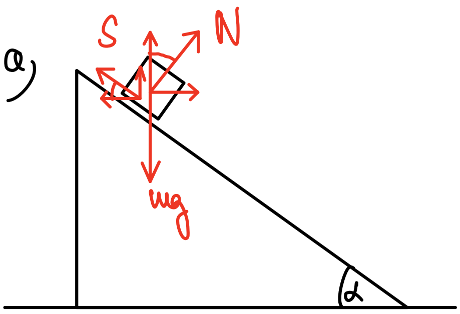
$$mg\sin\alpha - Dx_2 < \mu_0 mg\cos\alpha$$

b, rész eredményével: $\cancel{mg}\sin\alpha - 4\cancel{mg}\cdot\mu\cos\alpha < \mu_0\cancel{mg}\cos\alpha$

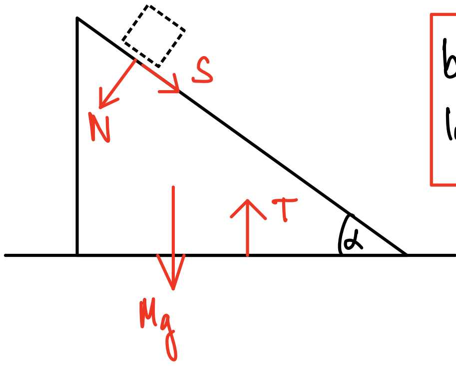
$\mu_0 > \mathrm{tg}\,\alpha - 4\mu = 0$ → mgcosα és mgsinα azonos, valóban nem indul el.

teljesül

(F2.) a)



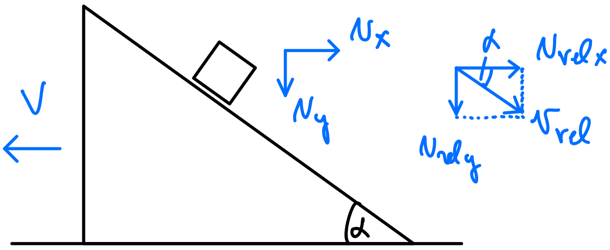
elindul, mert $\mu_0 < \mathrm{tg}\,\alpha \approx 0{,}57$



belső erők: S; N
külső erők: mg; Mg; T

b) Vízszintes irányban nincsenek külső erők, így ebben az irányban megmarad a lendület:

$$0 = m v_x - MV \Rightarrow \underline{v_x = \frac{M}{m} \cdot V}$$



$$\left.\begin{aligned} v_{rely} &= v_{rel} \cdot \sin\alpha = v_y \\ v_{relx} &= v_x + V = v_{rel} \cdot \cos\alpha \end{aligned}\right\} \Rightarrow$$

$$\Rightarrow v_x + V = \frac{v_y}{\sin\alpha} \cdot \cos\alpha = \frac{v_y}{\mathrm{tg}\,\alpha}$$

$$\underline{v_y} = (v_x + V) \cdot \mathrm{tg}\,\alpha = \left(\frac{M}{m} + 1\right) V \cdot \mathrm{tg}\,\alpha = \underline{\frac{m+M}{m} V \cdot \mathrm{tg}\,\alpha}$$

c) $$a_x = \frac{dv_x}{dt} \; ; \; a_y = \frac{dv_y}{dt} \; ; \; A = \frac{dV}{dt} \Rightarrow a_x = \frac{M}{m} A \; ; \; a_y = \frac{m+M}{m} A \cdot \mathrm{tg}\,\alpha$$

<u>kis testre:</u>

x: $N\sin\alpha - S\cdot\cos\alpha = m a_x$ (1)

y: $mg - N\cos\alpha - S\sin\alpha = m a_y$ (2)

$S = \mu N$ (3)

<u>ékre:</u>

x: $N\sin\alpha - S\cdot\cos\alpha = MA$ (4)

d) A mozgásegyenletek megoldása:

(2): $$mg - N\cos\alpha - \mu N \sin\alpha = \cancel{m} \cdot \frac{m+M}{\cancel{m}} A \cdot \mathrm{tg}\,\alpha \quad (2')$$

(4): $$N\sin\alpha - \mu N\cos\alpha = MA \Rightarrow N = \frac{MA}{\sin\alpha - \mu\cos\alpha}$$

Ezt beírva (2')-be:

$$mg - \frac{MA}{\sin\alpha - \mu\cos\alpha}(\cos\alpha + \mu\sin\alpha) = (m+M) A \cdot \mathrm{tg}\,\alpha$$

$$mg = A\cdot\left[(m+M)\cdot tg\alpha + M\cdot\frac{\cos\alpha+\mu\sin\alpha}{\sin\alpha-\mu\cos\alpha}\right]$$

$$mg = A\cdot \frac{m\cdot\frac{\sin\alpha}{\cos\alpha}\cdot\sin\alpha - m\cdot\frac{\sin\alpha}{\cancel{\cos\alpha}}\cdot\mu\cancel{\cos\alpha} + M\cdot\frac{\sin\alpha}{\cos\alpha}\cdot\sin\alpha - \cancel{M\frac{\sin\alpha}{\cos\alpha}\cdot\mu\cos\alpha} + M\cos\alpha + \cancel{\mu M\sin\alpha}}{\sin\alpha-\mu\cos\alpha}$$

$$mg = A\cdot\frac{(m+M)\cdot\frac{\sin^2\alpha}{\cos\alpha}+M\cos\alpha-\mu m\sin\alpha}{\sin\alpha-\mu\cos\alpha}$$

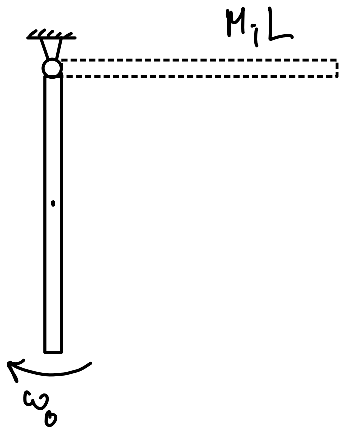
$$mg = A\cdot\frac{m\sin^2\alpha+M\sin^2\alpha+M\cos^2\alpha-\mu m\sin\alpha\cos\alpha}{\cos\alpha(\sin\alpha-\mu\cos\alpha)}$$

$$mg = A\cdot\frac{m\cdot\sin\alpha(\sin\alpha-\mu\cos\alpha)+M}{\cos\alpha(\sin\alpha-\mu\cos\alpha)}$$

$$A = g\cdot\frac{m\cos\alpha(\sin\alpha-\mu\cos\alpha)}{m\cdot\sin\alpha(\sin\alpha-\mu\cos\alpha)+M} = g\cdot\frac{m\cdot\cos\alpha}{m\cdot\sin\alpha+\frac{M}{\sin\alpha-\mu\cos\alpha}}\approx 0{,}05g$$

F3

a)



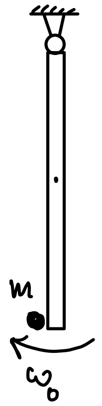
energiamegmaradás:

$$\cancel{M}g\frac{\cancel{L}}{\cancel{2}} = \frac{1}{\cancel{2}}\cdot\frac{1}{3}\cancel{M}L^{\cancel{2}}\omega_0^2$$

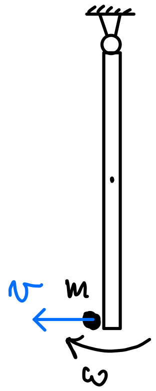
$$\omega_0=\sqrt{\frac{3g}{L}}$$

b)

előtte:



utána:



perdületmegmaradás a felfüggesztési pontra:

$$\frac{1}{3}ML^2\omega_0 = \frac{1}{3}ML^2\omega + mvL \quad (1)$$

energiamegmaradás:

$$Mg\frac{L}{2} = \frac{1}{2}\cdot\frac{1}{3}ML^2\omega^2+\frac{1}{2}mv^2 \quad (2)$$

c) $\underline{\omega = 0}$

$\rightarrow$ (1): $\frac{1}{3}ML^{\cancel{2}}\omega_0 = mv\cancel{L}$ $\rightarrow$ $v = \frac{ML\omega_0}{3m}$

$\rightarrow$ (2): $Mg\frac{L}{\cancel{2}} = \frac{1}{\cancel{2}}mv^2$

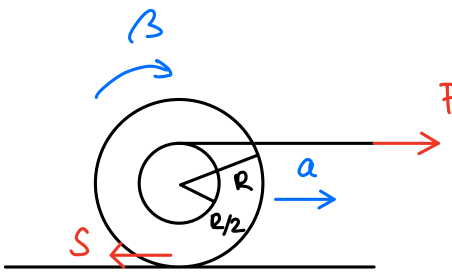
$$\cancel{M}g\cancel{L} = \cancel{m}\cdot\frac{M^{\cancel{2}}L^{\cancel{2}}\omega_0^2}{9m^{\cancel{2}}}$$

a, eredményével $\Rightarrow$

$$9m\cancel{g} = M\cancel{L}\cdot\frac{3\cancel{g}}{\cancel{L}}$$

$$\frac{m}{M} = \frac{1}{3}$$

F4.



a, $F - S = m\cdot a$ (1)

TK-ra: $F\cdot\frac{\cancel{R}}{2} + S\cancel{R} = \frac{1}{2}mR^{\cancel{2}}\beta$ (2)

tisztán gördülés miatt: $a = \beta R$ (3)

(1)+(2) és (3) $\Big\}$ $\Rightarrow$ $\frac{3}{2}F = \frac{3}{2}ma$ $\Rightarrow$ $a = \frac{F}{m} = 5\ \frac{m}{s^2}$; $\beta = \frac{a}{R} = \frac{F}{mR} = 100\ \frac{1}{s^2}$

b) $S \geq \mu_0 N = \mu_0 mg$; $S = F - ma = 0$ $\Rightarrow$ $\mu_0 \geq 0$